

MACQUARIE  
UNIVERSITY



FACULTY OF  
BUSINESS AND ECONOMICS

ACST851  
Mathematics of Finance

Semester 1, 2009

*Department of Actuarial Studies*

**MACQUARIE UNIVERSITY  
FACULTY OF BUSINESS AND ECONOMICS  
UNIT OUTLINE**

<b>Year and Semester:</b>	Semester 1, 2009
<b>Unit convenor:</b>	Jim Farmer
<b>Prerequisites:</b>	ACST101(Cr) and MATH133(P) and GPA of at least 2.50.

Students in this unit should read this unit outline carefully at the start of semester. It contains important information about the unit. If anything in it is unclear, please consult one of the teaching staff in the unit.

**ABOUT THIS UNIT**

Credit Points: 4

Unit Description:

Topics include the force of interest and its relationship with the annual rate of compound interest; loans repayable by instalments of principal and interest; the effect of income and capital gains tax; the use of discounted cash flow techniques in product appraisal; simple stochastic interest rate models; unit-pricing systems; yield curves, matching and immunisation; forward rate agreements; pricing using the no-arbitrage assumption. It is assumed students are able to use spreadsheets.

Unit rationale

Much of the work that actuaries do involves long term financial problems. For any problem spanning more than a few months, the effects of compound interest are significant. This unit is a study of compound interest, with a few very brief diversions into alternative systems of interest. As far as possible, the examples considered are real life problems, though in some cases we deliberately simplify scenarios so that we can concentrate on the interesting maths rather than getting slowed down by fiddly details.

**TEACHING STAFF**

The unit convenor and lecturer, Jim Farmer, can be contacted via the unit's web site. Questions about the unit material should be placed in the Discussion Area. Administrative questions which have not already been answered in this document or the Student Guide should be sent to the "Administration Inquiries" account using the mail tool in the unit's web site. Tutors cannot be contacted other than at tutorials.

## CLASSES

There are 5 hours of face-to-face teaching per week consisting of 3 hours of lectures and 2 hours of tutorial.

Class times can be found at: <http://www.timetables.mq.edu.au/>

## REQUIRED AND RECOMMENDED TEXTS AND/OR MATERIALS

The “ACST200/851 Unit Notes” are available from the University Co-Op Bookshop. Additional readings are available from the unit’s web site. A booklet of tutorial exercises will be distributed at the Week 1 tutorial.

The following is a list of all textbooks I am aware of that cover material from this unit, with their Macquarie University call numbers.

- Broverman, Samuel. “Mathematics of Investment & Credit” 3rd Edition. Mad River Books. (<http://www.actexamdriver.com>) HG4515.3.B76/2004. (2nd edition: HG4515.3.B76/1996.)
- There is an associated electronic workbook for the above textbook. Details are available at: <http://www.actexamdriver.com/productdetails.cfm?PC=1478> .
- Daniel, James W and Vaaler, Leslie J F. “Mathematical Interest Theory”. Pearson Prentice Hall. 2007. HB539.D33/2007.
- Donald D.W.A. “Compound Interest and Annuities Certain” 1970. Heinemann HG8790.D65/1970. Out of print.
- Kellison, Stephen G. “The Theory of Interest” Irwin, HB539.K28/1991
- Knox, Zima and Brown. “Mathematics of Finance” 2nd Edition. McGraw Hill. HF5691.K56/1999
- McCutcheon, J.J. and Scott W.F. “An Introduction to the Mathematics of Finance” Heinemann. 1986. HF5691.M27. (Available for purchase via <http://wam.actuaries.org.uk>.)
- Sherris, Michael. “Money and Capital Markets” 2nd Edition. HG4515.S54
- Course notes for the UK courses may be purchased from ActEd. (<http://www.acted.com.au>)

## UNIT WEB PAGE

The unit web site can be accessed via the logon facility at <http://learn.mq.edu.au>

## LEARNING OBJECTIVES AND OUTCOMES

By the end of the unit you should be able to demonstrate competence in the range of techniques described in the unit notes and lectures. Ideally you will be able to demonstrate an understanding of the techniques rather than simply demonstrating the ability to rote learn formulae without understanding. You should also be able to demonstrate ethical behaviour by complying with examination rules and by not colluding on assessment tasks.

In addition to the discipline-based learning objectives, all academic programs at Macquarie seek to develop students' generic skills in a range of areas. In this unit students may develop their written communication skills and problem-solving skills.

### TEACHING AND LEARNING STRATEGY

This unit is taught via lectures and tutorials. However, a significant amount of the lecture time will be spent on attempting problems. The emphasis is on learning by doing.

The order of topics has been shuffled to accommodate the class test.

Week	Week Begins	Topics Covered
1	23 Feb	1. Compound Interest & Discount; Forces of Interest
2	2 March	2. Inflation and Capital Gains Tax – Reading topic 3. Level Annuities
3	9 March	4. Varying Annuities
4	16 March	5. Loans
5	23 March	7. Unit Pricing Systems – Reading Topic Friday: Test on Topics 1 to 4.
6	30 March	6. Bonds
7	6 April	8. Yields on funds (Friday public holiday)
2-week study break		9. Project Appraisal – Reading Topic
8	27 April	10. Yield Curves
9	4 May	11. Forward Contracts
10	11 May	12. Bond Statistics
11	18 May	13. Immunisation
12	25 May	14. Stochastic Models
13	1 June	

#### Detailed list of topics

Compound interest; effective and nominal interest and discount rates; force of interest; accumulating and discounting at discretely changing and continuously changing interest rates; discrete and continuous cash flows; continuously removed interest.

Inflation; CPI; capital gains tax based on real and nominal gains.

Level annuities;  $a_{\overline{n}|}$ ,  $\ddot{a}_{\overline{n}|}$ ,  $\overline{a}_{\overline{n}|}$  and corresponding perpetuities;  $s_{\overline{n}|}$ ,  $\ddot{s}_{\overline{n}|}$ ,  $\overline{s}_{\overline{n}|}$ ;  $a_{\overline{n}|}^{(p)}$ ,  $\ddot{a}_{\overline{n}|}^{(p)}$ ,  $s_{\overline{n}|}^{(p)}$ ,  $\ddot{s}_{\overline{n}|}^{(p)}$ ; limit properties linking these to continuous annuities; dealing with changing interest

rates; use of  $\frac{i}{i^{(p)}}$ ,  $\frac{i}{d^{(p)}}$  and  $\frac{i}{\delta}$  factors to adjust timing of cash flows.

Varying annuities;  $(Ia)_{\overline{n}|}$ ,  $(I\ddot{a})_{\overline{n}|}$ ,  $(I\overline{a})_{\overline{n}|}$ ,  $(\overline{I\overline{a}})_{\overline{n}|}$ ,  $(Is)_{\overline{n}|}$ ,  $(I\ddot{s})_{\overline{n}|}$ ,  $(I\overline{s})_{\overline{n}|}$ ,  $(\overline{I\overline{s}})_{\overline{n}|}$ ; Geometrically varying annuities.

Loans; Interest only loans; Reducible Rate loans; Loan repayment schedules; Finding instalments and loan outstanding; dealing with changes in interest rates by adjusting repayments or by adjusting the term of the loan.

Bonds: Face value, coupon rate, valuing with and without allowance for tax; indexed (capital) bonds; purchasing cum and ex interest.

Unit pricing systems; income-inclusive vs income distributing; calculation of unit price for a simple income-inclusive fund.

Yields on Funds; money weighted rates of return and time weighted rates of return; calculation from accounts and from unit-pricing system data; Hardy's formula; linked rates of return.

Project Appraisal; IRR; NPV; discounted payback period; problems involving different interest rates on loans and deposits; unique or multiple solutions for IRR; deficiencies of IRR for project assessment.

Yield Curves; spot rates and forward rates; calculations using the no-arbitrage assumption; valuation of bonds using spot rates; spot rate yield curve; yield to maturity yield curve; par bond yield curve;

Forward contracts; derivation of formula for the forward price & the value of an existing contract under the no-arbitrage assumption; allowance for fixed dollar income on the security prior to delivery date; assumptions and limitations of the no-arbitrage model.

Bond statistics; discounted mean term; duration; volatility; modified duration and formulae linking them. Convexity and Spread. Theoretical definitions and practical approximations.

Immunisation: Absolute immunisation and Redington immunisation; derivation of formula and worked examples with fixed and certain liabilities and government bonds.

Stochastic Models; derivation of mean and variance of accumulation of a single cash flow and of a discrete level annuity; application of the lognormal distribution; simulation.

## RELATIONSHIP BETWEEN ASSESSMENT AND LEARNING OUTCOMES

The assessment tasks are designed to test your understanding of the unit material. The range of assessment has been constrained by the need to ensure that the unit attains its accreditation status with the Institute of Actuaries of Australia.

Blackboard Quizzes	10%
Class Test	20%
Examination	70%

- The Blackboard Quizzes cover approximately the first half of the unit and provide you with an early low risk diagnostic tool. The due dates can be viewed on the unit's web site. The first Quiz becomes available in the 2<sup>nd</sup> week of semester. No extensions of deadline are available, with the exception that we reserve the right to extend deadlines or remove quizzes from the assessments in the event of catastrophic failure of the relevant computers systems.
- In respect of the class test and exam, except where explicitly stated otherwise, we are not just assessing the correctness of your final answer. You will also be judged on your ability to provide a coherent solution and to provide clear verbal explanations.
- Class tests will be returned at a convenient class after the marking has been completed. If you miss the relevant class, you can collect your test from E4B 106 - Business and Economics Student Services (BESS).

The final exam consists of two papers, each of two hours length, to be held during the University Examination period.

The University Examination period in First Half Year 2009 is from 10 June 2009 to 26 June 2009.

You are expected to present yourself for examination at the time and place designated in the University Examination Timetable. The timetable will be available in Draft form approximately eight weeks before the commencement of the examinations and in Final form approximately four weeks before the commencement of the examinations. The timetable will be available at:

<http://www.timetables.mq.edu.au/exam>

The only exception to not sitting an examination at the designated time is because of documented illness or unavoidable disruption. In these circumstances you may wish to consider applying for Special Consideration. Information about unavoidable disruption and the special consideration process is available at

<http://www.reg.mq.edu.au/Forms/APSCon.pdf>

If a Supplementary Examination is granted as a result of the Special Consideration process the examination will be scheduled after the conclusion of the official examination period.

You are advised that it is Macquarie University policy not to set early examinations for individuals or groups of students. All students are expected to ensure that they are available until the end of the teaching semester, that is, the final day of the official examination period.

## **PLAGIARISM**

The University defines plagiarism in its rules: "Plagiarism involves using the work of another person and presenting it as one's own." Plagiarism is a serious breach of the University's rules and carries significant penalties. You must read the University's practices and procedures on plagiarism. These can be found in the *Handbook of Undergraduate Studies* or on the web at: <http://www.student.mq.edu.au/plagiarism/>

The policies and procedures explain what plagiarism is, how to avoid it, the procedures that will be taken in cases of suspected plagiarism, and the penalties if you are found guilty. Penalties may include a deduction of marks, failure in the unit, and/or referral to the University Discipline Committee.

## **STUDENT SUPPORT SERVICES**

Macquarie University provides a range of Academic Student Support Services. Details of these services can be accessed at <http://www.student.mq.edu.au>.