Microinsurance Pricing under the Microscope: Findings from the Philippines

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Abstract

Microinsurance has considerable potential in addressing issues such as poverty alleviation and social protection in less developed countries. From an actuarial perspective it faces a distinct set of challenges due to tradeoffs between economic and social objectives such as financial sustainability and outreach. This paper investigates the characteristics of microinsurance using empirical evidence from the Philippines, covering a microinsurance scheme between 2003 and 2011. Individual short-term microinsurance is found to be feasible with no evidence of high claim rates. Major actuarial challenges are high lapse rates, adverse selection contingent on duration, and covariate-specific risk factors. Accurate pricing is required to support sustainability and outreach. Traditional simple actuarial pricing of microinsurance is found to be inadequate given this evidence. A simple duration-oriented multi-state model based on continuous-time Markov chains is proposed for valuation. Target population-specific transition probabilities are derived empirically with the Aalen-Johansen estimator. The model is applied and yields lower and actuarially fair premiums.

Keywords

Microinsurance, actuarial pricing, risk evaluation,

Markov multi-state model, Asia, Philippines.
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1. INTRODUCTION

The aim of this paper is to analyze the feasibility and risk patterns of microinsurance empirically, point out challenges in pricing and provide a simple yet actuarially fair pricing approach. Microinsurance is commonly defined as insurance aimed at persons with low income being subject to generally accepted insurance principles, most importantly that the insurance is funded by premiums reflecting risks (Churchill & McCord, 2012; IAIS-CGAP, 2007). Microinsurance markets have developed dynamically in many countries (Churchill & McCord, 2012; Roth et al., 2007) but experience considerable limitations in achieving both wide-spread outreach within the low income population and financial sustainability (Biener & Eling, 2012).

Financial sustainability requires financial institutions to operate efficiently and price risks adequately. This competes with widespread outreach due to high information and transaction costs inherent in the provision of financial services to persons with low income. There is well-observed evidence of a tradeoff between financial sustainability and outreach for microfinance institutions (Cull et al., 2011; Hermes et al., 2011). While there is a trend towards more sustainability, most microfinance institutions are not financially sustainable (Ayayi & Sene, 2010; Hermes & Lensink, 2011). Given the positive economic and social impact of microfinancial services, e.g. with regard to poverty alleviation\(^1\), overcoming this trade-off is crucial.

This is especially challenging for the provision of microinsurance. In addition to high costs, microinsurance institutions face numerous impediments in achieving outreach and financially sustainable services. There is extensive
literature investigating the feasibility of microinsurance. The most relevant problems include: First, information asymmetries resulting in moral hazard, adverse selection and fraud (Biener & Eling, 2012; Wang et al., 2006). Second, limited willingness to pay of insureds and too large insurance premiums (Cohen & Sebstad, 2005; Dong et al., 2004). Third, high lapse rates and low take-up rates (Ito & Kono, 2010; Sinha et al., 2007). Fourth, inability to adequately evaluate and price risks, especially due to a lack of data and actuarial expertise (McCord & Osinde, 2005). This results in inability to derive actuarially fair premiums.

Pricing of microinsurance is pivotal to achieving outreach and financial sustainability. If microinsurance institutions are not able to accurately evaluate and price risks, loadings for uncertainty have to be included in risk premiums that may well exceed the norm. The uncertainty can result from risk evaluation or actuarial pricing itself. Risk evaluation refers to the evaluation of underlying risks with regard to, for example, randomness of loss occurrence or adverse selection (Biener & Eling, 2012; Brown & Churchill, 2000; Wipf & Garand, 2006). Actuarial pricing refers to the derivation of prices on the basis of evaluated risks using an actuarial pricing model. Given limited willingness to pay of the target population, the loading for uncertainty impedes take-up and increases lapse rates (Churchill & McCord, 2012; Cohen & Sebstad, 2005; Dror & Armstrong, 2006), therefore limiting outreach. Not accounting for uncertainty by lowering premiums to match willingness to pay would endanger financial sustainability and the economic viability of the insurance transaction in terms of risk transfer (Biener, 2012; Vaté & Dror, 2002).
A complementary approach to loading for uncertainty is its mitigation and avoidance (risk control). For microinsurance, this encompasses especially product design, for example restrictive coverage and restrictive policy conditions. While being important, risk control has limitations. It reduces the acceptance of microinsurance and also endangers its economic viability in terms of risk transfer (Biener, 2012; Biener & Eling, 2012; Cohen et al., 2005).

Despite its importance, only limited research has been conducted on pricing of microinsurance. The attention of literature to date primarily focuses on the issue of risk evaluation and risk control. The early work of Brown & Churchill (2000) on the provision of microinsurance products also reviewed best practice and experience in pricing. Auray & Fonteneau (2002) discuss approaches for risk evaluation for health microinsurance. Wipf & Garand (2006) provide a non-technical overview of insurance pricing. They also review case studies and investigate the issue of database design as a prerequisite to risk evaluation. Garand et al. (2012) introduce a process for pricing of microinsurance; emphasizing its iterative nature with continuous improvement of risk evaluation, actuarial pricing and risk control based on experience. Biener (2012) discusses problems in the area of risk evaluation and risk control, focusing on the best use and acquisition of information on risks given its limited availability. He reviews techniques regarding data acquisition, data analyses and risk management as facilitators in pricing.

Actuarial pricing models for microinsurance particularly have not been subject to extensive research to date. Pricing approaches are reported as being actuarially simple or without an actuarial basis at all (Barbin et al.,
2002). Authors emphasize the general importance of actuarial fair pricing and that traditional actuarial pricing approaches can regularly not be applied in a microinsurance context due to its characteristic challenges. However, only limited guidance on feasible models is given (Biener, 2012; Brown & Churchill, 2000; Garand et al., 2012; Wipf & Garand, 2006). We argue overly simplified actuarial pricing models themselves lead to higher loadings for uncertainty than necessary. Access and use of sufficient internal or external actuarial expertise for pricing is crucial (Garand et al., 2012; Wipf & Garand, 2006). It is pivotal for long-term financial sustainability and outreach to apply suitable actuarial models. The soaring number of large commercial insurers in the microinsurance market and its increasing maturity resulting in experience and more sophisticated products also facilitate the application of advanced pricing approaches (Churchill & McCord, 2012).

The contribution of this paper is twofold. First, the risk patterns of microinsurance and its feasibility are investigated in both an experience study and a statistical estimation of risk forces using empirical data from the Philippines. In the experience study, the microinsurance scheme’s sources of profit and loss are analyzed, resulting in insights to impediments to achieving outreach and financial sustainability. In the statistical estimation of risk forces, potential drivers of microinsurance risks and observable risk patterns are determined. Second, given the evidence from the experience study, a simple duration-oriented pricing approach based on Markov multi-state models using empirically estimated probabilities is proposed and applied for the derivation of actuarially fair premiums. The model can account for insured-specific discrete covariates for risk-discrimination (e.g., by age). It is emphasized
that both the experience study and actuarial model focus on (technical) risk premiums, including premium refunds. Other components of gross premiums (e.g., loading for overhead expenses) are discussed to a lesser extent.

Our results show that the provision of microinsurance for the scheme studied is feasible. Despite the microinsurance scheme under consideration offering short-term policies that provide voluntary individual coverage, we find no evidence of overall high claim rates for this type of insurance as reported in the literature (Wipf et al., 2006). We find that the main challenges appear to be high lapse rates, duration-contingent adverse selection due to short-term insurability, and overpricing due to uncertainty with regard to both risks and adequate actuarial pricing. Moreover, (Philippine) regulatory premium refund requirements appear to be of limited value; it increases premiums and creates additional uncertainty. These findings partly add to available evidence discussed in the literature (Biener, 2012; Ito & Kono, 2010). Risks of microinsurance appear to be driven considerably by policy duration, and simple traditional actuarial pricing is found to be unable to adequately account for this effect. The application of a proposed duration-oriented pricing approach yields lower premiums and accounts for adverse-selection and lapse in pricing. This can be expected to support take-up and outreach and to create disincentives for lapse. The Aalen-Johansen estimator from event analysis is found to be able to adequately estimate risks in a typical microinsurance setting with respect to risk factors and data. While the paper discusses short-term microinsurance for life and health risks, many results can potentially be transferred to other types of microinsurance. The findings should not be considered as generalizable for the microinsurance industry.
However, they provide intuition of relevant risk patterns and feasible pricing and risk estimation approaches and can be regarded as a first step for more extensive analysis.

The remainder of the paper is structured as follows. Section 2 introduces the microinsurance institution under consideration and investigates the feasibility of microinsurance and its risk characteristics in an experience study. Given this evidence, Section 3 introduces a feasible pricing model. This is then applied on the microinsurance institution under consideration in Section 4, where risk patterns with regard to duration and covariates are also investigated, and an actuarially fair premium is derived. Section 5 summarizes the findings and limitations of this paper and gives scope for future research.

2. CASE STUDY FROM THE PHILIPPINES: PMBA

(a) Description of the scheme

The data under consideration is derived from the database of PMBA, a Philippine microinsurance institution. It is a mutual benefit association founded in 2002 as a microinsurance service provider for a peasant confederation. It offers two microinsurance products. First, an insurance product offering voluntary individual coverage of primarily mortality and permanent disability of the insured with payment of a lump sum upon decrement (henceforth, base insurance product). The base insurance product also provides temporary hospitalization benefits and funeral benefits. However, these are insignificant in size. Due to minor importance and lack of data, funeral benefits are not separately accounted for in the following. The term of the base
insurance product is annual with option to renew. Coverage sum and number of persons insured depends on the type of policy chosen. While each policy is attached to the policyholder, certain types of policies can also cover additional persons (most importantly dependent family members). Premiums are paid (semi-)annually, quarterly or monthly in advance. The product design also includes some well-designed risk control features, most importantly a six months waiting period for eligibility of claims for hospitalization.

Second, an insurance product combining the base insurance product with additional loan protection for microfinance institutions (henceforth, loan protection product). Only a small minority of total policies were related to this product. In case of death of the insured, the lending institution is paid (part of) the outstanding loan balance and principal. The term of the policy is the remaining life of the loan. The loan protection product was offered in cooperation with microfinance institutions and its coverage was partly obligatory to obtain loans. Due to the loan protection product being a base insurance product with an additional protection component for the microfinance lender, it will not be treated separately. It will be regarded in the following as as a base insurance product in terms of exposure to risks. In the experience study the loan protection component is taken into account in terms of premium income and claim costs.

The base insurance product offers voluntary individual coverage; allowing insureds to freely opt in or out. This type of microinsurance product is much less common compared to group covers and is widely reported in literature to be difficult to provide in a financially sustainable way (Churchill & McCord, 2012; Wipf et al., 2006). Given that the importance of these products is
increasing, an investigation of risk patterns promises insights with regard to potential adverse selection issues. Given the products’ other policy conditions and risks covered, PMBA’s products can be considered as being exemplary of contemporary microinsurance.

The Philippines microinsurance market exhibited dynamic development in the past with a large number of operating microinsurance institutions and strong growth of the insured population. This is partly credited to favorable and flexible microinsurance regulation. However, it also experiences low take-up, high lapse and low ability and willingness to pay (Llanto et al., 2008, 2009).

(i) Description of the data set

PMBA used insurance administration software to provide consistent membership and claims data on a per policy basis. Table 1 illustrates the development of the insurance scheme in terms of policies from 2003 to 2011. The data set encompasses a total of 21,975 years of policy exposure. Note that one policy can cover multiple insureds. A high number of annual lapses of policies can be observed. The considerable decrease in new policies from 2010 is due to a discretionarily made management decision resulting in a decrease in marketing efforts. Overall, PMBA has to be considered as being a small microinsurance provider within the Philippine microinsurance market (see Llanto et al. (2009) for a market overview).

[Table 1 about here.]
Comparing the lapses with the number of claims as shown in Table 2, it becomes evident that the insurance population of the scheme under consideration exhibits high annual lapse rates, while the number of claims due to decrements is low. High lapse rates are no phenomenon specific to PMBA but a general problem in the provision of microinsurance (see, for example, Llanto et al. (2006); Sinha et al. (2007) for evidence in the Philippines). Mortality is dominant in terms of benefits paid out, while other decrements are of minor importance.

[Table 2 about here.]

The data set provides additional information on the situation of insureds. Such information can be used for risk discrimination in pricing. Only easily observable covariates are taken into consideration to ensure required low transaction costs (Ito & Kono, 2010). Besides age, the data provides reliable information on gender. The use of covariates in pricing is analyzed in Section e.

The data set partly lacks reliable information on secondary persons being covered by policies. For empirical estimation of risk patterns in Section 4, the data are therefore analyzed from 2007 and only the primary insureds are accounted for in terms of claims to provide consistent estimates. This results in a considerable reduction of the data available. However, most of the claims attributable to secondary insureds are related to infant mortality and hospitalization. This is not a serious limitation given that infant mortality and hospitalization are rather unimportant decrements that are covered only with minor benefits. Hence, a total of 9,546 years of insureds’ exposure
are available for investigation of risk patterns, still sufficient for credible estimates.

(ii) Original pricing and actuarial basis

At inception of the scheme PMBA faced uncertainty in risk evaluation and actuarial pricing. The scheme’s target population is the informal sector of the Philippines with a focus on small farmers and rural workers. Geographically the scheme was present in 24 of the 80 Philippine provinces (Almazan, 2005). Hence, the scheme’s population was a socially and geographically defined subgroup of the overall Philippine population. Discrepancies between the risk patterns of the target population and the overall Philippine population were expected. No reliable risk data for this social subgroup in the Philippines was available. These constraints are typical in the context of pricing in microinsurance markets (Biener, 2012).

The premium, in the discussion that follows, is defined as including all premium components related to expected claim and lapse costs of insureds. PMBA priced the base insurance product using a simple actuarial present value approach. For the base insurance product, reliable information on pricing is only available for the life insurance component. Other risks have not been priced explicitly.

Based on an assumed median age of the target population on enrolment of 45 years, a corresponding conservative annual mortality rate was assumed. It was derived from experience rates from the Philippine social security system. The assumed discrete interest basis reflected expected investment income. Assuming mid-year discounting and uniform annual distribution of
mortality claim costs, the risk premium was then derived by computing the present value of the expected claim costs. The Philippine insurance code requires refunding of 50% of the total gross premium paid on lapse given a minimum of three years’ of coverage. The premium was therefore conservatively loaded with a premium refund provision of 50% of total gross premium paid. Other expenses such as overhead expenses and commissions were also loaded, but these components are disregarded due to lack of reliable experience data. These loadings were removed from the premiums reported in the experience study. See Appendix A for a brief illustration of the original pricing approach.

For determination of financial sustainability, PMBA used sensitivity analyses applied on a five-year actuarial projection of the fund assuming 10% lapse per year. The loading for uncertainty is reflected in actuarial conservatism with regard to assumed mortality forces in the population and premium refund provisioning. Such simple and heuristic actuarial pricing of the microinsurance scheme is found widely in the Philippines microinsurance market (see Barbin et al. (2002) for evidence in the Philippines).

(b) Experience study

In this section, we analyze the insurance experience of PMBA. It must be emphasized that the discussed premiums do not include any profits or overhead expenses except net reinsurance expenses; respective loadings are removed from premiums. Given the empirical data, the scheme was modeled on a policy level since inception in 2003 to September 2011.

The experience study follows the general methodology of an actuarial ex-
perience study. However, to allow decomposition of premiums and derivation of sources of profits and losses in absence of information on investment income and to facilitate interpretation, this was modified. The methodology of the experience study is explained in Appendix A.\textsuperscript{9} Table 3 gives a break-up of premiums by actual experience in terms of time value as of inception.

[Table 3 about here.]

PMBA was profitable to a considerable extent with regard to claims costs and premium refunds. There is no indication of unexpectedly high claim rates resulting from adverse selection. Given the magnitude of profitability, the microinsurance was potentially overpriced. Premium refunds paid have been considerably higher than claim costs.\textsuperscript{10} The scheme’s management emphasized in interviews that the eligible premium refunds were rarely collected by insureds in practice.

In the following the experience of PMBA is compared with the implicit expectations reflected in original pricing of the microinsurance as introduced in Section ii. The simple original pricing approach allows one to decompose the scheme’s experience into its single components and compare it against its implicit expectations from pricing, thereby deriving sources of profit and loss.\textsuperscript{11} Based on pricing, PMBA expected small profits to arise due to actuarial conservatism. However, in actual experience, considerable profits were achieved.

Table 4 illustrates the decomposition of the present value of the profits and losses of the actual experience over the expectations reflected in pricing as of inception. PMBA realized a considerable gain on unexpected low claim
costs and early lapse of insureds. Losses on decrements other than mortality result from these other decrements not being priced explicitly. While not being priced, these risks incurred only minor claim costs. Reinsurance was also not priced originally as an expense.\textsuperscript{12}

[Table 4 about here.]

The high profits over expected total claims costs imply that the population either has a unexpected low risk profile or that the microinsurance was considerably overpriced due to a loading for uncertainty. The significant profits on early lapse resulted from full provisioning for premium refunds. Though early lapse was not accounted for in premium pricing and high early lapse rates were experienced.

To investigate whether the considerable profits on expected claim costs arose from overpricing or a low risk population, the mortality gain over age-dependent mortality rates from Philippine intercompany mortality experience rates were derived.\textsuperscript{13} Table 4 also provides the mortality gain on Philippine intercompany mortality experience.

Mortality gain on intercompany mortality experience is smaller compared to mortality gain on pricing assumptions, but remains substantial. This has two implications. First, the assumed mortality rate for the target population in original pricing was significantly too high. This resulted in overpricing and reflects the loading for uncertainty both with regard to risk evaluation and actuarial pricing. Second, as PMBA experienced a considerable gain on intercompany mortality experience a generally low risk target population with the absence of overall adverse selection is implied.
Microinsurance provision of PAKISAMA is found to be feasible with considerable profitability. High profitability results from low claim rates, high profits on early withdrawal and significant overpricing due to loading for uncertainty. While several authors argue that individual voluntary coverage is difficult to provide due to expected high claim rates (Biener & Eling, 2012; Wipf & Garand, 2006; Wipf et al., 2006), the target population exhibited no indication of elevated mortality claim rates compared to the general Philippine population. Low take-up rates resulting in slow growth and high lapse rates have been the main challenges of PMBA, which are also widely supported by literature (Ito & Kono, 2010; Sinha et al., 2007).

PMBA’s experience also questions regulatory required premium refunds, especially given the short term nature of microinsurance and its social and economic goals. Required provisioning for premium refunds increases premiums, creates additional uncertainty in pricing and endangers outreach and take-up. However, such cash-back benefits can also increase perceived value of the microinsurance to insureds and therefore support take-up (Wipf et al., 2006). Yet the high early lapse rates indicate that it is neither perceived as an savings instrument by the target population nor suitable. Hence, if such a cash-back benefit is included in a microinsurance policy to support take-up, the awareness for it has to be increased in the insured population. This creates economic disincentives for lapse and thus supports outreach.

It must be noted that PMBA experienced high overhead expenses, primarily related to distribution, that rendered the scheme financially unsustainable. High overhead expenses are typical for microinsurance institutions due to high transaction and information costs (Werner, 2009). However,
these costs are especially pronounced given provision of individual voluntary microinsurance (Wipf et al., 2006). The high overhead costs of PMBA are potentially a result of the rather small population of insureds. This further emphasizes the importance of reduced loadings for uncertainty and achieving outreach, which facilitates the apportion of overhead expenses through scale (Garand et al., 2012).

Actuarial pricing of PMBA is found not to be actuarially fair due to typical constraints to microinsurance markets. A significant loading for uncertainty can be observed, both with regard to risk evaluation and actuarial pricing. The resulting overpriced premiums impede take-up and facilitate lapse, hence limiting outreach. While the issue of low outreach also has important implications for education, organizational issues and marketing (Sinha et al., 2007), we argue that actuarially fair risk evaluation and sophisticated actuarial pricing resulting in lower premiums is crucial. In the following a simple pricing approach for short-term microinsurance as described above is proposed.

3. PRICING MICROINSURANCE

(a) Mathematical and economic framework

The proposed approach relies on the mathematical framework of Markov multi-state models. This framework is well-developed and both analytically rigorous and general in modeling. While being a flexible modeling tool suited for analyzing complex insurance products, its application to microinsurance products of limited complexity is relatively straightforward in terms
of modeling and estimation.

The model exhibits the following characteristics: First, the model is designed for pricing short-term microinsurance policies like PMBA’s products. Microinsurance products are mostly short-term with a coverage term of 12 months or less. This product design decision is driven by several factors. It generally yields lower premiums due to smaller loading for uncertainty. Short-term coverage also decreases required risk management and actuarial expertise and allows more frequent adjustment of pricing based on experience (Biener, 2012; Wipf et al., 2006). It also reduces the complexity of rigorous derivation of fair premiums and provisioning.

Second, the model is duration-oriented. Fair premiums are priced contingent on duration of a policy. Relevant risk factors of microinsurance are to a significant extent driven by duration. This allows one to capture the effect of lapse, which is typically contingent on duration. As lapse impacts competing risk factors and can itself affect profitability significantly, it has to be adequately reflected in pricing (Wipf & Garand, 2006). Duration-oriented pricing can also capture potential adverse selection arising from short-term coverage. This complements other approaches for mitigation of adverse selection in product design, such as waiting periods or lower benefits for new insureds. However, these limit the value of the product to insureds (Biener, 2012; Wipf et al., 2006). Charging premiums contingent on duration may not be desirable due to complexity, community preferences or even regulation. Be that as it may, such a pricing process allows insurers to attain a solid understanding of the target population’s risk patterns.

Third, the model can price microinsurance subject to competing risks.
Microinsurance policies insuring multiple risks are both common (Churchill & McCord, 2012; Wipf et al., 2006) and demanded as it allows insureds to manage multiple risks simpler and at lower costs (Cohen et al., 2005; Cohen & Sebstad, 2005). A competing risk approach also adequately captures the effect of lapse, which can have significant impact on fair premiums (Wipf & Garand, 2006).

Fourth, the model can account for covariates in risk evaluation. This allows one to capture insured-specific risk factors within the population and discriminate different risks accordingly. The approach can only be applied for a limited number of discrete covariates. However, this is no serious limitation. While discrimination is theoretically attractive from a risk pricing efficiency perspective, it is only of secondary importance in practice. Discrimination of individual insureds increases policy complexity, and risk factors are difficult to capture and costly to verify for individual insureds (Biener, 2012; Garand et al., 2012). Given the objectives of microinsurance institutions, cross-subsidization of high risks is also partly preferred by microinsurance institutions and the target population (Garand et al., 2012; Wipf et al., 2006).

In the following, the pricing model for derivation of fair premiums and its probabilistic structure are introduced. We will not investigate other components of gross premiums. Their derivation is usually a less quantitative exercise and difficult to generalize. Amongst others, Garand et al. (2012); Wipf & Garand (2006) discuss approaches and best practices.
(b) Multi-state model

Let $Z = (Z_1, ..., Z_k)$ be a vector of $k$ observable discrete covariates reflecting the riskiness of an insured. Each insured is then described by a vector of possible covariate values $z = (z_1, ..., z_k)$ from the covariates’ domains. Following the notation of actuarial multi-state models (see Haberman & Pitacco (1999); Wolthuis (2003)), let the policy-relevant status of an insured be modeled by discrete states. Assume that the state of an insured exhibiting covariate values $Z = z$ follows a well-behaved time-inhomogeneous Markov chain $\{S_{t,z}\}_{t \in [0, \infty)}$ with right-continuous sample paths. Hence, at time $t$, the insured is in state $S_{t,z}$. Time unit is one year. The model’s discrete state space is given by the ordered finite set $N = \{0, ..., K\}$, $K \geq 1$. The state-space is policy-specific and is modeled in actuarial terms of coverage and exposure to decrements.

Using Markov chain state classification, state 0 is the transient starting state for all insureds upon inception of the policy. It models active membership of the insured without decrements at time $t$. States $\{1, ..., K\}$ are states accessible from state 0, each of which models relevant decrement- and membership-related components of the policy. These states can either be transient for temporary decrements (e.g., in health insurance) or absorbing for permanent decrements (e.g., in life insurance). The possible transitions of $\{S_{t,z}\}$ are illustrated in Figure 1, for the exemplary case where states 1 and $K$ are defined as absorbing states.

[Figure 1 about here.]
Let

\[ P_{ij}(s,t) = \Pr(S_t = j \mid S_s = i) \], for \( i, j \in N, s \leq t, Z = z \)

be the conditional probability that an insured is in state \( j \) at time \( t \), given \( S_s = i \) and reflecting covariate values \( Z = z \). The transitions between states are then governed by time-dependent and covariate-dependent intensities \( \Lambda_z(t) = (q_{ij}(t), i \in N, j \in N, Z = z) \). Hence,

\[ q_{ij}(t) = \lim_{u \to t} \frac{P_{ij}(t,u)}{u - t} \] for all \( i \neq j \in N, Z = z \),

is the instantaneous probability of moving from state \( i \) to state \( j \), given that the insured exhibits covariate values \( Z = z \). The model’s \((K + 1) \times (K + 1)\) transition intensity matrix \( \Lambda_z(t) \) takes the general form

\[
\Lambda_z(t) = \begin{pmatrix}
q_{00}(t) & q_{01}(t) & \cdots & q_{0K}(t) \\
q_{10}(t) & q_{11}(t) & 0 & \cdots \\
\vdots & \vdots & \ddots & 0 \\
q_{K0}(t) & 0 & 0 & q_{KK}(t)
\end{pmatrix},
\]

where transitions with corresponding matrix entries of zero are not allowed.

The cumulative transition intensities are given by \( Q_{ij,z}(t) = \int_0^t q_{ij,z}(u)du \) for \( i \neq j \) and by \( Q_{ii,z}(t) = -\sum_{i \neq j} Q_{ij,z}(t) \). Let \( Q_z(t) \) denote the corresponding \((K + 1) \times (K + 1)\) cumulative transition intensity matrix, whose structure is equal to the structure of \( \Lambda_z(t) \) shown above.
Then the \((K+1) \times (K+1)\) matrix of conditional transition probabilities

\[
P_z(s, t) = \prod_{(s,t)} (I + dQ_z(t))
\]  

(1)
can be derived through product integration for given \(Q_z(t)\), whereas \(P_z(s, t)\) is the unique solution to the Kolmogorov forward equations

\[
\frac{\partial}{\partial t} P_z(s, t) = P_z(s, t)Q_z(t),
\]
given initial condition \(P_z(s, s) = I\) (Aalen & Johansen, 1978). Given this probabilistic structure a simple duration-oriented microinsurance pricing model is introduced.

(c) Valuation formula

Let \(\tau \in \mathbb{R}^+_0\) denote the duration of an insured’s policy in annual terms. Let \(\Delta \in \mathbb{R}^+\) be the interval between premium payments in annual terms. (e.g., for quarterly payment \(1/4\)). Let

\[
b_{j,z}(t), j \in B \subset N
\]
denote the rate of a continuous benefit payable to an insured exhibiting \(Z = z\) while occupying state \(j\) at time \(t\). Let

\[
c_{j,z}(t), j \in C \subset N
\]
denote the lump sum payment payable to an insured exhibiting covariate values $Z = z$ due to a transition to state $j$ at time $t$. Sets $B$ and $C$ do not have to be disjoint and both continuous and one-time benefits can be modeled for one decrement (e.g., lump sum payment and annuity for disablement). Assume a continuously compounding constant deterministic rate of interest $\delta$ in annual terms. Let $v = e^{-\delta}$ be the time value of $\$1$ payable in one year, hence $v$ is the annual discount factor.

Assuming payment of the premium in advance\textsuperscript{17} at time $\tau$ and given $S_{\tau,z} = 0$ and covariate values $Z = z$, the fair premium $\Pi_{\tau,z}$ for coverage in time period $(\tau, \tau + \Delta)$ is\textsuperscript{18}

$$
\Pi_{\tau,z} = \int_\tau^{\tau+\Delta} v^{u-\tau} \left[ \sum_{j \in B} P_{0j,z}(\tau,u) b_{j,z}(u) \right] du + \int_\tau^{\tau+\Delta} v^{u-\tau} \left[ \sum_{j \in C} P_{00j,z}(\tau,u) q_{0j,z}(u) c_{j,z}(u) \right] du.
$$

The fair premium is found to equal the time $\tau$ value of the expected claims arising from coverage from time $\tau$ to time $\tau + \Delta$. Hence $\tau + \Delta$ is the total length of the insurance coverage after paying $\Pi_{\tau,z}$. The fair premium is contingent on the risks of decrements represented by transition intensities $\Lambda_z(t)$ and on risk factors captured by duration $\tau$ and covariates $Z$. Note the interpretation of $\int_\tau^{\tau+\Delta} P_{0j,z}(\tau,u) du$ as probability of occupancy of state $j \in B$ from time $\tau$ to $\tau + \Delta$. If coverage between $\tau$ and $\tau + \Delta$ results in potential claims at a time later than $\tau + \Delta$ the equation can be adjusted easily to reflect this. This is illustrated in Section 4.

In microinsurance markets, there are significant limitations with regard to
risk evaluation, empirical estimation abilities, and actuarial expertise. Given additional assumptions, the complexity of applying equation (2) can be significantly reduced. Let 

\[ d_{j,z}(t), j \in D \subset N \]

denote the lump sum payment payable to an insured exhibiting covariates \( Z = z \) due to occupation of state \( j \) at time \( t \), whereas set \( D \) contains only absorbing states. Transition intensities are assumed to be uniformly distributed within each \( \Delta \) coverage period, resulting in piecewise-constant transition intensities contingent on \( \tau \) and the viability of mid-\( \Delta \) discounting. Given \( S_{\tau,z} = 0 \) and \( Z = z \), the fair premium \( \Pi_{\tau,z} \) for coverage in time period \((\tau, \tau + \Delta)\) is then given as

\[
\Pi_{\tau,z} = \left[ \int_{\tau}^{\tau+\Delta} \left( \sum_{j \in B} P_{0j,z} (\tau, u) b_{j,z}(u) \right) du + \sum_{j \in D} P_{0j,z} (\tau, \tau + \Delta) d_{j,z} (\tau + \Delta) \right] v^{\frac{\Delta}{2}}.
\]

The additional assumptions are not serious limitations, especially given typically short \( \Delta \) periods. However, they allow analytically rigorous pricing with transition probabilities and discount factors that can easily be derived empirically. It also facilitates the consistent use of subjective transition probabilities that have not been empirically estimated from the target population, e.g., from pre-existing actuarial tables or expert estimates.
4. MODEL APPLICATION

In the following, the model is applied to the base insurance product for investigation of risk patterns of the target population and derivation of an actuarially fair premium. The base insurance product is priced using covariate-contingent transition probabilities empirically estimated from experience data. Thus, the risk patterns of the target population are accounted for and hence investigated.

A robust and simple approach from survival analysis requiring minimal data is used for estimation purposes. The approach requires only exposure and claim information. This data should be available in microinsurance institutions’ databases set up according to best practice (see, e.g., McCord & Osinde (2005); Wipf & Garand (2006) for discussion). Empirical evidence provided by other authors partly hints at the existence of adequate systems and data and their importance is emphasized. However, data exploitation for risk evaluation is still lacking in practice (McCord & Osinde, 2005). Even given very limited experience data of the target population can potentially be leveraged and still be used for derivation of feasible estimates using bootstrap methodologies (Biener, 2012).

Estimation of empirical probabilities from experience with the target population is often preferable (Biener, 2012; Wipf & Garand, 2006) but not required. Given lack of experience or unavailability of appropriate data also independently assumed subjective transition probabilities reflecting the risk characteristics of the population can be used for (a subset of) the relevant transitions, e.g., from pre-existing actuarial tables or expert estimates. Bi-
ener (2012) discusses transition approaches that aim to adapt available risk data to a specific target population and credibility models that condense risk data from several sources in a microinsurance context.

The application of subjective transition probabilities can also be preferable to empirical estimation even if experience data is available. This is especially the case if empirical estimates have low credibility. In practical application, the trade-off between credibility and suitability for the target population has to be considered for all decrements. Discrepancies between assumed or estimated transition probabilities and true transition probabilities of the target population are not as critical as in pricing of traditional insurance products due to the short-term coverage allowing regular adjustments based on experience (Wipf et al., 2006).

(a) Pricing model for the base insurance product

The pricing approach introduced above is applied to the PMBA base insurance product. The base insurance product’s discrete state space is given by \( N = \{0, 1, 2, 3, 4\} \), where the states are defined by

\[
0 := \text{“Active membership”},
\]
\[
1 := \text{“Permanent disability”},
\]
\[
2 := \text{“Death”},
\]
\[
3 := \text{“Lapse”}, \quad \text{and}
\]
\[
4 := \text{“Hospitalization”}.
\]

The possible transitions within state space \( N \) are illustrated in Figure 2 and are given in terms of the transitions intensity matrix \( \Lambda_z(t) \). We apply
equation (3), noting that states \(\{1, 2, 3\}\) are absorbing states and state 4 is a transient state.

[Figure 2 about here.]

Given coverage at time \(\tau\) due to initial premium payment \((S_{\tau, z} = 0)\), the fair premium of the base insurance product for an insured exhibiting covariates \(Z = z\) is then given as

\[
\Pi_{\tau, z} = \left[ \int_{\tau}^{\tau + \Delta} \left( P_{04, z}(\tau, u) b_{4, z}(u) \right) du 
+ \sum_{j=1}^{3} P_{0j, z}(\tau, \tau + \Delta) d_{j, z}(\tau + \Delta) \right] v^{\Delta}. 
\]  

(4)

Pricing microinsurance on the basis of empirically estimated transition probabilities already implicitly includes policy conditions. Hence, explicit modeling of policy conditions (e.g., waiting periods) is not necessary. This is a significant advantage, given that rigorous mathematical modeling of policy conditions would have to be implemented on a per policy basis, resulting in a considerable increase in complexity in practical application (Barbin et al., 2002).\(^{19}\)

Risks that arise from coverage between times \(\tau\) to \(\tau + \Delta\) due to decrements after time \(\tau + \Delta\), which result in the need for provisioning, have to be modeled explicitly. A typical example is the partial premium refund requirement of the Philippine insurance code. Assume, for \(\theta \in \mathbb{R}_0^+\), that after a policy duration of \(\Delta \theta\) years the insured has a premium refund claim on a share of \(\alpha \in [0, 1]\) of a premium \(\Pi_{\tau, z}\) previously paid. The expected time \(\tau + \Delta/2\) value of the premium refund is given implicitly in terms of the lump sum

28
amount \( d_{3,z}(\tau + \Delta) \) as

\[
d_{3,z}(\tau + \Delta) = \alpha \Pi_{\tau,z} \times P_{00,z}(\min(\tau, \Delta \theta), \Delta \theta) \times \\
\sum_{i=\max(\tau, \Delta \theta)}^{\infty} P_{00,z}(\max(\tau, \Delta \theta), i) P_{03,z}(i, i + \Delta) e^{i-\tau} \frac{P_{03,z}(\tau, \tau + \Delta)}{P_{03,z}(\tau, \tau + \Delta)},
\]

and where \( i \) is a multiple of \( \Delta \). The amount \( d_{3,z}(\tau + \Delta) \) is thus the time \( \tau + \Delta / 2 \) value of the premium refund provision for premium \( \Pi_{\tau,z} \). The first term models the amount of premium refunds implicitly in terms of premium \( \Pi_{\tau,z} \). The second term captures the probability of insureds remaining active until being eligible for the premium refund, thus effectively eliminating profit on early lapse. The third term gives the expected probability of future premium refunds, given that the insured does not become inactive before being eligible and discounted to time \( \tau + \Delta / 2 \). The probability in its denominator is a conditioning adjustment with regard to equation (4). It is required as the risk from lapse does not result necessarily from a transition only within time period \((\tau, \tau + \Delta)\).

(b) Empirical estimation approach

In the following the non-parametric Aalen-Johansen estimator for the transition probability matrix \( \hat{P}_z(s, t) \) is introduced. It is frequently used in survival analysis and more generally in event history analysis (Andersen et al., 1993). Empirical estimation in survival analysis and microinsurance pricing face similar challenges, especially because of the small data sets for estimation, the dynamic nature of the data, and the empirical limitations resulting from incomplete observations of the population.
More specifically, the estimator generally has to be robust with regard to truncation, censoring and tied transition times. Actuarial experience data is left-truncated if it is only available from a specific point in time, with exposure from active policies existing earlier. Reliable claims data for exposure analysis is available only from 2007. The experience data under consideration is hence left-truncated with regard to the claims history of insureds before 2007. The data under consideration is right-censored with regard to transitions from the point in time of data collection in September 2011. The estimation approach also has to allow for tied times of transition. Tied transitions are observable to a significant extent in insurance data due to cycles in policy renewal. The Aalen-Johansen estimator can estimate transition probabilities non-parametrically given these limitations and is therefore applied in the microinsurance context (Andersen et al., 1993).

The variable $Y_{ij,z}(t)$ denotes the number of direct transitions from state $i$ to state $j$ that have been observed until time $t$ of insureds exhibiting $Z = z$. Let $X_{i,z}$ be the number of observed insureds in state $i$ before time $t$ given covariates $Z = z$. Assuming that censoring and truncation are independent, $\hat{Q}_z(t)$ can be estimated non-parametrically with the Nelson-Aalen estimator, given by

$$\hat{Q}_{ij,z}(t) = \int_0^t X_{i,z}(u)^{-1}dY_{ij,z}(u) \text{ for } i \neq j, Z = z,$$

and $\hat{Q}_{ii,z}(t) = -\sum_{j \neq i} \hat{Q}_{ij,z}(t)$ (Andersen et al., 1993). The cumulative intensity functions $\hat{Q}_{ij,z}(t)$ are then step-functions.

Using this estimate with equation (1) and taking into account that the product integral can be written as a matrix product due to the estimate
being a matrix of step functions with a finite number of jumps, \( \hat{P}_z(s,t) \) can be estimated with the Aalen-Johansen estimator given by \(^{20}\)

\[
\hat{P}_z(s,t) = \prod_{s < t_k \leq t} \left( I + \Delta \hat{Q}_z(t_k) \right).
\]

Note that this estimation approach results in the Markov process being a discrete-time Markov chain with a finite number of jumps at times \( t_k \).

Estimators for the covariance matrix of \( \hat{P}_z(s,t) \) can be derived for estimation of confidence intervals and investigation of statistical significance. The Greenwood-type covariance matrix estimator

\[
\hat{\text{cov}} \left( \hat{P}_z(s,t) \right) = \int_s^t \hat{P}_z(u,t)^\top \otimes \hat{P}_z(s,u^-) \hat{\text{cov}} \left( d\hat{Q}_z(u) \right) \hat{P}_z(u,t) \otimes \hat{P}_z(s,u^-)^\top
\]

can be applied on \( \hat{P}_z(s,t) \). The estimator is applicable given the described empirical limitations and Nelson-Aalen-estimated discrete cumulative intensity functions. \(^{21}\) As the Aalen-Johansen estimator converges to a Gaussian process in distribution, the covariance estimates can be used for estimation of pointwise confidence intervals (Andersen et al., 1993).

For practical empirical estimation, we must ensure occupation of only one state at time \( t \) of \( \{S_{t,z} \}_{t \in [0,\infty)} \). Respective problems resulting from product design can be overcome by splitting the state space, modeling multiple successive states, or by independent pricing of selected policy components. \(^{22}\)

Covariates \( Z \) can be accounted for using stratification according to discrete covariate value vectors \( z \). This approach is also used in the context of
joint Cox models (Andersen, 1986). While being straightforward, stratification results in the need for a large data set for estimation given the inclusion of multiple covariates and corresponding values. This aspect is generally not as limiting in an insurance setting compared to, for example classical survival analysis in patient studies. Due to the limited number of relevant and observable risk factors as described above, this is especially true in evaluating microinsurance.

The implementation of the pricing model and empirical estimation of transition probabilities were conducted in the open source statistical software environment R (R Development Core Team, 2012). Recently released free standard software from Allignol et al. (2011) can be used in this environment for empirical estimation of the transition probability matrix $\hat{P}_z(s,t)$ as described above, emphasizing the practical relevance of the approach in a microinsurance context.

(c) Empirical transition probabilities

The empirical transition matrix $\hat{P}(s,t)$ is estimated from the data set described in Section a for investigation of risk patterns. At this stage, insureds are not discriminated according to covariates, hence $Z$ is omitted. This allows comparison with the original actuarial pricing approach.

Figure 3 illustrates the estimated transition probabilities $\hat{P}_{0j}(0,t)$ for all relevant transitions and $t \in (0,3]$. Table 5 provides the corresponding estimated annual transition probabilities $\hat{P}_{0j}(s,t)$ for all relevant transitions. Probabilities of hospitalization $\hat{P}_{04}(s,t)$ are given integrated over $(s,t)$ to allow interpretation with regard to pricing, as hospitalization benefits are paid
as continuous annuities. While the estimates are reported, the probabilities of the permanent disability decrement $P_{01}(s, t)$ will not be accounted for in interpretation due to low significance and data sparsity.

[Figure 3 about here.]

[Table 5 about here.]

Probability of persisting active membership $\hat{P}_{00}(s, t)$ is found to be small, especially after duration $\tau = 1$. This is almost entirely attributable to high lapse probabilities $\hat{P}_{03}(s, t)$ that significantly increase after the initial policy term from duration $\tau = 1$. Figure 3 illustrates that especially pronounced lapse rates can be observed at the annual policy renewal intervals.\textsuperscript{23}

Table 6 illustrates differences between the estimated probability of death $\hat{P}_{02}(s, t)$ and the conservatively assumed annual probability of death of the target population in the original pricing $\tilde{P}_{02}$. Estimated probability of death in the first year of policy duration $\hat{P}_{02}(0, 1)$ is found to be close to $\tilde{P}_{02}$ with no statistically significant difference. Estimated probability of transition to death state $\hat{P}_{02}(s, t)$ is found to be significantly decreasing with duration. The reduction of probability of death $\hat{P}_{02}(s, t)$ from $\tau = 1$ is partly attributable to lapse as a competing risk. However, given the size of the effect, this is also an indicator of potential adverse selection within the population. The differences between the estimated $\hat{P}_{02}(s, t)$ and originally assumed probabilities of death $\tilde{P}_{02}$ are highly statistically significant from duration $\tau = 1$.

[Table 6 about here.]
Probability of occupancy of hospitalization state ∫₀ᵗ ˆP₀₄(s,u)du is found to be low. This results primarily from hospitalization being a transient state with short occupancy periods. The lack of transitions in the first six months of duration as observed in Figure 3 is due to a policy condition requiring a waiting period of a half year for eligible hospitalization claims. This illustrates how policy conditions are reflected in empirically estimated probabilities given experience with a policy. Probability of hospitalization ˆP₀₄(s,t) appears to be also negatively related with duration. However the effect is less pronounced than for probability of death ˆP₀₂(s,t) (see Table 5). This is potentially a result of the implemented waiting period as a risk control instrument regarding adverse selection.

The observed risk patterns might also be potentially caused by fraud, especially regarding relatively high mortality claim rates during the initial policy term. Fraud is a frequently observed challenge in the provision of microinsurance (Biener & Eling, 2012; Dercon et al., 2006; McCord & Osinde, 2005). PMBA had claim verification mechanisms in place. Our data set reveals that a significant share of total claims initially filed were unjustified and subsequently disapproved, e.g. due to non-payment of premiums or ineligible claimants. Disapproved claims have been dropped before estimation. It is not possible to identify or control for potentially remaining fraudulent claims in the data set. Like adverse selection, unidentifiable fraud is implicitly accounted for in empirical estimation of risks. The limits to claim verification and prevalence of fraud thus emphasize the importance of target population-specific and duration-oriented estimation of risks.

Several risk patterns of PMBA’s target population become evident. First,
insureds that opt for long-term coverage exhibit low riskiness. Provision of microinsurance appears to be feasible. Second, lapse has a dominant influence on risks and is contingent on duration. As lapse rates are dependent on the characteristics of a specific target population, this emphasizes the need for empirical estimation on the basis of experience. The pronounced lapse rates at policy renewal intervals emphasize the importance of actively following up insureds at these times (Sinha et al., 2007). Third, there is evidence of potential short-term adverse selection. This supports the literature emphasizing the challenges in offering individual short-term coverage (Wipf et al., 2006). Fourth, risk control measures such as waiting periods appear to be able to mitigate the impact of adverse selection through disincentives for individuals with an elevated riskiness over the short term (Wipf et al., 2006).

(d) Pricing results

Given the empirically estimated transition probability matrix $\hat{P}(s, t)$, an actuarially fair premium of the base insurance product will be derived using equation (4). Assume a lump sum payment payable on death and permanent disability decrements of $d_1(t) = d_2(t) = 1$. Assume a continuous annuity payment for occupancy of hospitalization state of $b_4(t) = 1$ in annual terms. The annual deterministic continuously compounding interest rate is given by $\delta = 0.0770^{24}$ and microinsurance premiums are paid annually in advance (i.e., $\Delta = 1$). Assume eligibility for premium refunds after policy duration $\Delta \theta = 3$ and a premium refund share of $\alpha = 1/2$, as required by Philippine insurance code. The estimated annual fair premium of microinsurance $\hat{\Pi}_r$ is then given by Table 7.
Estimated fair premiums \( \hat{\Pi}_\tau \) are considerably lower than the corresponding premium \( \tilde{\Pi} \) from the original pricing approach introduced in Section a. The estimated premium \( \hat{\Pi}_\tau \) decreases after duration \( \tau = 1 \) which primarily reflects the decreasing riskiness of longer-term insureds. However, as duration \( \tau \) approaches time \( \Delta \theta \), fair premiums increase as the probability of occurrence of eligible premium refunds is increasing. Hospitalization benefits are only of minor importance until \( \tau = 1 \). However, as the premiums decrease with bad mortality risks dropping out after the initial coverage term, hospitalization has a significant share of the total premium. The described relationships also remain pertinent if fair premiums are derived on the basis of upper confidence intervals of the empirically estimated transition probabilities \( \hat{P}(s,t) \) at a confidence level of 95%.

It must be noted that pricing microinsurance using duration-contingent empirically estimated transition probabilities has to account for potential trends in underlying risk forces, either due to fundamental changes of the risks of the target population or behavioral changes (Wipf & Garand, 2006). Duration-oriented pricing with decreasing premiums as discussed above creates economic incentives for longer-term coverage and disincentives for adverse selection. The probable results include a decrease in lapse rates, a decrease in claim costs and, if relevant, a decrease in profits on early withdrawal. Such expectations have to be accounted for; and the pricing model introduced above facilitates simple subjective adjustments of estimated transition probabilities for (a subset of) the transitions. Potential trends in the
risk patterns of the target population also emphasize the importance of continuous monitoring of the target population and subsequent regular adjustments of the premium based on experience (Garand et al., 2012).

To summarize the findings from probabilities estimation and pricing; risks underlying the microinsurance under consideration appear to be driven primarily by the risk patterns of the target population and duration. Given appropriate data, the duration-oriented pricing approach introduced above allows one to derive actuarially fair premiums while capturing the risk characteristics of the target population and accounting for both duration-contingent lapse and short-term adverse selection. It facilitates outreach through lower premiums and it provides disincentives for lapse by rewarding longer-term insureds. As evidenced by the second line of Table 7, the impact of regulatory required premium refunds on the size of the premium can be mitigated.

(e) Covariate analyses

Pricing based on a covariate vector $\mathbf{Z}$ can capture additional risk factors of the target population and therefore allow pricing risks more precisely. Age and gender of the insured are widely used as covariates for risk assessment in a life insurance context. Data on these covariates can be available and their verification can be feasible in a microinsurance context. Let $\mathbf{Z} = (Z_G, Z_A)$ be the insured-specific covariate vector capturing gender and age at inception of the policy, where their domains are given by $z_G \in \{z_f := \text{“Female”}, z_m := \text{“Male”}\}$ for gender and by positive integers for age.

The data set under consideration provides reliable insured-specific infor-
mation on values of the covariate vector $z$. Gender $z_G$ of an insured potentially affects attitudes to risk management, degree of risk aversion, and exposure to risks (Banthia et al., 2012). Hence, riskiness in terms of decrements and lapse might be affected. In the following, the results of an isolated analysis regarding the gender covariate $Z_G$ are reported—hence $Z = Z_G$. Due to the limited size of the data set, the analysis will not be conducted for permanent disability (i.e., transitions to state 1). Fair premiums will not be derived explicitly. However, pricing with covariates based on $\hat{P}_z(s, t)$ is equivalent to the approach discussed above, whereas data are stratified according to $Z = Z_G$.

Figure 4 illustrates probabilities $\hat{P}_{0j,z_G}(0, t)$ for transitions to states $j \in \{2, 3, 4\}$ and a three year duration with 95% confidence intervals, stratified according to $Z_G$. Table 8 gives annual transition probabilities $\hat{P}_{0j,z_G}(s, t)$ for transitions to absorbing states $j \in \{2, 3\}$. Male insureds contributed 35% of underlying total exposure, while female insureds contributed 65%.

[Figure 4 about here.]

[Table 8 about here.]

Higher death probabilities can be observed for male insureds in the target population. The difference is statistically significant until duration $\tau = 1$. From duration $\tau = 1$, the data suffer from sparsity, which results in the lack of potential statistical significance due to wide confidence intervals. While males may have greater exposure to risks causing mortality, given the size of the difference, this is a potential indicator of adverse selection within the population of male insureds. On the other hand, females exhibit higher
estimated probability of hospitalization, indicating higher utilization rates of health benefits (see Figure 4).

Male insureds also exhibit statistically significant higher lapse rates until duration $\tau = 1$. This might be an indicator of gender-specific attitudes to risk management, with women focusing more on long-term risk management compared to men (Banthia et al., 2012). Female insureds in the target population demonstrate higher lapse rates from duration $\tau = 2$ compared to male insureds.

The numeric variable age at inception $Z_A$ can be analyzed by partitioning it into value ranges and assigning these to discrete variable values. Analyses with two value ranges and varying split points have been conducted. However, these did not yield statistically significant results with regard to mortality. While this could be a result of the limited size of the data set, this might also indicate that other factors (e.g., gender) have dominant explanatory power with regard to mortality risk.

Transition intensities appear to be driven to a significant extent by gender — and the underlying risk patterns are complex. Driver for the gender-specific risk patterns are unknown and could be, e.g., gender-specific risk management strategies, adverse selection, or risk exposure. This shows that the use of covariates can facilitate understanding of the risk patterns of a target population and improve risk adequate pricing considerably. However, any implementation of respective risk discrimination in pricing has to consider social aims, community preferences, information availability, transaction costs, and the regulatory environment (Biener, 2012; Garand et al., 2012; Wipf et al., 2006).
5. CONCLUSION

This paper contributes to the field of microinsurance both empirically and theoretically. Empirically, risk patterns of microinsurance and its feasibility are analyzed in an experience study and in statistical analyses of risk forces using data from a Philippine microinsurance institution. Most research to date in this area is based on accounting data or surveys, which are by nature unreliable (see, e.g., Angove & Tande (2012)). Due to availability of a detailed data set, we are able to model the microinsurance scheme on a policy basis and investigate the observable risk patterns and decompose the premium to derive sources of profit and loss. Moreover, risks are empirically estimated and the relevance of policy duration and other covariates for explaining risks is investigated. To our knowledge, this is the first such empirical investigation of the risk patterns and feasibility of microinsurance on the level of individual policies and insureds.

Theoretically, an actuarial pricing model is proposed. The model relies on a general mathematical framework while being simple for pricing in practical application and research. It uses empirically estimated risks from the target population. To our knowledge, this is the first attempt to propose an actuarial pricing approach specifically meant for application in a microinsurance context with its distinct and prevalent challenges.

Provision of microinsurance is found to be feasible with no evidence of high claim rates. However, microinsurance institutions are found to face complex target population-specific and duration-contingent risk patterns, both with regard to riskiness of the population and lapse. Simple actuarial pricing
was found to be unable to arrive at actuarially fair premiums and results in overpricing if financial sustainability is to be achieved.

Given this evidence, we propose a simple duration-oriented pricing model based upon continuous-time Markov chains that allows actuarially fair pricing using empirical transition probabilities. It captures competing risks contingent on fundamental risk factors, covariates, and duration. It is shown that application of the pricing model yields lower and actuarially fair premiums, thereby supporting outreach while ensuring financial sustainability. The application of the proposed non-parametrical Aalen-Johansen approach from survival analysis for empirical estimation of risks shows that it requires minimal information. It can be applied given data sets of limited size, with respective empirical shortcomings. Moreover, it is shown how pricing on the basis of empirically estimated transition probabilities decreases actuarial complexity. Both explicit modeling of policy conditions and actuarial assumptions with regard to risk patterns (e.g., unobservable fraud or adverse selection) are not required.

The main limitation of this paper is its case study nature focusing on the empirical data of one particular microinsurance institution. Hence, the findings of this paper should not be regarded as being generalizable to overall microinsurance markets. However, our findings add to existing literature, and the underlying statistical analyses increases their credibility. Our empirical findings also show that microinsurance risks are target population-specific and that pricing has to be conducted accordingly. The proposed actuarial pricing model and the related empirical estimation approach of risk forces are no panacea to the challenging issue of pricing in microinsurance markets, es-
pecially given their requirement of available experience data and this model’s focus on pricing short-term insurance. However, the pricing approach and its application show that technically rigorous pricing of microinsurance using empirically estimated probabilities is required and feasible, and the model is generalizable.

Other limitations of this paper are as follows. First, the limited size of the data set results in limited statistical significance. The small size of the scheme also potentially limits generalization of findings to larger schemes, especially with regard to high overhead expenses. Second, use of stratification for covariate analyses is straightforward but requires large data sets in practice and has clear limitations in terms of size and complexity of the covariate vector. An alternative approach facilitating analyses of larger covariate vectors in complex state spaces is a regression approach as described by, for example, Andersen & Keiding (2002).

The results of this paper emphasize the importance of actuarial pricing and risk evaluation for achieving outreach and financial sustainability of microinsurance. However, this paper also shows the importance of adequate administrative systems and insurance databases (see also, e.g., McCord & Osinde (2005); Wipf & Garand (2006). These systems can act as a significant lever through improvement operational efficiency, risk evaluation, and actuarial pricing. Provision of respective support could be advisable for policymakers in the future.

This paper is a first step for future research in the area of actuarial pricing models tailored to a microinsurance context. A logical next step for more extensive analysis would be an application of the actuarial pricing model using
larger data sets. Through cross-validation of estimated transition probabilities, the impact of corresponding pricing on financial sustainability could be determined, thus the wider practical feasibility of this risk estimation and pricing approach could be investigated. Moreover, using techniques facilitating risk evaluation such as bootstrapping and credibility models as described by Biener (2012), along with the proposed pricing approach, promises an increase of practical feasibility. The results of such research would also contribute to the general understanding of risk patterns in microinsurance markets and serve as a lever to related areas in the field of microinsurance.

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Notes

1See Hermes & Lensink (2011) for a review of recent empirical evidence.

2See Biener & Eling (2012) for a recent comprehensive overview.

3PMBA was founded as PAKISAMA Mutual Benefit Association, Inc. Its name was eventually changed to Partnership MBA, Inc. For a detailed discussion of PMBA, see Almazan (2005).

4See, for example, IAIS-CGAP (2007); Roth et al. (2007) for a detailed survey of contemporary microinsurance concepts and products.

5Benefits for other risks have been added without an explicit actuarial basis or on the basis of reinsurance considerations. These risks only resulted in minor claims and were subsidized through actuarial conservatism in both pricing of mortality claim costs and premium refund provisioning.

6Additionally, the median age of the population was estimated conservatively at 45 years while actual median age of the population at policy inception was 40 years.

7See sections 227 (f) (1) and 230 (f) of the 1974 Philippine insurance code.

8This is necessary as no reliable information on overhead and profit ex-
Due to data limitations the following assumptions have been made. First, premium payments are not based on observed cash flows but were derived from membership data concerning annual premiums, coverage amounts and terms of coverage assuming a uniform distribution of payment of premiums over terms of coverage. If annual premiums were unavailable for a specific policy, the arithmetic average of the annual premium of the population was allocated. Second, as a prudent approximation, claims are assumed to be paid at the time a claim is initially filed. Third, as reliable information on actual payments related to reinsurance was missing, net cost of reinsurance has been modeled on the basis of the reinsurance terms. Fourth, for derivation of time value the same interest basis as in original actuarial pricing is assumed. These assumptions also hold with respect to the remainder of the experience study.

Due to lack of information on payment of eligible premium refunds, in the following full payment is assumed at time of withdrawal of eligible insureds.

No reliable information on the exact number of insureds and coverage amount per policy was available for part of the population for the experience studies. Expected claims have been derived for these policies assuming the average number of insureds per policy and coverage amounts in the population. Sensitivity analyzes on these assumptions did not yield materially different results.

Low reinsurance expenses given low claim experience is a result of a favorable profit sharing clause in the reinsurance agreement.
An intercompany mortality table frequently used in actuarial pricing in the Philippines as provided by the Actuarial Society of the Philippines was used. Due to lack of risk data on the other decrements this analysis was conducted only for mortality. Note, that intercompany experience rates are from companies in the traditional insurance sector. Hence, overall population mortality rates including exposure of uninsured lives can be expected to be even more pronounced. This supports the findings reported in the following.

Combining short-term life insurance with long-term savings components is not uncommon, but these instruments are often reported to be perceived as being of limited value to insureds (IAIS-CGAP, 2007).


The structure of the model could easily be generalized to allow multiple successive states. This is generally not required for microinsurance pricing and would increase complexity. For general actuarial Markov multi-state models see Haberman & Pitacco (1999); Wolthuis (2003).

Microinsurance premiums are usually periodically charged in advance due to short-term coverage. However, equation (2) could easily be adjusted to allow continuous or discrete payment of premiums during coverage, resulting in a differential equation (Haberman & Pitacco, 1999).

See Haberman & Pitacco (1999) for a general representation in terms of Markov multi-state models.
Hence, estimates from one type of policy can only be applied to other types of policies following adjustments and given detailed knowledge of policy conditions.

See Aalen & Johansen (1978); Andersen et al. (1993).

See Andersen et al. (1993, section IV.4.1) for a detailed discussion and interpretation of the estimator.

An example would be simultaneous allowance of multiple temporary benefits, e.g., independent sickness and hospitalization benefits. This can be remedied by introducing an additional state capturing simultaneous hospitalization and sickness.

The definition of the transition probability matrix $P(s, t)$ above results in lapses occurring at a policy renewal interval $t$ counting towards this $P(s, t)$. These lapses therefore do not bias the succeeding transition probability matrix $P(t, t + \Delta)$.

This corresponds to the discrete interest basis in original pricing of PMBA to allow comparison.

For many target populations even the verification of age is reported as being difficult (IAIS-CGAP, 2007).

Note that the discretization could be finer; given a large data set, annual age intervals could be used.
Figure 1: Illustration of possible transitions within state space $N$. The state space comprises a transient starting state and $K$ transient or absorbing decrement- and membership-related states.
Figure 2: Illustration of possible transitions within state space $N$. The state space comprises two transient states and three absorbing states.
Figure 3: Transition probabilities $\hat{P}_{0j}(0, t)$ with confidence intervals at 95% level for all relevant transitions for three years’ duration. Estimated transition probabilities are found to be contingent on duration. The importance of lapse as a competing risk is starkly evident.
Figure 4: Transition probabilities $\hat{P}_{0j,z}(0, t)$ by gender with confidence intervals at 95% confidence level for three years’ duration. Transition probabilities appear to be gender-specific.
Table 1: Development of PMBA in terms of number of policies. Slow take-up and high lapse rates are observable, thus outreach and sustainability are limited.

<table>
<thead>
<tr>
<th>Year</th>
<th>Total year end</th>
<th>New policies</th>
<th>Lapses</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>1 705</td>
<td>1 705</td>
<td>0</td>
</tr>
<tr>
<td>2004</td>
<td>4 033</td>
<td>2 333</td>
<td>5</td>
</tr>
<tr>
<td>2005</td>
<td>4 745</td>
<td>1 424</td>
<td>712</td>
</tr>
<tr>
<td>2006</td>
<td>3 319</td>
<td>312</td>
<td>1 738</td>
</tr>
<tr>
<td>2007</td>
<td>3 175</td>
<td>2 128</td>
<td>2 272</td>
</tr>
<tr>
<td>2008</td>
<td>2 667</td>
<td>1 536</td>
<td>2 044</td>
</tr>
<tr>
<td>2009</td>
<td>900</td>
<td>245</td>
<td>2 012</td>
</tr>
<tr>
<td>2010</td>
<td>701</td>
<td>280</td>
<td>479</td>
</tr>
<tr>
<td>2011(^a)</td>
<td>404</td>
<td>12</td>
<td>309</td>
</tr>
</tbody>
</table>

\(^a\)2011 year end as of September 13, 2011.
Table 2: Development of PMBA in terms of number of valid claims and percentage share in total benefit paid. Claim numbers appear to be relatively stable and strongly related to underlying exposure. Mortality is dominant in terms of benefits paid out.

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Mortality</th>
<th>Disability</th>
<th>Hospitalization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#</td>
<td>%</td>
<td>#</td>
<td>%</td>
</tr>
<tr>
<td>2003</td>
<td>7</td>
<td>54</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2004</td>
<td>46</td>
<td>12</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>2005</td>
<td>38</td>
<td>74</td>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>2006</td>
<td>30</td>
<td>90</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2007</td>
<td>18</td>
<td>88</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2008</td>
<td>25</td>
<td>84</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>2009</td>
<td>37</td>
<td>93</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2010</td>
<td>22</td>
<td>85</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2011a</td>
<td>9</td>
<td>84</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>232</td>
<td>73</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

*a2011 year end as of September 13, 2011.*
Table 3: Experience of PMBA, discounted to inception. Significant profitability can be observed. This can imply both overpricing and a population with an unexpectedly low risk profile.

<table>
<thead>
<tr>
<th>Time value in % of premium as of inception</th>
<th>Claim costs by decrement</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Premium</strong></td>
<td>100.0</td>
</tr>
<tr>
<td>Total claim costs</td>
<td>5.0</td>
</tr>
<tr>
<td><em>Mortality</em></td>
<td>4.1</td>
</tr>
<tr>
<td><em>Disability</em></td>
<td>0.2</td>
</tr>
<tr>
<td><em>Hospitalization</em></td>
<td>0.7</td>
</tr>
<tr>
<td>Reinsurance expenses, net</td>
<td>4.5</td>
</tr>
<tr>
<td>Premium refunds paid</td>
<td>17.5</td>
</tr>
<tr>
<td>Remaining premium</td>
<td>3.4</td>
</tr>
<tr>
<td>refund entitlement</td>
<td></td>
</tr>
<tr>
<td><strong>Gross Profit</strong></td>
<td>69.7</td>
</tr>
</tbody>
</table>
Table 4: Decomposition of profits and losses over expectations reflected in pricing and discounted to time of inception. Gain on early lapse due to high lapse rates and mortality gain are the dominant sources of profit. Reduced mortality gain on Philippine inter-company mortality implies considerable overpricing due to uncertainty. The persistence of mortality gain implies presence of a population with a general low risk profile.

<table>
<thead>
<tr>
<th>Realized profit or loss due to</th>
<th>Time value in % of premium as of inception</th>
<th>Gain by decrement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total claim gain</td>
<td>39.4</td>
<td></td>
</tr>
<tr>
<td>Mortality gain</td>
<td>40.3</td>
<td></td>
</tr>
<tr>
<td>Disability gain</td>
<td>-0.2</td>
<td></td>
</tr>
<tr>
<td>Hospitalization gain</td>
<td>-0.7</td>
<td></td>
</tr>
<tr>
<td>Reinsurance cost, net</td>
<td>-4.5</td>
<td></td>
</tr>
<tr>
<td>Gain on early lapse</td>
<td>29.2</td>
<td></td>
</tr>
<tr>
<td><strong>Total profit or loss</strong></td>
<td><strong>64.1</strong></td>
<td></td>
</tr>
<tr>
<td>Mortality gain on intercompany mortality</td>
<td></td>
<td>26.2</td>
</tr>
</tbody>
</table>
Table 5: Annual estimated transition probabilities $\hat{P}_{0j}(s,t)$ for all relevant transitions for three years’ duration. Potential adverse selection is observable resulting from individual short-term coverage.

<table>
<thead>
<tr>
<th>Transition probability</th>
<th>$(s,t)$</th>
<th>$(0,1)$</th>
<th>$(1,2)$</th>
<th>$(2,3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{P}_{00}(s,t)$ (Persisting active membership)</td>
<td></td>
<td>0.7806</td>
<td>0.2729</td>
<td>0.3167</td>
</tr>
<tr>
<td>$\hat{P}_{01}(s,t)$ (Permanent disability)</td>
<td></td>
<td>0.0002</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\hat{P}_{02}(s,t)$ (Death)</td>
<td></td>
<td>0.0044</td>
<td>0.0005</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\hat{P}_{03}(s,t)$ (Lapse)</td>
<td></td>
<td>0.2148</td>
<td>0.7266</td>
<td>0.6829</td>
</tr>
<tr>
<td>$\int_s^t \hat{P}_{04}(s,u)du$ (Hospitalization)</td>
<td></td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
</tbody>
</table>
Table 6: Annual estimated transition probabilities of death $\hat{P}_{02}(s, t)$ for three years’ duration. Comparably high probabilities of death are observed during the initial policy term with a significant subsequent reduction. The difference with the assumed death probability in original pricing $\tilde{P}_{02}$ is statistically significant after the initial policy term.

<table>
<thead>
<tr>
<th>Transition probability</th>
<th>$(0,1)$</th>
<th>$(1,2)$</th>
<th>$(2,3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{02}(s, t)$</td>
<td>0.0044</td>
<td>0.0005***</td>
<td>0.0005***</td>
</tr>
<tr>
<td>$\tilde{P}_{02}$</td>
<td>0.0049</td>
<td>0.0049</td>
<td>0.0049</td>
</tr>
</tbody>
</table>

*Difference significant at the 0.10 level.
**Difference significant at the 0.05 level.
***Difference significant at the 0.01 level.
Table 7: Estimated fair premium $\hat{\Pi}_\tau$. The fair premium is found to be considerably lower than from original pricing. The model can price competing risks of a specific target population actuarially fairly and capture the explanatory power of duration.

<table>
<thead>
<tr>
<th></th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Estimated fair premium $\hat{\Pi}_\tau$</td>
<td>0.0046</td>
</tr>
<tr>
<td>Premium refund provision share in $\hat{\Pi}_\tau$</td>
<td>0.0250</td>
</tr>
<tr>
<td>Hospitalization benefit share in $\hat{\Pi}_\tau$</td>
<td>0.0109</td>
</tr>
<tr>
<td>$\hat{\Pi}_\tau$ at upper 95% confidence intervals$^a$</td>
<td>0.0071</td>
</tr>
<tr>
<td>Equivalent original pricing premium $\tilde{\Pi}$</td>
<td>0.0094</td>
</tr>
</tbody>
</table>

$^a$The fair premium was derived using upper confidence intervals of the transition probabilities $\hat{P}(s,t)$ for decrement-related transitions and for persistence of active membership until policy duration $\Delta \theta$, therefore estimating a fair premium reflecting maximal expected claims cost and minimal profit on early lapse at a confidence level of 95%.
Table 8: Annual estimated transition probabilities $\hat{P}_{0j,2}(s,t), j \in \{2,3\}$ by gender for three years’ duration. Differences in transition probabilities potentially reflect gender-specific risk patterns and are partly statistically significant.

<table>
<thead>
<tr>
<th>Transition probability</th>
<th>(s, t)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0,1)</td>
<td>(1,2)</td>
<td>(2,3)</td>
<td></td>
</tr>
<tr>
<td>$\hat{P}_{02,zm}(s,t)$</td>
<td>Death males</td>
<td>0.0116***</td>
<td>0.0013</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\hat{P}_{02,zf}(s,t)$</td>
<td>Death females</td>
<td>0.0010</td>
<td>0.0003</td>
<td>0.0007</td>
</tr>
<tr>
<td>$\hat{P}_{03,zm}(s,t)$</td>
<td>Lapse males</td>
<td>0.4587***</td>
<td>0.7009</td>
<td>0.5336***</td>
</tr>
<tr>
<td>$\hat{P}_{03,zf}(s,t)$</td>
<td>Lapse females</td>
<td>0.1005</td>
<td>0.7322</td>
<td>0.7626</td>
</tr>
</tbody>
</table>

*Difference significant at the 0.10 level.
**Difference significant at the 0.05 level.
***Difference significant at the 0.01 level.
Appendix

A. Experience study methodology

Below we describe the methodology of the experience study. Assume pricing of the base insurance product as described in Section 2. Let \( \tilde{q} \) be the originally assumed annual mortality rate. Assume uniform distribution of claims over the coverage terms and mid-year discounting. Let \( \tilde{v} \) be the corresponding discount factor. Also, a share of \( \alpha \in [0, 1] \) of premium is provisioned for premium refunds.

According to the scheme’s actuarial report and ignoring expenses, the original premium \( \tilde{\Pi} \) for coverage of mortality with a lump sum benefit of \( \tilde{c} \) was then derived as

\[
\tilde{\Pi} = \frac{\text{Expected mortality claims cost}}{\text{Premium refund provision}} \times \tilde{c} \tilde{q} \tilde{v} + \alpha \tilde{\Pi},
\]

implying \( \tilde{\Pi} = (\tilde{c} \tilde{q} \tilde{v}) / (1 - \alpha) \). Due to the short-term nature of coverage, this premium can be decomposed into the expected claims cost and the premium refund provision for the given period based on experience.

Actuaries have developed methodologies for analyzing the ex-post profit or loss of insurance products (see, e.g., Easton & Harris (1999)). We follow the basic experience study approach, but it is modified as the data from the scheme lacked reliable information on investment income. We therefore cannot provide any results on investment profit or loss (interest gain). Thus, as premium pricing reflects expected investment income these expectations have to be sorted out of premiums. Hence, the actual cash flows were discounted
with the actuarially assumed discounting factor $\tilde{v}$ assuming uniform annual distribution of cash flows. Moreover, premiums are decomposed based on experience into their allocation to actual costs or profit and loss at the time of occurrence of the initial premium payment.

Our method to analyze profit or loss is as follows. Let $m \in \{0, \alpha\}$ be the share of actual premium refunded and $n \in \{0, \alpha\}$ be the share of actual premium still provisioned for refunding. Note that for a given individual insured at least $m$ or $n$ must be zero. Given an actual premium $\tilde{\Pi}$ in a past period for a given insured, $\tilde{\Pi}$ can be decomposed based on experience into

$$\tilde{\Pi} = (\text{Actual mortality claims cost} +$$

$$+ \max (m, n) \tilde{\Pi} +$$

$$+ \text{other costs not reflected in pricing} +$$

$$+ \text{gross profit or loss}) \tilde{v}.$$  

This decomposition is given above in Table 3 aggregated over the insurance population in terms of time value as of inception. Time values were derived using $\tilde{v}$.

The sources of profits and losses over expectations for a given period can then be determined by comparing expectations with actuals using

$$\text{Profit or loss} = (\tilde{c} \tilde{q} - \text{actual mortality claims cost}) +$$

$$+ \left(\tilde{\Pi} (\alpha - \max (m, n)) \right) -$$

$$- \text{other costs not reflected in pricing.}$$
This decomposition of profits and losses compared to expectations (reflected in pricing) is given above in Table 4 aggregated over the insurance population. Mortality gain assuming population mortality was derived using the same methodology. However, the mortality rate from pricing \( \tilde{q} \) was replaced with Philippine population mortality rates \( q_x \), which are dependent on the age \( x \) of the insured at mid-term of the period under consideration. The result is also provided in Table 4.

Note that as a result of this methodology, the premium refunds paid, the profits on early lapse and the premium refund entitlements are not given in terms of time of their economic occurrence but in terms of time of occurrence of the corresponding premium payment \( \Pi \).