Fee for service, outperformance or assets under management?

Indications from generalized log utility*

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We investigate efficient fee structures for actively managed funds when both the investor-principal and the prospective manager-agent have utility functions of the generalized log variety. Efficient fees include a fixed component that reflects the manager’s protected consumption. This is a new rationale for a flat component of fees analogous to fee-for-service. Also new is our result that participation is not all or nothing. Specifically, the amount placed with the active manager is equal to the investor’s wealth less the present value of the total protected consumption of the investor and the manager. Efficient fees also include variable components reflecting total assets under management and performance relative to a passive benchmark.

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1. Introduction

Most mutual funds are actively managed. This gives rise to an adverse-selection problem as investors cannot easily assess manager skills or diligence. Managers may also be susceptible to moral hazard arising from insufficient effort on behalf of the client. We investigate fee structures that mitigate agency problems when both the principal and the agent have utility functions of the generalized log variety. One new result is that efficient fees include a fixed component reflecting the agent’s protected consumption. This represents a new rationale for a flat component of fees analogous to fee-for-service. This fixed component could take the form of a proportional asset fee in conjunction with some minimum investible amount stipulated in the manager’s prospectus, as we commonly observe. Another new result is that from both an investor and manager standpoint the participation decision is not all-or-nothing. Specifically, the amount placed with the active manager is equal to the investor’s wealth less the present value of the total protected consumption of the investor and the manager. Remaining wealth is allocated to safe assets.

Notwithstanding these new results our formal analytical structure holds few surprises for readers of Dybvig et al (2010). Accordingly, our efficient fee structure retains variable components reflecting assets under management and performance relative to a passive benchmark, as highlighted by those authors. They derive optimal contracts under the assumption that both the investor-principal and the manager-agent have log utility. Efficient fees are typically proportional to assets under management. When agency problems are present, fees include a component of remuneration proportional to actual performance less a
passive benchmark return. Benchmarking serves to mitigate the problem of closet indexing. The participation decision in their analysis is all-or-nothing in the sense that either no wealth or the entire wealth of the investor is placed with the active manager, in contrast to the participation decision here.

Generalized log utility viewed as a model was abbreviated to GLUM by Rubinstein (1976), who described it as the premier one-parameter utility function for financial models of households. In contrast to its quadratic, log and exponential competitors, generalized log utility has the realistic implication that relative risk aversion falls as wealth rises. For example, we typically see a greater proportionate investment in stocks as wealth rises. Generalized log utility turns out to be the simplest and most appealing of the three special cases of Hyperbolic Absolute Risk Aversion (HARA) that were investigated numerically by Bateman et al. (2007) in the context of retirement planning. It is the simplest way to capture habit-dependent utility whereby a retiree is concerned to prevent her living standard falling below some pre-determined level. To the extent the habit paradigm makes sense at any stage of life, retirement is that stage. Equally, generalized log utility is consistent with a desire to ‘keep up with the Joneses’. It can rationalise conservative asset allocations on the cusp of retirement, in contrast to simple log utility, which generates aggressive allocations, corresponding to the familiar ‘growth-optimal’ property of log utility. Finally, generalized log utility captures the concern of some investors with preventing shortfalls in wealth below some subjective reference level. In particular, the present value of protected consumption is the natural interpretation of that level.
2. The model

This section generalizes the model of Dybvig et al (2010). Their three optimization problems impose successively tighter constraints on the objective function, corresponding to increasingly severe agency problems. In the first-best case the manager’s choice of action and portfolio can be dictated. Agency problems are absent. In the second-best case the manager reveals truthfully the observed signal but has private information about her effort level. In the third-best case the adverse selection problem and the moral hazard problem are both present.

In all cases the manager’s effort $\varepsilon \ (0 \leq \varepsilon \leq 1)$ generates a signal $s \in S$ about future market states $\omega \in \Omega$. The joint density of $s$ and $\omega$ conditional on effort $\varepsilon$ is given by the mixture model

$$f(s, \omega; \varepsilon) = \varepsilon f^I(s, \omega) + (1 - \varepsilon) f^U(s, \omega).$$

(1)

Here $f^I$ is the informed (effort-conditioned) distribution of portfolio returns, and $f^u$ and $f^s$ are the marginal distributions of the uninformed density $f^U$ with respect to $\omega$ and $s$, with $f^U(s, \omega) = f^s(s)f^u(\omega)$.

**First-best** Choose the utility of the investor, $u_i(s, \omega) = \ln(C_i(s, \omega) - c)$, the utility of the manager $u_m(s, \omega) = \ln(C_m(s, \omega) - c_m)$, and the manager’s effort level $\varepsilon$ to maximise the investor’s expected utility,

$$\max_{u_i(s, \omega), u_m(s, \omega), \varepsilon} \iint u_i(s, \omega)(\varepsilon f^I(\omega)s) + (1 - \varepsilon)f^u(\omega)f^s(s)d\omega ds,$$

subject to the budget constraint
\[(\forall s \in S) \int (\exp(u_i(s, \omega)) + \exp(u_m(s, \omega)))p(\omega)d\omega = w_0 - C, \quad (3)\]

and a participation constraint on the manager,

\[\iiint u_m(s, \omega)(\epsilon f^{\prime}(\omega)s) + (1 - \epsilon)f^{\prime\prime}(\omega))f^{\prime}(s)d\omega ds - c(\epsilon) = u_0. \quad (4)\]

Here \(C_i(s, \omega)\) is the consumption of the investor, \(\underline{C}_i\) is the protected consumption of the investor, \(C_m(s, \omega)\) is the consumption of the manager, \(\underline{C}_m\) is the protected consumption of the manager, \(p(\omega)\) is the pricing density for a claim that pays a dollar in state \(\omega\), \(w_0\) is the initial portfolio value, \(\underline{C} = \int (C_i + C_m) p(\omega) d\omega\) is the present value of total protected consumption, \(c(\epsilon)\) is the cost of manager effort, and \(u_0\) is the manager’s reservation utility level.

**Second-best** Add a constraint for the incentive-compatibility of effort:

\[\epsilon = \arg \max \int u_m(s, \omega)(\epsilon f^{\prime}(\omega)s) + (1 - \epsilon)f^{\prime\prime}(\omega))f^{\prime}(s)d\omega ds - c(\epsilon'). \quad (5)\]

**Third-best** Instead of the constraint (4) (or (5)) add a constraint for simultaneous incentive-compatibility of effort and truthful signal reporting:

\[\{\epsilon, s\} = \arg \max \int u_m(\rho(s), \omega)(\epsilon f^{\prime}(\omega)s) + (1 - \epsilon)f^{\prime\prime}(\omega))f^{\prime}(s)d\omega ds - c(\epsilon') \quad (6)\]

The utility function of the investor can be rewritten as \(u_i = \log(V - \underline{C}_i)\) and the utility function of the manager can be rewritten as \(u_m = \log(\phi - \underline{C}_m)\) where \(V\) the value of what remains in the portfolio after the fee has been paid and \(\phi\) is the manager’s fee.
The differences between our three problems and their counterparts due to Dybvig et al. are the two protected consumptions $C_i$ and $C_m$. The existence of solutions to our modified problems presupposes suitable upper bounds on these parameters, including $C < w_0$. There may also be conceptual issues in interpreting the manager’s protected consumption $C_m$.¹

Dybvig et al. come up with a lemma that enables replacement of $u_i(s, \omega)$ in the above problems by the investor’s indirect utility. We need a slight extension of it to the case of protected consumptions. In particular, and in the solution of all three forms of the investor’s problem, the expected utility conditional on $s$ for the investor turns out to be

$$\log \left( B_i(s) \frac{f^m(\omega) + \varepsilon(f^i(\omega)|s) - f^m(\omega)}{p(\omega)} \right)$$

(7)

where

$$B_i(s) = w_0 - C - \int \exp(u_m(s, \omega)p(\omega)d\omega$$

(8)

is the investor’s share of the budge net of the present value of total protected consumption.

¹ The interpretation of terms like $C_i$ has been familiar since Rubinstein (1976) but the interpretation of $C_m$ may be more problematic as management costs are already registered in the cost of manager effort $c(\varepsilon)$ and the opportunity-cost term $u_0$. Since $\partial^2 u(\phi, C_m)/\partial \phi \partial C_m = -1/(\phi - C_m)^2 < 0$, one possible interpretation of $C_m$ is that it captures a nonseparable (and fixed) component of the manager’s total costs.
The proof of (7) follows Dybvig et al. The optimal solution must satisfy the subproblem of maximizing (2) subject to (3). Differentiating the Langrangean for this problem with respect to \( u_i(s, \omega) \) gives

\[
[\varepsilon f^i(s, \omega) + (1 - \varepsilon) f^\omega(\omega)] f^i(s) = \lambda_y(s) p(\omega) \exp(u_i(s, \omega))
\]

(9)

where \( \lambda_y(s) \) is the multiplier to the budget constraint. Integrate equation (9) with respect to \( \omega \), and rearrange, to get

\[
\lambda_y(s) = \frac{f^i(s)}{B_i(s)}.
\]

Substitute this into equation (9) to get (7).

For future reference and again following Dybvig et al., we set out three definitions of equilibrium returns. The gross portfolio return conditional on observing \( s \) is

\[
R^p \equiv \frac{\varepsilon f^i(\omega|s) + (1 - \varepsilon) f^\omega(\omega)}{p(\omega)}.
\]

(10)

The gross portfolio return without observing \( s \) is termed the benchmark return and is given by

\[
R^b \equiv \frac{f^\omega(\omega)}{p(\omega)}.
\]

(11)

Finally, the return under maximum effort (\( \varepsilon = 1 \)) is

\[
R^l \equiv \frac{f^i(\omega|s)}{p(\omega)}.
\]

(12)
These definitions give the intuitive decomposition $R^P = \varepsilon R^I + (1-\varepsilon)R^g$.

Equation (7) enables computation of the investor’s expected utility, namely

$$\int \log\left(w_0 - C - \int \exp(u_m(s, \omega)p(\omega)d\omega) f^s(s) ds\right)$$

$$+ \int \log\left(\frac{\varepsilon f'(s, \omega) + (1-\varepsilon) f''(s, \omega))}{p(\omega)}\right)\left(\varepsilon f'(s, \omega) + (1-\varepsilon) f''(s, \omega) f^s(s)\right) dsd\omega.$$  \hspace{1cm} (13)

Following Dybvig et al. the second term in this expression can be written as $K(\varepsilon)$ as a reminder of its independence from the two utility functions.

3. Optimal contracts

**First-best** In this case we differentiate the Langrangean associated with the problem of maximising (13) with respect to $u_m(s, \omega)$ and subject to (4). This gives the first-order condition

$$\frac{\exp(u_m(s, \omega))p(\omega)}{B_i(s)} = \lambda_R(f''(s, \omega) + \varepsilon(f'(s, \omega) - f''(\omega)))$$  \hspace{1cm} (14)

where $\lambda_R$ is the Lagrange multiplier on (4). Multiply both sides by $B_i(s)$ and integrate both sides with respect to $\omega$ to get an expression for the manager’s share of the budget net of the present value of total protected consumption, namely, $B_m(s) = w_0 - C - B_i(s)$:
Apply (4) and (15) to get

\[ B_i(s) = \frac{w_0 - C}{1 + \lambda_R}. \]  \hspace{1cm} (16)

Equations (14) and (16) together imply

\[ u_m(s, \omega) = \log \left( \frac{(w_0 - C) \lambda_R f^\omega(\omega) + \varepsilon(f^I(\omega s) - f^\omega(\omega))}{1 + \lambda_R p(\omega)} \right), \]  \hspace{1cm} (17)

so the manager’s fee is

\[ \phi(s, \omega) = C_m + \frac{(w_0 - C) \lambda_R}{1 + \lambda_R} R^p. \]  \hspace{1cm} (18)

Compared with the corresponding equation in Dybvig et al., generalised log utility changes the first-best contract in two ways. First, the optimal fee structure incorporates a flat component $C_m$. This component might take the indirect form of a minimum investible amount stipulated by the manager, in conjunction with a proportional asset fee. Indeed we typically see such minima in the prospectuses of actively managed funds. The flat component can be interpreted as the fixed cost of operating an account. Second, the proportional component $(\lambda_R / (1 + \lambda_R))R$ is not based on the investor’s entire wealth $w_0$ but on an amount net of the present value of total protected consumption $C$. In this way the optimal contract protects core interests of the investor and manager alike.
**Second-best** In this case the investor does not necessarily observe the manager’s effort so we replace equation (4) with a condition that induces best effort:

\[
\int u_m(s, \omega)(f^i(\omega|s) - f^u(\omega))f^i(s)d\omega ds - c'(\epsilon) = 0. 
\]  
(19)

**Proposition 1 (pace Dybvig et al.)** The second-best contract gives the manager a payoff that consists of a fixed component plus a component proportional to the investor’s payoff plus a bonus that is proportional to the excess return of the portfolio over a benchmark:

\[
\phi(s, \omega) = C_m + B_m(R^p + k(R^p - R^a))
\]

where \(B_m\) and \(k\) are non-negative constants.

**Proof.** The second-best contract follows from maximizing (13) with respect to \(u_m(s, \omega)\) and subject to (4) and (19). This gives the first-order condition

\[
\exp\left(u_m(s, \omega)\right) p(\omega) = \lambda^p \left( f^u(\omega) + \epsilon(f^i(\omega|s) - f^u(\omega)) \right)
\]

\[
+ \lambda^e \left( f^i(\omega|s) - f^u(\omega) \right)
\]

(20)

where \(\lambda^e\) is the Lagrange multiplier to (19). Paralleling the derivation of (17) and (18) we get

\[
u_m(s, \omega) = \log \left( \frac{w_0 - C)(\lambda^p f^u(\omega) + (w_0 - C) \frac{\lambda^e}{\lambda^p} (f^i(\omega|s) - f^u(\omega))}{p(\omega)} \right). \]

(21)

Taking exponentials of both sides shows that the manager’s fee includes one fixed and two proportional components. The first proportional component is an asset fee and the second is a
bonus or penalty depending on whether the active manager performs better or worse than a passive (zero-effort) benchmark:

\[ \phi(s, \omega) = C_m + B_m (R^p + k(R^p - R^B)), \]  

(22)

where

\[ k = \frac{\lambda_s}{\omega \lambda_n} \geq 0. \]  

(23)

Equation (22) helps explain the fact that we seldom see ‘symmetric’ or ‘fulcrum’ elements in contracts to the extent that the incentive fee introduces states in which the investor receives a payment from the manager.\(^2\) For such a payment the penalty component of a total fee must exceed not only its asset component, as in Dybvig et al., but the sum of its asset component and its flat component \(C_m\).

**Third-best** In this case the investor can observe neither effort nor the signal generated by effort. Still following Dybvig et al. we add the following condition to motivate truthful reporting of the signal:

\[ (\forall s \in S) \int \frac{\partial u_m(s, \omega)}{\partial s} (\varepsilon f^1(\omega | s) + (1 - \varepsilon) f^0(\omega)) f^\gamma(s) d\omega = 0. \]  

(24)

The third-best contract follows from maximizing (13) with respect to \(u_m(s, \omega)\) and subject to (4), (19) and (24). Differentiate with respect to \(u_m(s, \omega)\) to get the first-order condition

\(^2\) On symmetric and fulcrum contracts see Gomez and Sharma (2006).
\[ \frac{\exp(u_m(s, \omega)) p(\omega)}{B_i(s)} = \lambda_r (f^o(\omega) + \varepsilon(f^l(\omega|s) - f^o(\omega))) + \lambda_x f^l(\omega|s) - f^o(\omega) - \lambda'(s)(f^o(\omega) + \varepsilon(f^l(\omega|s) - f^o(\omega))) - \varepsilon \lambda'(s) \frac{\partial f^l(\omega|s)}{\partial s} \]

where \( \lambda'(s) \) is the Lagrange multiplier on (24).

As in Dybvig et al. the manager’s share of the budget (here net of the present value of total protected consumption) is no longer constant but depends on the signal. Specifically,

\[ B_i(s) = \frac{w_0 - C}{1 + \lambda_r - \frac{\lambda'(s)}{f^l(s)}} \]

so that

\[ B_i(s) = \frac{(w_0 - C)(\lambda_r - \frac{\lambda'(s)}{f^l(s)})}{1 + \lambda_r - \frac{\lambda'(s)}{f^l(s)}}. \]

4. Numerical analysis

This section gives a numerical analysis of optimal contracts. For brevity we confine attention to the cases of the first best and the second best. Paralleling Dybvig et al. we made the following assumptions for \( f^e(s), f^o(\omega), f^l(\omega|s) \) and \( p(\omega) \), where the market state \( \omega \) and
signal $s$ have zero means and standard deviations $\sigma$ and have correlation $\rho > 0$. In what follows, $r$ is the riskfree rate and $\mu$ is the mean return on the market:

\[
f^s(s) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{s^2}{2\sigma^2}\right) \tag{28}\]

\[
f^\omega(\omega) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{\omega^2}{2\sigma^2}\right) \tag{29}\]

\[
f^i(\omega \mid s) = \frac{1}{\sigma \sqrt{2\pi (1 - \rho^2)}} \exp\left(-\frac{(\omega - \rho s)^2}{2\sigma^2 (1 - \rho^2)}\right) \tag{30}\]

\[
p(\omega) = e^{-r} \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(\omega + \mu - r)^2}{2\sigma^2}\right). \tag{31}\]

We set $\mu = 0.1$, $\sigma = 0.2$, $\rho = 0.5$, $r = 0.05$, $\varepsilon = 0.5$, $w_0 = 100$ and, initially, $C_i = C_m = 5$. By choosing positive Lagrange multipliers $\lambda_R$ and $\lambda_c$, we can plot the investor’s wealth and the manager’s fee for the first best and second best problems.

Figure 1 portrays the investor’s wealth and the manager’s fee in the case of the first best. As expected, high signals and market states lead to good outcomes for investor and manager alike.

Figure 2 portrays the effects on first-best outcomes of increasing $C_m$ from 5 to 10 while keeping $C_i$ constant at 5. Specifically, the figure shows the difference in the manager’s fee. To the extent signal and market are both high, moving to a higher level of protected
consumption leads to less difference in the fee. The intuition is straightforward; protected consumption has less effect on outcomes to the extent returns are both forecastable and high.

Figure 3 portrays similar results for the effects on first-best outcomes of increasing $C_i$ from 5 to 10 while keeping $C_m$ constant at 5. When the investor’s protected consumption is high, less initial wealth invested in the risky portfolio. As a consequence the investor and manager alike enjoy less upside from high forecastability and market states.

To compare the manager’s fee in first-best and second-best we examine changes in the manager’s fee as we move from first-best to second-best; Figure 4 shows the fee in second-best minus first-best. The manager is rewarded when signal and market are both high and is therefore induced to exert effort.

However, following an increase in the manager’s protected consumption the manager is less rewarded when signal and market are high, as shown in Figure 5. This indicates that higher protected consumption leads to less effort.
Fig. 1. Manager’s fee (left) and investor’s wealth (right) in first-best
Fig. 2. Difference in manager’s fee in first-best
Fig. 3. Difference in investor’s wealth in first-best
Fig. 4 Manager’s payoff: Second-best minus first-best levels
5. Conclusion

We extended the influential model of fees due to Dybvig et al. to the case of generalized log utility. This extension is inconsequential from a mathematical standpoint but is significant for answering the questions of whether fees should contain a flat component and the extent to which an investor should entrust her entire wealth to a single active manager.

References

