



Division of Economic and Financial Studies

Department of Actuarial Studies

## **ACST101 : TECHNIQUES AND ELEMENTS OF FINANCE**

### **UNIT OUTLINE : Semester 2, 2006**

#### **Teaching Staff Involved in the Unit**

The staff member involved in the teaching of this unit is

	Room	Telephone	Email
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Questions relating to the administration of the unit should be directed to the Unit Co-ordinator. Questions relating to the unit content should be directed to your tutor at your tutorial. Consultation hours for the Unit Co-ordinator and the tutors will be shown on the ACST101 website. Instructions for accessing the ACST101 website are on page 4.

#### **Unit Prerequisites**

There are no prerequisites or corequisites, however a background of HSC mathematics or equivalent numerical competency is desirable.

#### **Unit Description and Objectives**

Students will gain skills in the pricing of financial instruments in the Techniques section and knowledge of financial institutions, instruments and markets in the Elements section. ACST101 is a prerequisite for further study in the areas of actuarial studies and finance.

#### **Techniques**

The basic methods of financial mathematics (present value and accumulated value) are applied in valuing a range of financial transactions including the purchase of promissory notes, bank bills, bonds and debentures, the analysis of mortgage loans, personal loans and investment proposals.

#### **Elements**

The basic functions of the Australian financial system, the financial institutions (banks, insurance companies, finance companies, credit unions, etc.), the financial instruments (bills, bonds, debentures, shares, etc.) and the financial markets are discussed.

## Learning Strategy

**It is essential that you work steadily and consistently over the whole semester;** in particular attend tutorials and keep up with the weekly assignments. You should revise the previous week's techniques lecture before you attend your weekly tutorial. It is extremely difficult to catch up if you fall behind. Each topic builds on the previous one.

**Understanding of the concepts is required rather than memorisation of formulae.** Success in this unit requires logical thinking, reasoning and problem solving skills.

The Academic Senate of the University has set the average workload as three hours total work per credit point per week. (ie 9 hours per week for ACST101). Total work includes time for private study and reading as well as attending classes and performing set tasks.

## Assessment

The following table gives the relative weighting of the assessment components:

Weekly Assignments (11)	10%
Class Tests (3)	20%
Final Examination	70%

Assignments, Final Examination and Tutorial Attendance have minimum requirements.

## Weekly Assignments

**A satisfactory attempt by the due date is required for at least eight assignments.**

There are 11 weekly assignments, each based upon a "techniques" topic. For each assignment you will use the website to obtain the questions and to enter your answers.

To access Assignment 1 you must score at least 80% in **both** the Maths Revision Exercises and the Practice Assignment which are due early in Week 2. The former will give you practice in the types of algebraic operations required in ACST101.

The marks for all 11 assignments are used to calculate the component of the final assessment based on the assignments. **Assignments 10 and 11 will be given triple weighting.** Full details of the computerised assignments are given on pages 9 and 10.

## Class Tests

There are three class tests scheduled for the following dates in Weeks 5, 8 and 12.

Test One	Wednesday 30 August	10 am	Macquarie Theatre
	Wednesday 30 August	12 pm	Mason Theatre
Test Two	Wednesday 4 October	10 am	Macquarie Theatre
	Wednesday 4 October	12 pm	Mason Theatre
Test Three	Wednesday 1 November	10 am	Macquarie Theatre
	Wednesday 1 November	12 pm	Mason Theatre

The topics to be examined in each test are shown on page 6.

Students must attend at the lecture time for which they are enrolled.

Full details of Class Tests will be given on the ACST101 website under Announcements.

Tests will be returned to students at the tutorial in the week following the test.

## Final Examination

**To pass this unit a satisfactory performance is required in the final examination.**

The final examination will contain questions from all techniques and all elements lectures. It will be a three-hour written paper with ten minutes reading time.

The University examination period is between Wed. 15 Nov. and Fri. 1 Dec. 2006.

Part A: Forty-five multiple choice questions - twenty-two based on "techniques" and twenty-three based on "elements". Marked out of 45.

Part B: Three questions requiring application of "techniques" to the solution of practical problems. Marked out of 30.

The list of basic formulae shown at the end of this Unit Outline will be supplied.

The multiple choice questions are answered by marking (in pencil) a computer readable answer sheet. Bring **TWO 2B Pencils**, and an eraser, into the examination with you.

## Grading

Macquarie University uses the grades HD, D, Cr, P, PC and F. Each symbol is explained in the Bachelor Degree Rules on page 96 of the 2006 Undergraduate Studies Handbook.

The numerical marks resulting from assessment of your work in this unit will be used as an initial indicator of the quality of your learning and understanding. The use of these numerical marks is, however, only a starting point in determining the appropriate grade. Note that the mark ranges mentioned in the Grades section on page 43 of the Handbook are not the raw marks. To obtain a grade you must satisfy the qualitative definition of that grade. Once your grade has been determined, you are allocated a standardised mark indicating your approximate position amongst students assigned that grade.

## Textbooks

The textbooks are available as a package from the Macquarie University Co-op Bookshop.

Knox D M, Zima P and Brown R L, *Mathematics of Finance*, 2nd Edition, McGraw-Hill (1999)

Viney C, *McGrath's Financial Institutions, Instruments and Markets*, 4th Edition, McGraw-Hill (2003) (Custom edition of selected chapters)

## Calculators

Calculators will be allowed in the class tests and the final examination but a clear indication of the steps involved in every calculation must be shown.

**Calculators that have a text-retrieval capacity are not allowed.**

**Calculators that have a full alphabet on the keyboard are not allowed.**

You will need a calculator which has  $x^y$  or  $\wedge$ ,  $1/x$  and log or ln functions, and a memory.

## Numeracy Centre C5A225

Students who lack the knowledge of mathematics needed for ACST101 are encouraged to seek the help of the Centre. Consultations are free of charge. Staff will recommend work to fill gaps in background knowledge of mathematics.

## Lectures

The **Techniques** lecture is held at the following time:

Day	Time	Location
Wednesday	10 am	Macquarie Theatre
Wednesday	12 pm	Mason Theatre

The **Elements** lecture is held at the following time:

Day	Time	Location
Thursday	11 am	Macquarie Theatre
Thursday	12 pm	Mason Theatre

You should attend your allocated techniques lecture and elements lecture each week.

Detailed lecture notes for techniques and a summary of elements lectures are available from the ACST101 website.

## Tutorials

**Tutorial attendance is compulsory.** Students must attend and fully participate in at least 9 tutorials, otherwise they may be excluded from the unit.

**Tutorials will commence in the second week of the semester.**

**To prepare for each weekly tutorial,** attempt at least the first few questions from the **Revision Exercises** for the previous week's Techniques lecture eg for the Week 2 tutorial you should attempt the Revision Exercises on Week 1. Revision Exercises are accessed from the website in the same way as the assignments. (see page 9)

**Tutorial exercises** will be provided at each tutorial so that you can practise applying the results developed in lectures. Your tutor is available to help you sort out the things that are not immediately obvious or prove to be a bit tricky.

**Tutorial Room** locations are shown on your enrolment printout. The tutorial list will also be shown under the Lecture Announcements icon of the ACST101 website on the Monday of the second week of classes. **You must attend your allocated tutorial.**

**Tutorial enrolment or change of tutorial** can be made through eStudent on the web or at a change of program session held in E7B in the **first two weeks** of the semester. No forms are required to change a tutorial. **No tutorial changes are allowed after Week 2.**

## ACST101 Website

You may access the ACST101 website from your home or work computer if you are connected to the internet. You can also access it from the student laboratories located in E7B146 or E4B and from computers in the Library. See under Technical Information at the login address given below for details of recommended browsers.

If you are not already familiar with using a web browser you can get assistance through the Library Information Technology Help Desk (level 1).

The login address is <http://online.mq.edu.au> (There is no www in the address.)  
Then click on the login button.

You will be required to enter a username and password.

Your username will be your 8-digit Macquarie Student ID Number.

Your password will be your myMQ Student Portal password. This will be the original MQID password that was sent to you on enrolment (2 random characters followed by your date of birth in ddmmyy format), unless you have already changed your password in the myMQ Student Portal. You can look up your original MQID password at <http://www.library.mq.edu.au/help/ithelp/mqid.html> . Passwords are case sensitive.

The above address gives you access to all of your online units. Just click on the name of the unit you want to work on. When you want to change from one unit to another click on MYWEBCT at the top right of the screen.

If you do not attend a lecture , you should consult the Lecture Announcements section of the website to see what information, if any, you have missed.

When moving around the website the path that you have followed is displayed below the ACST101 Techniques and Elements of Finance line. To move back to a previous page, click on the title of that page. In particular to move back to the opening page, click on Homepage. An example is: Homepage > Tutorial Solutions > Tutorial Exercises on Week 1

If you wish to contact the unit co-ordinator, you can use the ACST101 website. Click on Private Mail, then on Compose Mail Message and send to the username of ACST101.

The following are available on the website:

1. Lecture notes and self-test exercises for "Techniques".
2. Lecture summary, self-test exercises and internet exercises for "Elements".
3. Tutorial Exercise solutions.
4. Assignments and Revision Exercises.
5. Class Test solutions for the past two semesters and the current semester.
6. Final Examination Part A specimen multiple choice paper and solutions.
7. Final Examination Part B specimen papers and solutions.

## **Special Consideration**

If the quality of your work in this unit is adversely affected by illness, accident or other form of unavoidable disruption, you should acquaint yourself with the Unavoidable Disruption section on page 41 of the 2006 Undergraduate Studies Handbook and the Special Consideration section on page 43 of the Handbook.

**All requests for special consideration should be made in writing to the Registrar's Office and include full supporting documentation.**

- Notification of absence from a Class Test should be made within 2 weeks of the test.
- Requests for special consideration for the final examination should be made within 5 working days after the date of the examination or the day after the end of the examination period whichever ever is sooner. The Professional Authority Form which is required if you wish to request special consideration for the final examination due to illness can be found at <http://www.reg.mq.edu.au/Forms/APSCons.pdf>

Special consideration will NOT be granted where a student has unsatisfactory class test marks, unsatisfactory assignment marks or unsatisfactory tutorial attendance. The exam content and/or assessment standards of supplementary examinations will be made more stringent to allow for the extra time available for prior study.

Further details about Special Consideration and arrangements for Supplementary Examinations will be posted on the ACST101 website in the last week of the semester under the Announcements icon.

## Unit Timetable

Week Number	Week Beginning	Techniques Wed 10am / Wed 12pm	Elements Thu 11am / Thu 12pm	Class Test
1	31 July	Simple interest & simple discount	Information about Assignments	-
2	7 August	Compound interest	Overview	-
3	14 August	Compound interest	Banks and RBA	-
4	21 August	Annuities	Banks and RBA	-
5	28 August	Annuities	-	1
6	4 September	Annuities	Non-bank institutions	-
7	11 September	Mortgage loans	Non-bank institutions	-
STUDY	18 September	STUDY	STUDY	
BREAK	25 September	BREAK	BREAK	
8	2 October	Flat rate loans, NPV, IRR	-	2
9	9 October	Bonds & debentures	Government finances and instruments	-
10	16 October	Tax on bonds	Corporate finances and instruments	-
11	23 October	Varying annuities	Financial markets	-
12	30 October	Sinking funds and capitalised costs	-	3
13	6 November	Revision	-	-

Any alterations will be advised in lectures and via the ACST101 website.

- Class Tests will be based on the following lecture topics:

	Techniques	Elements
Test 1	Weeks 1,2,3	Weeks 2,3,4
Test 2	Weeks 4,5,6	Weeks 4,6,7
Test 3	Weeks 7,8,9,10	Weeks 9,10,11

- At each Class Test,
  - the formula sheet will be displayed on the overhead projector
  - normal examination rules will apply - see page 43 of the 2006 Undergraduate Studies Handbook. Students are responsible for familiarising themselves with these rules prior to the class tests.
- In weeks where there is a Class Test
  - the Class Test will be on Wednesday
  - the Techniques lecture will be on Thursday
  - there will be no Elements lecture.

## Techniques Topics and Textbook References

### Textbook

Knox D M, Zima P and Brown R L, *Mathematics of Finance*, 2nd edition, McGraw-Hill (1999). The answers to the even-numbered exercises start on page 297, the answers to the odd-numbered exercises can be found on the website under the Tutorial Solutions icon.

Week	Techniques Topic	Textbook Reference
1	Simple Interest and Simple Discount	Chapter 1 (exclude 1.4 and 1.5)
2	Compound Interest	Chapter 2, 2.1 to 2.3 (exclude 2.4)
3	Compound Interest, Logarithms and Linear Interpolation	Chapter 2, 2.5 to 2.8 Appendices A & C
4	Valuation of Annuities	Chapter 3, 3.1 to 3.3 Chapter 4, Section 4.2
5	Valuation of Annuities	Chapter 3, 3.4 to 3.7 Chapter 4, 4.3 and 4.5
6	Valuation of Annuities	Chapter 4, Section 4.4 Chapter 5 (exclude 5.4)
7	Mortgage Loans	Chapter 6, 6.1 to 6.4
8	Flat Rate Loans Net Present Value and Internal Rate of Return	Chapter 6, Section 6.6 (exclude Rule of 78) Chapter 8, 8.1 and 8.2
9	Bonds and Debentures	Chapter 7, 7.1 to 7.4
10	Tax on Bonds	Chapter 7, 7.5 and 7.7 (exclude section 7.6 and pages 207 - 211)
11	Varying Annuities (The approach taken will be different to that of the textbook)	Chapter 4, Section 4.6
12	Sinking Funds and Capitalised Costs	Sections 6.5, 7.8 and 8.3

### Notes

- 1 Other sections of the textbook not referred to above are outside the scope of this unit and are NOT examinable.
- 2 The "Part A" exercises in the textbook are ideal for practice in applying the "techniques" to solve financial problems, but some of the "Part B" exercises which involve mathematical proofs are beyond the scope of this unit.

## Elements Topics and Textbook References

### Textbook

Viney C, *McGrath's Financial Institutions, Instruments and Markets*, 4th Edition, McGraw-Hill (2003)

### References

Crane R, Fraser I and Martin T, *Financial Institutions, Markets and Instruments*, 5th edition, LBC Information Services (2001)

Valentine T, Ford G and Copp R, *Financial Markets and Institutions in Australia*, Prentice Hall (2003)

Hunt B and Terry C, *Financial Institutions and Markets*, 4th edition, Thomson (2005)

In addition the *Reserve Bank of Australia Bulletin* contains articles of current interest and statistical information. The "elements" tutorial exercises will contain a link to the RBA website which contains much of this information.

### Topics and Recommended Reading from Textbook

#### Topic 1 Overview of the Financial System

Week 2 Chapter 1 (exclude 1.7)

#### Topic 2 Banks and RBA

Week 3 Chapter 2 (2.1 to 2.5)

Week 4 Chapter 2 (2.6)

#### Topic 3 Non-Bank Institutions

Week 6 Chapter 3 (3.1, 3.3 to 3.5)

Week 7 Chapter 3 (3.2, 3.6 to 3.9) (exclude 3.10)

#### Topic 4 Government Finances and Instruments

Week 9 Chapter 12

#### Topic 5 Corporate Finances and Instruments

Week 10 Chapter 5 (5.4, 5.6 only), 9 (9.4, 9.6, 9.7 only) and 10 (10.4, 10.6 only)

#### Topic 6 Financial Markets

Week 11 Chapter 18

#### ERRATA to Knox, Zima & Brown *Mathematics of Finance* second edition

Page 7	Example 2	Answer should be \$8.91 not \$6.51
Page 10	Example 4	The bill was purchased on 2 May not 3 May
Page 52	Solution Example 2	In the line beginning <i>Step 1</i> , 1000 should be 10000
Page 64	Example 2	The interest rate is $j_4 = 12\%$ not $j_4 = 3\%$
Page 227	Formula for $i$	Numerator should be $F_0 + F_1 + F_2 + F_3 + \dots + F_n$
Page 297	Exercise 1.6 Q4	Answer should be \$1025.28 not \$810.66
Page 299	Exercise 3.6 A Q2	Answer should be \$4291.72 not \$2262.56
Page 300	Exercise 6.5 A Q2	Answer should be sinking fund by \$302.25 not \$1090.80

<p><b>ACST101 : TECHNIQUES AND ELEMENTS OF FINANCE</b></p> <p><b>WEEKLY ASSIGNMENTS; REVISION EXERCISES</b></p>
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This is a brief outline of how to complete your weekly assignments in ACST101 Techniques and Elements of Finance. The revision exercises are accessed in a similar manner.

- **A satisfactory attempt by the due date is required for at least eight assignments.**
- **Assignments 10 and 11 will be given triple weighting.**
- **The marks for all 11 assignments are used in the final assessment.**

**Overview** There are eleven assignments to be completed, each of which is based on the corresponding Techniques topic; e.g. Assignment 1 is based on the Week 1 Techniques topic.

You will use the ACST101 website to get your copy of each assignment, and subsequently enter your answers for marking. (Instructions for accessing the ACST101 website are on page 4.)

Your version of each assignment will be personalised - it will be different to that of each other student in ACST101. Your version will have the same number of problems as everyone else's, and each problem in your version will be of the same TYPE as the corresponding problem in everyone else's, BUT the numerical values (like monetary amounts, interest rates, dates, etc.) in your version will be different to those in everyone else's.

**What do you have to do to complete an assignment?**

When you have logged in to the ACST101 website, **click the left mouse button on the icon for Assignments**. The next screen gives you information about the availability of each assignment and the number of attempts that you are allowed. You are allowed 1 attempt at each assignment except the Practice Assignment for which you are allowed 3 attempts.

When you **click on the name of the assignment**, general information about the assignment will appear. When you click **Begin quiz** the assignment questions will appear on your screen. To scroll up or down through the questions click on the ▲ or ▼ arrows which appear on the screen. To **print** an assignment place the cursor in the frame containing the assignment questions and click the **right** mouse button. In Netscape the frame that appears on the screen contains an option to Open Frame in New Window. If you click on this you can then use the browser Print button to print the assignment. Close the window by clicking the  in the top right hand corner of the window. In Internet Explorer the frame that appears gives an option to Print. To exit from the assignment click on Home. Do NOT click on the Finish button at this stage. To close your connection click on LOGOUT at the top right of the screen.

When you have worked out the answers to your assignment questions (which may be several days later), login, click on Assignments then on the name of the appropriate assignment as before. The **answer** for Question 1 is **entered** in the answer box at the end of Question 1. Click the cursor into the answer box then type in the answer. Then when you have checked that the answer was typed in correctly click the Save Answer box. Repeat this for each subsequent question, scrolling between questions as indicated in the previous paragraph or by using the Tab key. When all answers have been entered and Save Answer has been clicked for each question click on the Finish button so that your assignment can be marked.

When your assignment has been marked click on the View Results button. A list of your answer and the correct answer is given for each question and an overall mark is given for the assignment.

**Full Worked Solutions** to each question (not just the numerical answers) will appear each Wednesday after answers can no longer be entered. To view the solutions, click on the View scores link for the assignment. On the next page click on 1 under Number.

To move back to a previous screen click the appropriate title in the path below the ACST101 Techniques and Elements of Finance line. To close your connection click on LOGOUT at the top right of the screen.

### **Revision Exercises**

For each techniques lecture there is a set of revision exercises. You should attempt these before completing the corresponding assignment. The revision exercises do not count towards the final assessment but are part of the practice required to reinforce the concepts. You are allowed 3 attempts. They are accessed in the same way and from the same webpage as the assignments.

**Maths Revision Exercises** - Due Tuesday 8 August. (Completion time about 20 minutes)

**You must score at least 80% in the Maths Revision Exercises to have access to the Practice Assignment.** If you score less than 80% make another attempt. The Maths Revision Exercises will give you practice in the types of algebraic operations required for ACST101. This is important if you are unsure about your maths or haven't done any maths for a long time. Each revision exercise is preceded by a worked solution of a similar type problem. Students may consult the Numeracy Centre C5A225.

**Practice Assignment** - Due Tuesday 8 August. (Completion time about 20 minutes)

**You must score at least 80% in the Practice Assignment to have access to Assignment 1.** If you score less than 80% make another attempt. The Practice Assignment, which does not count towards the final assessment, tests calculator and basic algebra skills.

### **Assignment Availability Dates**

Answers to the assignments should be entered by midnight on the following dates. Answers will be accepted until 9 a.m. the next morning to make allowance for any last minute computer problems. Deadlines will not be extended any further.

<u>Assignment</u>	<u>Due Date</u>	<u>Assignment</u>	<u>Due Date</u>
Maths Rev.	Tue 8 August		
Practice	Tue 8 August	6	Tue 3 October
1	Tue 15 August	7	Tue 10 October
2	Tue 22 August	8	Tue 17 October
3	Tue 29 August	9	Tue 24 October
4	Tue 5 September	10	Tue 31 October
5	Tue 12 September	11	Fri 10 November

An assignment is due on the Tuesday of each teaching week of the semester from Week 2. (Assignment 11 is due on a Friday.) No assignments are due in the mid-semester break.

### **Confidentiality**

You should read the Confidentiality section under Privacy Information at the login address. It sets out the types of information collected about student use of the site.

## Spreadsheets

Students enrolling in ACST101 are not required to use a computer spreadsheet. However some students will have learnt to use a spreadsheet in 100-level COMP or ISYS units or in other studies. Spreadsheets such as Microsoft Excel can be used to perform ACST101 calculations and the following notes may be useful to students who wish to extend their learning.

### Excel Commands

The Help topics have a financial functions category.

Among the functions that are of use in ACST101 are:

$FV(i,n,-R,-P,0)$	which calculates $Rs\frac{i}{n} + P(1+i)^n$
$PV(i,n,-I,-C,0)$	which calculates $Ia\frac{i}{n} + C(1+i)^{-n}$
$RATE(n,-I,P,-C,0)$	which calculates the value of $i$ for which $P = Ia\frac{i}{n} + C(1+i)^{-n}$
$NPER(i,-R,L,0,0)$	which calculates the value of $n$ for which $L = Ra\frac{i}{n}$
$PMT(i,n,L,0,0)$	which calculates the value of $R$ for which $L = Ra\frac{i}{n}$

Some of the parameters in the above expressions have negative signs. The convention is adopted that payments out of your pocket should have a negative sign and money coming in should have a positive sign.

The last entry in the above commands is 0 or 1. The indicator 0 means that the annuity payments are at the end of each period or (in arrear) and the indicator 1 means that the annuity payments are at the start of each period or (in advance).

$NPV(i,A1:A5)$	which calculates the net present value at rate $i$ of the set of cash flows in cells say A1,A2,...,A5 assuming A1 occurs at the end of the first time period. If A1 occurs at the start of the first time period, it must be added separately.
$IRR(A1:A5,Guess)$	which calculates the internal rate of return of the cash flows in say A1,A2,...,A5. Guess is a rough estimate of the IRR. The numerical value of A1 is usually negative.

Another useful function is the **Solver** function or the **Goal Seek** function in the Tools menu. As an example you might use the spreadsheet to set up a loan repayment schedule (studied in Week 7) for a given loan but using an estimate of the instalment. The Solver function allows you to find the instalment which will make the loan outstanding at the end of the term equal to zero.

**FORMULAE FOR USE IN EXAMINATIONS**

- 1 Future value at simple interest

$$S = P(1 + rt)$$

- 2 Present value at simple interest

$$P = S(1 + rt)^{-1}$$

- 3 Present value at simple discount

$$P = S(1 - dt)$$

- 4 Future value at compound interest

$$S = P(1 + i)^n$$

- 5 Present value at compound interest

$$P = S(1 + i)^{-n}$$

- 6 Future value of  $n$  payments of  $R$  at compound rate  $i$

$$S = R s_{\overline{n}|i} = R \left[ \frac{(1 + i)^n - 1}{i} \right]$$

- 7 Present value of  $n$  payments of  $R$  at compound rate  $i$

$$P = R a_{\overline{n}|i} = R \left[ \frac{1 - (1 + i)^{-n}}{i} \right]$$

- 8 Approximation to bond or debenture yield for given price

$$i \approx \frac{I + \frac{1}{n}(C - P)}{\frac{1}{2}(C + P)}$$

- 9 Present value of an annuity with payments increasing in arithmetic progression

$$P = R[(1 + i)^{-1} + 2(1 + i)^{-2} + \dots + n(1 + i)^{-n}]$$

$$= R \left[ \frac{(1 + i)a_{\overline{n}|i} - n(1 + i)^{-n}}{i} \right]$$

- 10 Future value of an annuity with payments increasing in arithmetic progression

$$S = R \left[ \frac{(1 + i)s_{\overline{n}|i} - n}{i} \right]$$

- 11 Present value of an annuity with payments increasing in geometric progression

$$P = R[(1 + i)^{-1} + (1 + r)(1 + i)^{-2} + \dots + (1 + r)^{n-1}(1 + i)^{-n}]$$

$$= R(1 + r)^{-1} a_{\overline{n}|j} \text{ where } j = \frac{i - r}{1 + r}$$

- 12 Future value of an annuity with payments increasing in geometric progression

$$S = R(1 + r)^{n-1} s_{\overline{n}|j} \text{ where } j = \frac{i - r}{1 + r}$$