

# Diversification and Risk in Superannuation Funds.

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Yuri Salazar<sup>1</sup>, Robert Bianchi<sup>1</sup>, Michael Drew<sup>1</sup>, Stefan Trück<sup>1,\*</sup>

<sup>a</sup>*Faculty of Business and Economics, Macquarie University, NSW, Australia, 2109*

<sup>b</sup>*Griffith Business School, Griffith University, QLD, Australia, 4111*

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## Abstract

This paper deals with the question of which factors have the highest impact on the wealth outcome at retirement in a superannuation context. We focus on the last 10 years prior to retirement as these are the most crucial in determining the wealth outcome. We evaluate the performance of different lifecycle investment strategies for superannuation funds under different scenarios of market conditions, the initial accumulated value of the portfolio, salary and salary growth rates, interest rates and contribution levels. Our analysis is based on modelling the dependence structure of the different risk factors. We apply parametric and non-parametric techniques to evaluate different strategies by examining wealth outcomes and risk-adjusted performance measures. Lifecycle strategies have become increasingly popular for defined contribution plans and have the advantage of adjusting the portfolio composition towards the age of retirement. Optimal strategies are also able to consider different scenarios.

*Keywords:* Risk Management, Portfolio Optimisation, Copula, Nonparametric Estimation, Superannuation Funds

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<sup>\*</sup>Corresponding author. Stefan Trück Address: Center for Financial Risk, Faculty of Business and Economics, Macquarie University, NSW, Australia, 2109, Telephone number: +61 2 9850 8483, Email: stefan.trueck@mq.edu.au

*Email addresses:* [yuri.salazar@mq.edu.au](mailto:yuri.salazar@mq.edu.au) (Yuri Salazar), [r.bianchi@griffith.edu.au](mailto:r.bianchi@griffith.edu.au) (Robert Bianchi), [michael.drew@griffith.edu.au](mailto:michael.drew@griffith.edu.au) (Michael Drew), [stefan.trueck@mq.edu.au](mailto:stefan.trueck@mq.edu.au) (Stefan Trück), (Stefan Trück)

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## 1. Introduction

The adequate estimation of risk related measures is fundamental in the analysis of strategies for superannuation funds. In particular, it is crucial that these measures are reliable when dealing with extreme losses. To this end, it is necessary to have a good understanding of the different risk factors affecting the portfolio as well as their corresponding dependence structure. In this work we focus on the analysis of lifecycle strategies and their performance during the last 10 years prior to retirement. To this end we consider parametric and a nonparametric models.

Lifecycle strategies have become increasingly popular for defined contribution plans. In the Australian context of MySuper products, these strategies have replaced the Target Risk Funds (TRFs), in which the weight given to different assets is always constant. Lifecycle strategies have the advantage of adjusting the portfolio composition towards the age of retirement. A basic example of Lifecycle strategies are the Target Date Funds (TDFs). TDFs typically switch from growth to defensive assets. The adjustment of the portfolio done by TDFs is known as “set it and forget it”. An example of a TDFs strategy for a portfolio of stocks and bonds is to give a weight of 40% to stocks during the first 20 years and then increase it linearly during the last 20 years to 80% at retirement. In recent times more dynamic lifecycle strategies have appeared, these strategies have the flexibility to take into account contribution levels, the accumulated value of the portfolio and current market conditions, see Basu and Drew (2009) and Basu et al. (2009).

There is an ongoing debate in literature regarding the optimal nature of Lifecycle strategies. Many authors advocate for the standard approach in which there is a switch from growth to conservative assets towards the age of retirement. They argue that is too risky to invest in growth assets in the final years before retirement. However, it has also been argued that by doing this the investor misses out on the opportunity to maximise his wealth when his balance is higher. see Basu and Drew (2009).

In this work we analyse the role of several factors in determining the final wealth at retirement during the last 10 years of contributions. We not only consider different Lifecycle strategies as other studies but also cover a wide range of scenarios of market conditions, accumulated values, salaries and salary growth rates as well as interest rates and contribution levels.

In order to analyse different strategies and scenarios we consider both the individual assets

as well as their dependence structure. The first models we consider are non-parametric and are based on historical simulation. For these models we consider different bootstrap and simulation methods. To analyse the individual assets we fit ARMA-GARCH models to account for the serial autocorrelation and the stochastic volatility. For the dependence structure amongst assets we consider a dynamic copula approach and fit several models that take into account the changing nature of dependence structure.

Our parametric approach enables us to incorporate the different factors that affect the final wealth at retirement. With this approach we aim to provide an adequate analysis of their role. Dynamic copula models are much more flexible than other parametric models such as the Gaussian distribution. They are able to address the “heavy tailedness” of the data and the changing nature of the dependence structure of assets in times of economic stress.

We consider several strategies in our study from TRFs to TDFs and other strategies that take into account the performance of the portfolio to determine the optimal weights. Roughly speaking the difference between the examples we consider is the weight given to growth and conservative assets.

The rest of this paper is divided into three sections. In Section 2 we discuss the methodology we follow. We include important results as well as examples of copula families and their corresponding tail copulas. At the end of this section we analyse the nonparametric estimation of the tail copula. In Section 3 we assess our methodology in an empirical application. In Section 4 we conclude and suggest future lines of research.

## **2. Methodology**

This section provides a brief review of approaches that will be used in the empirical analysis to model the dynamic behaviour of the returns of a superannuation portfolio. We distinguish between non-parametric and parametric approaches. For the former we apply a standard bootstrap, the block bootstrap and the stationary bootstrap to simulate the joint returns of the asset classes and an EWMA. For the latter we suggest a Dynamic Copula Model in combination with an ARMA-GARCH model for the marginal return series as well as a dynamic copula approach.

### *2.1. Nonparametric Approaches*

The first modelling method selected in this study is a form of block bootstrap simulation, see Kunsch (1989). We first consider a standard bootstrap resampling method. The empir-

ical monthly return vector of two asset classes in the data set are randomly resampled with replacement to generate asset class return vectors for each of the 10-year investment horizon confronting the different strategies. Since we randomly draw rows (representing years) from the matrix of asset class returns, we are able to retain cross-correlation between the asset class returns as given by the historical data series while assuming that returns for individual asset classes are independently distributed over time. Because the resampling is done with replacement, a particular data point from the original dataset can appear multiple times in a given bootstrap sample. This is particularly important in examining the probability distribution of future outcomes. For example, September, 1987 is the worst month for the stock market in our 44-year dataset. In that year the return from stocks was -42.13%, while bonds produced returns of 1%. Although this is only one observation in a 44 years' worth of data, a bootstrap sample of 10 years of monthly returns can include the return observation for September, 1987 many times in a sequence. Similarly, return observations for other months, good or bad, can also be repeated a number of times within a bootstrap sample. Because this method allows for inclusion of such extreme possibilities –such as a -42.13% return concurring a number of times in a particular 10-year return path- a much wider range of future possibilities can be captured. We consider two approaches to this model. In the first approach we assume that future scenarios follow a uniform distribution from previous scenarios. In the second approach we assign different weights to the scenarios. Under this weighting scheme we consider the probability of past events decline exponentially as we go back in time. Under this model, given a sample of size  $n$ , sorted chronologically, the weight given to the  $i$ -th observation is

$$\frac{\lambda^{n-i}(1-\lambda)}{1-\lambda^n},$$

where  $0 < \lambda \leq 1$  is the weighting parameter. Note that the weight or probability assigned to the  $i$ -th scenario is  $\lambda$  times the weight assigned to the  $(i+1)$ -th. Also, when  $\lambda \rightarrow 1$  the weights tends to  $\frac{1}{n}$ , for all scenarios, so this is equal to the first approach. Following similar studies we set  $\lambda = 0.995$ .

The asset class return vectors obtained by bootstrap resampling are combined with their respective weightings under each asset allocation strategy to generate portfolio returns for each year in the 44 year horizon. The simulation trial is iterated a large number of times for lifecycle strategies.

In spite of being able to capture cross-correlation, a major drawback of this method is its incapability of capturing autocorrelation. Because of this in a second model we consider a

block bootstrap in which more than one element of the sample is taken to account for the serial autocorrelation. The block we consider is two years of data. This means that when a point is sampled a block of two years consisting of the point and the following values, allowing the individual autocorrelations to be taken into account.

In these two historical simulation methods we assume that each of the elements of the sample has the same probability to be sampled. To complement this we also consider an exponentially weighted moving average (EWMA). This model considers a higher probability to the most recent events. Models based on historical simulation have the advantage of letting the data speak for itself by avoiding a parametric assumption; however they do not allow for record values and tend to underestimate extreme events among other drawbacks.

## *2.2. Parametric Approaches*

Traditionally, parametric models for dependence have been based on the assumption of a multivariate distribution. This assumption restricts not only the dependence structure but also the marginal distributions. Further to this, these models generally rely on the correlation between the series to model dependence. The pitfalls of relying on the correlation coefficient when measuring dependence have been extensively reported in financial literature.

As a response to this, copula functions have been used more recently to quantify dependence in risk related contexts. These models have the flexibility of modelling the dependence structure without restricting the marginals, allowing for a separation of the two things, see e.g. Nelsen (2006). In more recent times dynamic copula models have emerged to also account for the changing nature of dependence structure through time.

The use of copula and dynamic copula models have proven to be very effective in modelling dependence. However, they are based on the assumption of having an independent and identically distributed random sample, an assumption that is often violated. In order to account for this models that account for stochastic volatility, such as GARCH, have been successfully used.

Our approach is based on a Dynamic Copula model with ARMA-GARCH innovations. We now describe the model into more detail.

### *2.2.1. ARMA-GARCH- Models for Individual Assets*

The first step in our model is to find an appropriate model for the marginals. Thus, we need to estimate the parameters for the conditional mean and conditional variance equations to account for their stochastic nature. We focus on different ARMA-GARCH specifications for

each of the considered series and abstain from using additional exogenous variables. In order to avoid overfitting, the best model is chosen based on Akaike's Information Criterion (AIC) and the Ljung Box Test  $p$ -values. Our model gives more flexibility to modelling conditional correlations as it involves less complicated calculations in comparison to e.g. the GARCH-BEKK model by Engle and Kroner (1995). Considering their effectiveness, we assume that the variance of the individual series of asset returns can be modelled by ARMA-GARCH models.

### *2.2.2. Dynamic Copula Models*

A copula is a function that combines marginal distributions to form a joint multivariate distribution. The concept was initially introduced by Sklar (1959) but has only gained strong popularity for use in modelling financial or economic variables in the last two decades. For an introduction to copulas see e.g. Joe (1997) or Nelsen (2006), for applications to various issues in financial economics and econometrics, see, e.g. Cherubini et al. (2004), Frey and McNeil (2003), Patton (2006), just to name a few. As shown by Cherubini and Luciano (2001), Jondeau and Rockinger (2006), Junker et al (2006) and Luciano and Marena (2003), the use of correlation usually does not appropriately describe the dependence structure between financial assets and this could lead to inadequate risk measurement. Ang and Chen (2002) and Longin and Solnick (2001) empirically demonstrate that, in general, asset returns are more highly correlated during volatile markets and during market downturns. Dowd (2004) suggests that the strength of the copula framework is attributable to not requiring strong assumptions about the joint distributions of financial assets in a portfolio. Jondeau and Rockinger (2006) and Patton (2006) illustrate that copulas can be applied, not only directly to the observed return series but also, for example, to vectors of innovations after fitting univariate GARCH models to the individual return series. Overall, the use of copulas offers the advantage that the nature of dependence can be modelled in a more general setting than using linear dependence only. Copulas also provide a technique for decomposing a multivariate joint distribution into marginal distributions and an appropriate functional form for modelling the dependence between the asset returns.

In the following paragraphs we will briefly summarize the basic ideas and properties of copulas. For a definition of copulas we refer e.g. to Sklar (1959) or Nelsen (2006). Let  $X_1$  and  $X_2$  be continuous random variables with distribution functions  $F_1$  and  $F_2$  and joint distribution function. Following Sklar (1959), there exists function  $C$  such that:

$$C(F_1(x_1), F_2(x_2)) = F(x_1, x_2)$$

Further setting  $\mathbf{U}$ , the function is the distribution of whose margins are uniform on  $[0, 1]$ . This function  $C$  is called a copula and denotes a joint cumulative density function (CDF) of the  $d$  independent,  $U \sim [0; 1]$  distribution functions. Moreover, if the marginal distributions  $F_1$  and  $F_2$  are continuous, the copula function  $C$  is unique, see Sklar (1959), and the copula is an indicator of the dependence between the variables  $X_1$  and  $X_2$ .

The literature suggests a wide range of different copulas, see e.g. Joe (1997) or Nelsen (2006) for an overview of the most common parametric families of copulas. In the following we will limit ourselves to a description of a number of copula families that will be used further on in the empirical analysis. Among the most commonly used copulas in finance are the Gaussian, Student t, Clayton and Gumbel copula.

Probably the most intensively applied copulas in financial applications are the Gaussian and Student t copula. The Gaussian copula is constructed using a multivariate normal distribution and is defined as

$$C(u_1, u_2) = \int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{|R|}} \exp\left(-\frac{\mathbf{x}'R^{-1}\mathbf{x}}{2}\right) d\mathbf{x},$$

with  $\Phi$  the standard univariate Gaussian distribution,  $\mathbf{u} = (u_1, \dots, u_2)$  and  $\mathbf{x} = (x_1, \dots, x_2)'$ . The normal copula correlates the random variables rather near the mean and not in the tails. Therefore, it fails to incorporate tail dependence which can often be observed in financial data. In order to add more dependence in the tails, alternatively, the Student t-copula can be applied. The Student's  $t$  copula is well known for accounting for stylised facts such as fat tail and the presence of tail dependence, see Joe (1997). The Student's  $t$  copula with  $\nu$  degrees of freedom and correlation matrix  $R$  is expressed in terms of integrals of its corresponding density  $t_{\nu,R}$ .

$$C(u_1, u_2) = \int_{-\infty}^{t_{\nu}^{-1}(u_1)} \int_{-\infty}^{t_{\nu}^{-1}(u_2)} \frac{\Gamma\left(\frac{\nu+2}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) \pi\nu\sqrt{|R|}} \left(1 + \frac{\mathbf{x}'R^{-1}\mathbf{x}}{\nu}\right)^{-\frac{\nu+2}{2}} d\mathbf{x},$$

with  $t_{\nu}$  the Student's  $t$  distribution with  $\nu$  degrees of freedom.

Both the Gaussian and Student t copula are symmetric. However, often financial variables are observed to exhibit tail-dependence in only one of the tails, either the upper right or lower left edge of the data. For example, tail-dependence in the lower left tail indicates that the two variables have a tendency to simultaneously yield high negative returns. However, in situations

where returns from one of the variables are highly positive the other financial variable may not be affected to the same extent. To model asymmetric tail-dependence, so-called Archimedean copulas can be used, see e.g. Cherubini et al (2004). In this work we use two of the most prominent members of the family of Archimedean copulas, the Clayton and Gumbel copula,. The Clayton copula is defined as

$$C_\theta(u_1, u_2) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-\frac{1}{\theta}},$$

$\theta > -1$ .

The Gumbel copula is defined as

$$C_\theta(u_1, u_2) = \exp\left(-\left[(-\log u_1)^\theta + (-\log u_2)^\theta\right]^{\frac{1}{\theta}}\right),$$

$\theta > 1$ . These copulas are both asymmetric. Note that the higher the value of parameter  $\theta$ , the greater is the degree of dependence between the considered variables. For further properties and examples of elliptical and Archimedean copulas and the construction of such copulas by using generator functions, we refer to Cherubini et al (2004) or Nelsen (2006).

Note that due to the possible heteroskedastic behaviour of the return series, in the empirical analysis we will not apply copula models to the observed returns directly. Instead the copula functions will be estimated using the vectors of innovations after fitting univariate ARMA-GARCH models to the individual return series e.g. Jondeau and Rockinger (2006) and Patton (2006).

After fitting the stochastic process for the marginal return series, a dynamic conditional copula model can be estimated to specify the dynamics of the copula dependence parameter. Patton (2006) proposes observation-driven copula models where the time-varying dependence parameter is a parametric function of transformations of the lagged data and an autoregressive term. Then, using the marginal distribution of the standardized residuals the dynamics of the parameters for the Gaussian, Student's  $t$ , Gumbel or Clayton copula can be specified. For the dynamics of the correlation for the Gaussian and Student's  $t$  copula, following Patton (2006), we apply the following model:

$$\rho_t = \Lambda \left\{ \omega + \beta \rho_{t-1} + \alpha \frac{1}{12} \sum_{j=1}^{12} F^{-1}(u_{t-j}) F^{-1}(v_{t-j}) \right\}, \quad (1)$$

with

$$\Lambda(x) = (1 + e^{-x})^{-1}.$$

In a similar manner, the model for the two Archimedean copulas can be specified as:

$$\tau_t = \Lambda \left\{ \omega + \beta \tau_{t-1}^U + \alpha \frac{1}{12} \sum_{j=1}^{12} |u_{t-j} - v_{t-j}| \right\}, \quad (2)$$

with

$$\Lambda(x) = \frac{(1 - e^{-x})}{(1 + e^{-x})}.$$

Note that in this specification, the previous value of the parameter is used as a regressor to capture the persistence in the dependence parameter, while the mean of the last 12 observations of the transformed variables and , previous observations are used to capture any variation in dependence between the innovation series. The link function  $\Lambda$  denotes a transformation which ensures that the correlation parameter will always be in the interval  $(-1, 1)$ . The suggested dynamic copula models can then be estimated using maximum likelihood. In the following section we use the methodology described so far to model an empirical application.

### 3. Empirical Application

In this section we use the methodology described so far to determine the effectiveness of four MySuper strategies, we now define the procedure into detail.

#### 3.1. The Data

In order to set up a comprehensive analysis of different superannuation investment strategies, it is of great importance to identify the most important assets used to construct the different portfolios. By having a simple portfolios of representative assets it is possible to focus on the main factors that affect the final wealth at retirement. Because of this, for our empirical analysis we consider a portfolio consisting of Australian stocks and bonds. These two assets are, generally speaking, the main constituents of MySuper investment strategies

In this section we investigate the dependence structure between logarithmic returns from Australian stocks and bonds. We will consider monthly, the time period extends from January, 1970 to December, 2013. Data was obtained from DataStream. The logarithmic returns we consider are calculated from the original price series. In the following we will refer to the return series simply as stocks and bonds.

One may assume that the relationship between stocks and bonds is particularly strong. The correlation between the two assets for the whole period is 25.62%. Later on this work we

<b>Series</b>	<b>Mean</b>	<b>Median</b>	<b>St. Dev.</b>	<b>Min.</b>	<b>Max.</b>	<b>Skew.</b>	<b>Kurtosis</b>
<b>Stocks</b>	0.0088	0.0132	0.1726	-0.5470	0.0540	-2.2167	24.0780
<b>Bonds</b>	0.0068	0.0074	0.1357	-0.1090	0.0189	-0.1715	12.5853

Table 1: Descriptive Statistics for Logarithmic Returns of Australian Stocks and Bonds from January 1970 to December 2013

will investigate the dependence structure between these assets into more detail. This includes analysing its changing nature. Table 1 provides descriptive statistics for the log-returns of the stocks and bonds. We consider the mean, median, standard deviation, minimum, maximum, skewness and kurtosis of the corresponding logarithmic returns. All of the descriptive statistics of stocks are much higher, in absolute terms, than those of the bonds. Table 1 exemplifies the high return and more volatile nature of stocks with respect to bonds. There are several factors that influence these results. We are not going to focus on these factors but rather on the portfolios generated with them.

### 3.2. Modelling the Marginals

To analyse the individual asset we implement a two-stage procedure. In the first stage, we fit ARMA-GARCH models to each series and obtain the standardized residuals for each series. These residuals are then assumed to be independent and identically distributed and the generalised inverse distribution is then used to generate a uniformly distributed sample, suitable for the copula analysis. Note that one could also model the dependence structure using the original return series. However, due to the heteroscedastic behaviour of financial returns, a conditional approach that models the dependence structure after filtering out autoregressive and heteroscedastic behaviour seems more appropriate as suggested by e.g. Grégoire et al (2008), Jondeau and Rockinger (2006) and Patton (2006).

#### 3.2.1. Estimation results for the ARMA-GARCH models

We focus on different ARMA-GARCH specifications for each of the considered series and abstain from using additional exogenous variables. In order to avoid overfitting, the best model is chosen based on the Akaike's Information Criterion (AIC) and the Ljung-Box Test p-value (LBTp). The LBTp values for lags 5, 10 and 15 for stocks before fitting the models are 0.3603, 0.6227 and 0.4035. In the case of bonds the values are 0.0022, 0.0001 and 0. Note that we reject the hypothesis of no serial autocorrelation if  $p$  is below the confidence level. In Table 2 we present the chosen models and their corresponding LBTp for lags 5, 10 and 15 for each of the considered time series. The obtained standardized residuals will then be used in the subsequent

Time Series	Chosen model	LBQ test p-value of residuals		
		lags = 5	lags = 10	lags = 15
Stocks	AR(1)-GARCH(2, 1)	0.933	0.780	0.646
Bonds	ARMA(1, 1)-GARCH(2, 1)	0.741	0.834	0.586

Table 2: p-values of the LBQ test of residuals for different lags of the ARMA-GARCH models. The choice is based on LBQ and AIC tests

empirical analysis.

### 3.3. Estimation Results for the Copula Functions

In a next stage we investigate the dependence structure between the logarithmic returns from Australian stocks and bonds. In the first step we fit the Gaussian, Student's t, Gumbel and Clayton copula to the standardised residuals. We estimate the dependence parameters for these copulas. Note that in the case of the Student's t copula, also the degrees of freedom parameter  $\nu$  needs to be estimated.

#### 3.3.1. Goodness of fit tests

One of the challenges is deciding on which copula provides the best fit to the actual dependence structure of the data. Berg and Bakken (2007) point out that information criteria such as e.g. Akaike's Information Criterion (AIC) are generally not able to provide any understanding about the power of the decision rule employed. Instead, goodness-of-fit (GOF) approaches are more powerful in deciding whether to reject or accept parametric copulas, making them the preferred choice in empirical applications, see e.g. Genest et al. (2006, 2009). Therefore, in our empirical analysis, for selecting the most appropriate among a set of copulas, we decided to use goodness-of-fit tests that investigate the distance between the estimated and the so-called empirical copula, see e.g. Genest et al. (2006, 2009). The empirical copula basically represents an observed frequency and is calculated from the empirical margins. The distance between estimated and empirical copula is then evaluated using the so-called Cramer-Von Mises (CVM) distance. To compare the different copula models we use this measure. The parametric copula that is closest to the empirical copula represents the most appropriate choice. The Cramer-Von Mises test statistic is based on the empirical copula, introduced by Deheuvels (1979) under the name of empirical dependence functions. The empirical copula is defined as

$$C_n^e(u_1, u_2) = F_n(F_{1,n}^{\leftarrow}(u_1), F_{2,n}^{\leftarrow}(u_2)),$$

where  $\leftarrow$  denotes the generalised inverse function. Note that the empirical marginal distribution converges towards the actual distribution function for approaching infinity. According

Copula Model	Cramer Von-Mises Statistic
Gaussian	0.0358
Student's $t$	0.0404
Clayton	0.0476
Gumbel	0.0292

Table 3: Cramer-Von Mises statistic for different copula models

to e.g. Tsukahara (2005), the empirical copula is a consistent estimator of the true copula and, thus, is a well-accepted benchmark for copula goodness-of-fit tests. Genest et al. (2009) provide various options for such tests by conducting a large Monte Carlo experiment and report particularly good results for the blanket tests using ranks or the Rosenblatt transform. The authors recommend the so-called Cramer-Von Mises statistic as a distance measure. Consider a sample  $(u_{1,i}, u_{2,i})$  for  $i = 1 \dots n$  and a parametric copula  $C_\theta$ , the statistic is defined as:

$$\sum_{i=1}^n (C_n^e(u_{1,i}, u_{2,i}) - C_\theta(u_{1,i}, u_{2,i}))^2$$

The procedure we follow as goodness of fit test has the following two steps.

1. Based on the empirical cdfs for the marginal series, estimate the empirical copula and the parametric copula.
2. Using the Cramer-Von Mises statistic, calculate the distance between the empirical and the estimated copula.

According to Table 3, the Von Mises distance the Gaussian and Gumbel copulas have the best performance of the models considered. In Figures 1 and 2 we have the time-varying behaviour of the parameters of this copula. The behaviour of the correlation of the Gaussian copula is very stable compared to that of the Gumbel copula. In the following section we use these two copulas to model the dependence structure between stocks and bonds. For this we consider fixed and time-varying parameters.

### 3.3.2. Estimation Results for the Fixed and Time-Varying Parameter Copulas

In order to analyse the dependence structure between stocks and bonds we first consider a fixed parameter for the whole period. After this, we consider the time-varying approach discussed in the previous section. We decided to choose a window length of 12 months or 1 year. Thus, the first twelve month period considers returns from February, 1970 to January, 1971 while the last window uses data from January to December, 2013.

Figures 1 and 2 show a plot of the estimated copula parameters for the Gaussian and the Gumbel copulas. For most of the considered series, we find that the estimated copula parameters

exhibit time-variation. From the Gaussian copula plot, we generally find that the dependence between stocks and bonds has decreased, particularly we see very steep decrease since the year 2000. The degree of time-variation, however, is considerably different for the parameter of the Gumbel copula. In this case we have several lapses of peaks and troughs around 1.15. This holds throughout the whole period. This indicates that joint upward movements of the two series occur considerably often during the studied period. Periods where the parameter of the Gumbel copula is approximately one indicate that the dependence is very weak. The relationship between stocks and bonds is found to be relatively strong for the entire time horizon. Note that conclusions as to whether there is a structural break or a significant change in the dependence structure during the considered period require further statistical tests as suggested by Patton (2006).

For comparison purposes, in Figures 3 and 4 we plot a standard rolling window for the parameters of the Gaussian and the Gumbel copulas. Under this approach we simply fit the copula parameter for each window. Again, the length of the window is 12 months. The behaviour found on these two figures somehow resembles that of Figures 1 and 2. However it is much less stable and for most of the period is similar to white noise. It would be much harder to draw conclusions based on these figures. We can conclude that the use of the “ARMA-like” equations of Patton’s models is more suitable for time-varying parameter estimation than standard estimation.

### *3.4. MySuper Strategies*

There are a myriad of asset allocation approaches currently implemented in approved MySuper products. As discussed in the previous section, we base our analysis in the last 10 years prior to retirement. Balanced or target risk funds (TRFs) maintain the same level of risk through time by holding a constant proportion of growth and defensive assets. TRFs are commonly employed in MySuper products at varying proportions of growth and defensive assets. TRFs strategies can range from 100% stocks to 100% bonds.

Further to TRFs the other type of strategies we consider are lifecycle or target date funds (TDFs) which have also since emerged in the MySuper universe. Typically, TDFs switch from growth to defensive assets according to a pre-determined glide-path as a worker approaches retirement. TDFs change the proportion of growth assets in the retirement portfolio as the worker approaches a retirement date using deterministic switching rules. TDFs have become a core product for investors saving for retirement, particularly in the US (Estrada, 2013).

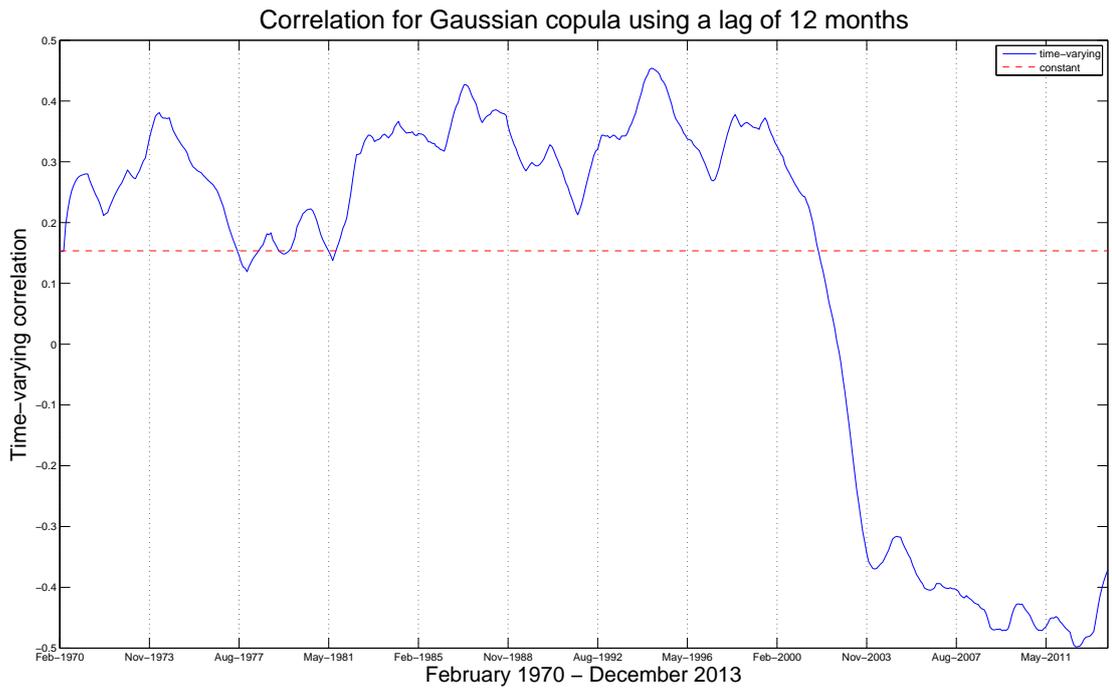


Figure 1: Moving parameter of Gaussian copula using Patton's model

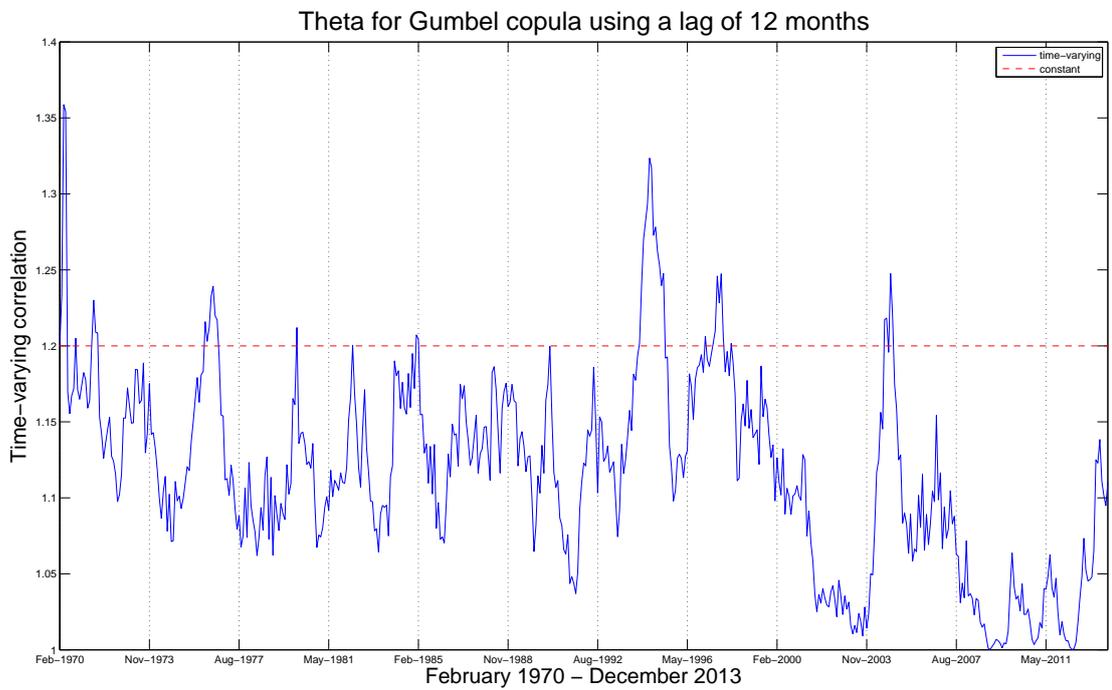


Figure 2: Moving parameter of Gumbel copula using Patton's model

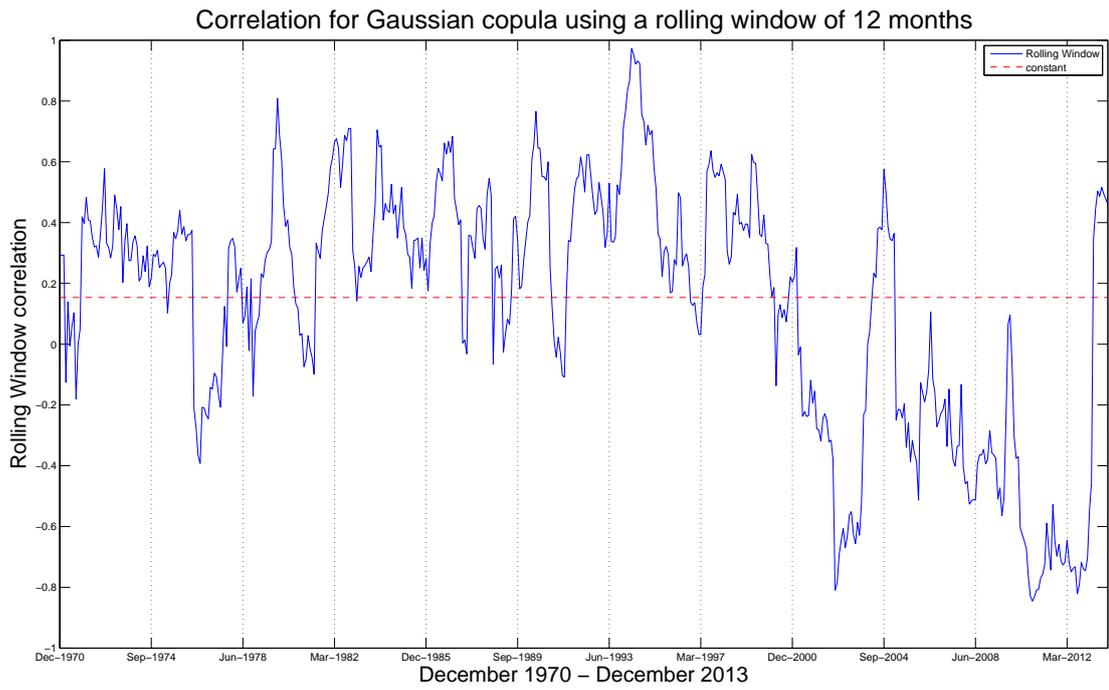


Figure 3: Moving parameter of Gaussian copula using standard model

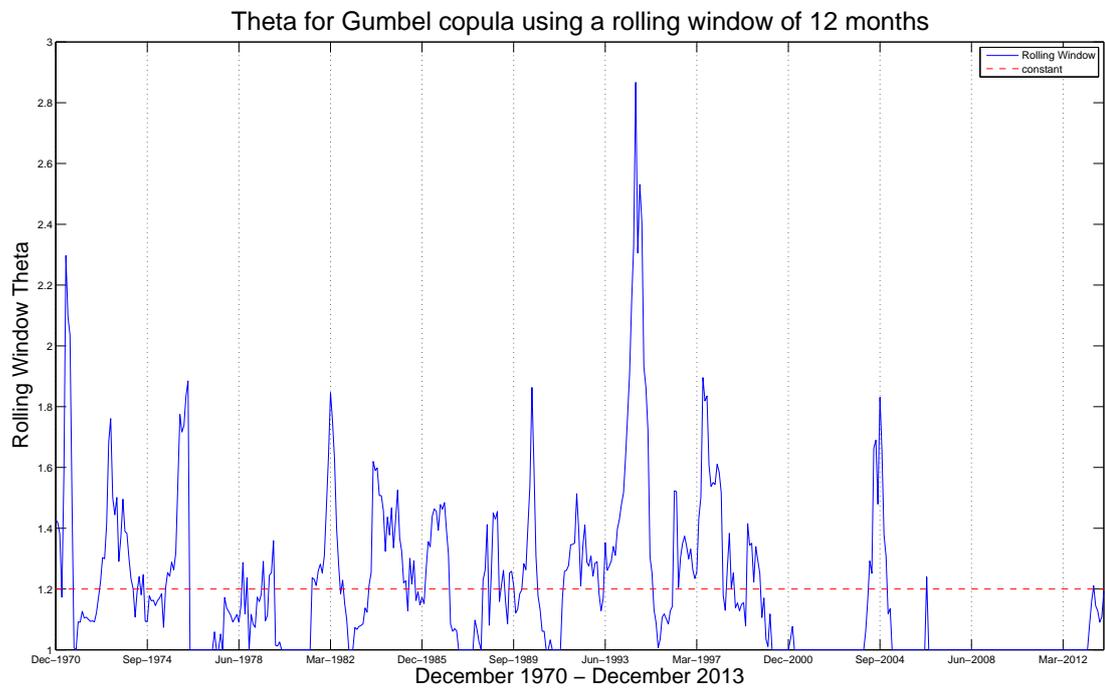


Figure 4: Moving parameter of Gumbel copula using standard model

We analyse four examples of asset allocation for MySuper products. Two target risk funds (TRFs):

- A portfolio of 80% stocks and 20% bonds
- A portfolio of 20% stocks and 80% bonds.

The other examples are two target date funds (TDFs) which have deterministic glidepaths.

- A portfolio that linearly switches from 80% stocks to 20% stocks
- A portfolio that linearly switches from 20% stocks to 80% stocks

### *3.5. The Retirement Wealth Ratio for MySuper Portfolios*

To evaluate asset allocation strategies and assess their appropriateness as default investment options in MySuper strategies, we need to make plausible assumptions about the rationale that may guide the selection of a specific asset allocation strategy as a default option from many competing candidates. The basic motivation behind instituting retirement savings plans is to generate adequate income for the participating employees after retirement. In that case, performance of MySuper strategies should be measured in terms of their ability to generate sufficient retirement income (Baker et al., 2005). Therefore, the principal investment objective of such plans would be to maximize the terminal value of plan assets at the point of retirement since that would directly determine the amount of annuity the retiring employees are able to purchase for sustenance during post-retirement life. Past studies have mainly considered the absolute value of the participant's accumulated assets at retirement. However, we employ a ratio which compares the terminal wealth of the participant's retirement account to their terminal income because it is very likely that the participant's post-retirement income expectations are closely linked to their immediate income before retirement. Basu and Drew (2010) call this measure the 'retirement wealth ratio' (RWR) and is defined as the wealth at retirement divided by the final yearly income. Higher estimates of different measures of RWR outcomes (like mean or median) do not automatically qualify a particular strategy to be selected as default option. The trustees also need to consider the risk associated with investment of plan assets since participants would want a better exploitation of trade-off between risk and reward. In this study we use the RWR (instead of variability of terminal wealth outcomes given by their standard deviation) as measure of risk. Since we assume that the ultimate goal of MySuper strategies is to attain a specific amount of wealth relative to their terminal income, we measure this using the target retirement wealth ratio (RWRT). The investment risk most relevant to participants is that of failure of their chosen asset allocation strategy to generate RWRT. A

MODEL	5						8						10											
							Weight given to stocks in portfolio																	
	0.8	0.2	0.2	0.8	0.8	0.2	0.8	0.2	0.2	0.8	0.8	0.2	0.2	0.8	0.8	0.2	0.2	0.8	0.8	0.2	0.2	0.8	0.8	
Gaussian copula with fixed parameter	0.835	0.812	0.836	0.847	0.847	0.201	0.376	0.379	0.303	0.053	0.180	0.178												
Gumbel copula with fixed parameter	0.813	0.768	0.793	0.806	0.497	0.211	0.381	0.384	0.311	0.075	0.194	0.188												
Gaussian copula with time-varying parameter	0.828	0.804	0.830	0.839	0.504	0.194	0.372	0.378	0.300	0.052	0.163	0.167												
Gumbel copula with time-varying parameter	0.821	0.782	0.815	0.820	0.498	0.208	0.370	0.373	0.306	0.063	0.187	0.181												
Historical simulation with block bootstrap method	0.820	0.885	0.857	0.865	0.497	0.251	0.410	0.410	0.300	0.072	0.182	0.162												
Historical simulation EWMA with block bootstrap method	0.771	0.876	0.814	0.829	0.366	0.109	0.248	0.238	0.186	0.018	0.082	0.070												

Table 4: Probabilities of reaching or exceeding TRWRs of 5, 8 and 10 for the six models. For each TRWR, we consider: a portfolio of 80% stocks and 20% bonds, a portfolio of 20% stocks and 80% bonds, a portfolio that linearly switches from 20% stocks to 80% stocks and a portfolio that linearly switches from 80% stocks to 20% stocks

value of 10 is sometimes deemed as adequate by practitioners, but there is no consensus on this matter.

### *3.6. Simulation procedure*

In order to assess the effectiveness of MySuper strategies in achieving TRWRs we follow a simulation procedure considering four parametric models and the two block bootstrap models based on historical simulations described in Section 2. The four copula models are the Gaussian and the Gumbel copulas with fixed and time-varying parameters. The methods used in our simulation procedure have already been thoroughly described throughout the paper. The corresponding steps, for the parametric models, are the following:

1.-We consider the logarithmic returns of stocks and bonds and fit the corresponding ARMA-GARCH models of Table 1.

2.-We use the inverse empirical distribution on the standardised residuals and consider this as a bivariate sample with uniform marginals, suitable for fitting the corresponding copula models

3.-We fit the corresponding copula models and generate 10,000 samples of size 120 (10 years). In the case of the time-varying parameter models, we generate one element of the sample at a time and use equations (1) and (2) to update the parameter.

4.-We filter the corresponding samples through the ARMA-GARCH models to generate 10,000 random samples of logarithmic returns which we then convert to discrete to use them for risk analysis purposes.

### *3.7. Preliminary results*

As mentioned earlier in this section the main interest in our analysis is to determine what factors affect whether or not the retiree has an adequate wealth at retirement. For this purpose we use the RWR. Using the 10,000 samples obtained with our simulation procedure we are able to estimate the probability of achieving or exceeding different TRWRs for the MySuper strategies according to the different models.

In order to make this analysis we consider a typical investor 10 years before retirement. The investor has \$500 monthly contributions increasing 4% annually, her superannuation contribution is 9.5% and her current balance is \$250,000. These values are representative of the Australian MySuper universe. Considering these values, the terminal yearly income is \$93,489.

This means that, for example, if we want to have a TRWR of 5, the terminal wealth must be \$467,445.

Table 4 presents the results of this analysis for TRWRs of 5, 8 and 10. According to this table the probabilities obtained by the block bootstrap are higher than for the parametric models. In contrast, the probabilities obtained using the EWMA block bootstrap are generally lower, particularly for high TRWRs. This makes sense given that in recent times returns have shown lower values with respect to previous times. This is an important result, it indicates that if stock and bond returns behave like they have done in recent times we can not expect to get high RWR values. This means investors need to adjust their expectations. This is fundamental for the planning of retirement outcomes. For the parametric models the values are closer together. In this case the values obtained using the time-varying parameters are generally between the values of the other parametric models. This seems to indicate a more stable behaviour.

As expected the probability of achieving or exceeding TRWRs decrease as they get higher. The rate of decreasing seems to be very close to linear. It is also expected how the probabilities are higher for strategies with high weight in stocks. Regarding the difference between the two TDFs, we have that switching from stocks to bonds is only more profitable for TRWR of 10. The fact that stands out the most from this table is how the strategy becomes increasingly important for high values of the TRWR. We find that investing in growth assets as we approach retirement highly improves the probability of achieving high RWR. This is not in line with general recommendations for lifecycle investment where switching to defensive assets is recommended. On the other hand, the EWMA approach indicates that, for an RWR of 5, defensive strategies are not outperformed by more aggressive ones.

To finalise this analysis, in a second step we also investigate the behaviour of the distribution of the different RWRs. In Figures 5 and 6 we present the corresponding histograms and the descriptive statistics for the different models. For this part, we consider the two TRFs portfolios of 80% stocks, 20% bonds and 20% bonds, 80% stocks.

For the portfolio of 80% stocks we find high RWRs with mean values around 8.5. The difference between means for different models seems to be in line with the analysis of the probabilities of the TRWRs. The standard deviation is similar amongst models except for the block bootstrap EWMA model which is much lower and the same applies to the skewness as well. In the case of the kurtosis, the values are generally around 5 with the exception of the Gumbel copula which has a much higher value.

HISTOGRAMS OF RWRs FOR DIFFERENT MODELS  
FOR A PORTFOLIO OF 80% STOCKS AND 20% BONDS

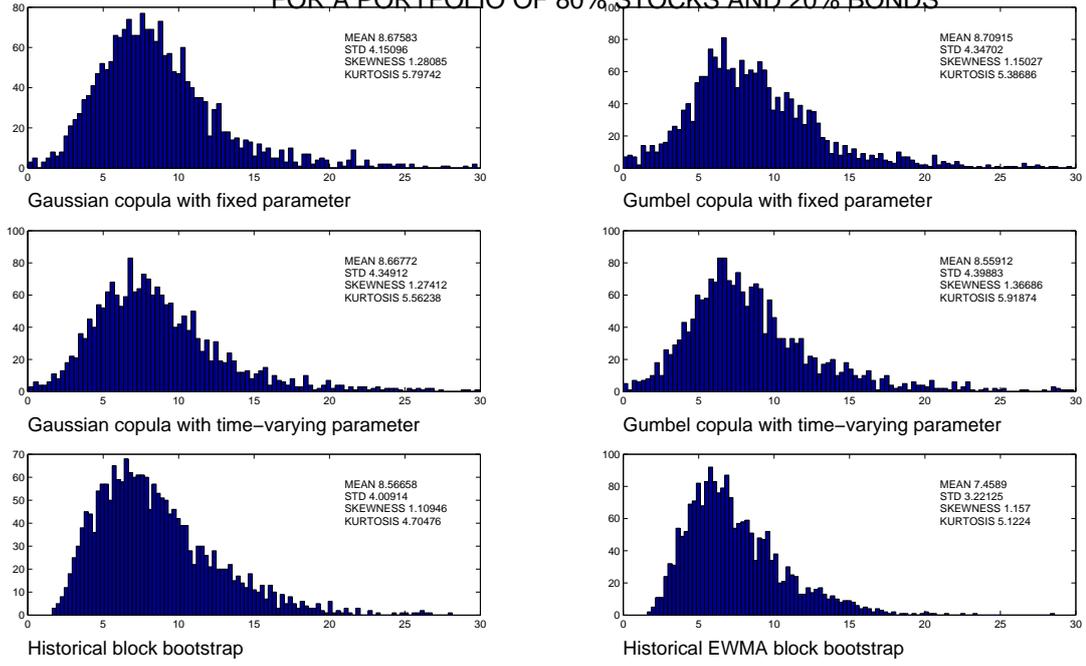


Figure 5: Histogram for different models for portfolio of 80% stocks, 20% bonds

HISTOGRAMS OF RWR FOR DIFFERENT MODELS  
FOR A PORTFOLIO OF 20% STOCKS AND 80% BONDS

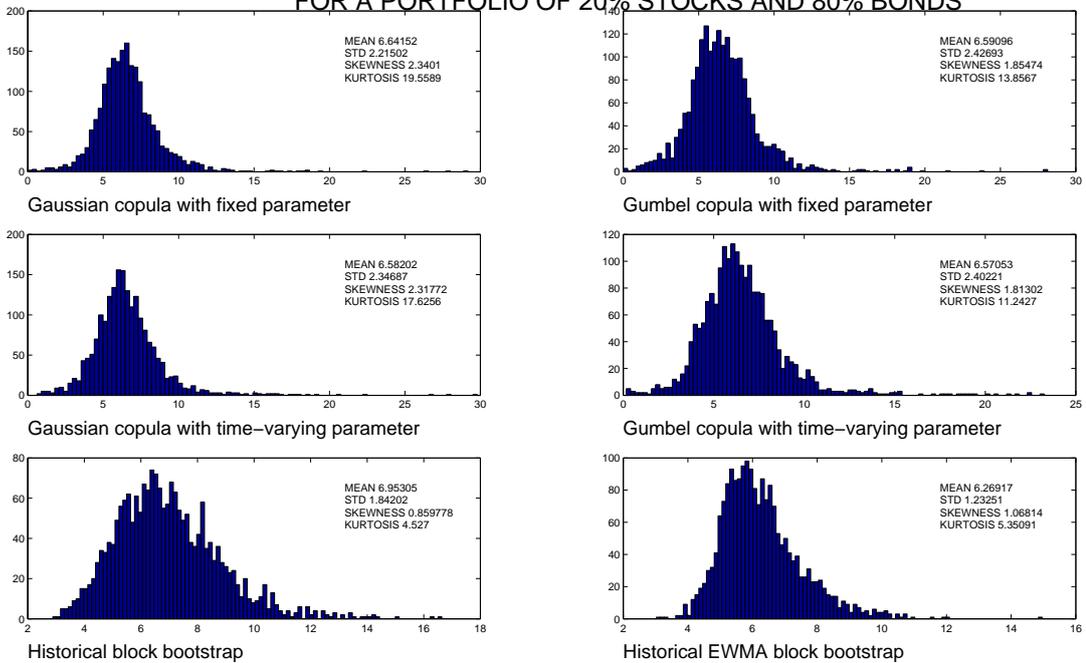


Figure 6: Histogram for different models for portfolio of 20% stocks, 80% bonds

In the case of the portfolio of 20% stocks we have mean values around 6.5 with a similar behaviour to the previous figure. In this case the standard deviation is lower. The skewness and kurtosis have more heterogeneous values. In this case the highest values are presented by the Gumbel copula with time-varying parameter. The difference for the descriptive statistics for these figures is also partly explained by the nature of the stocks and bonds that form the portfolios.

From the results obtained at this point we see that parametric models seem to be more conservative about the expectations of RWR. This seems to be in line with the general movement of the Australian financial market.

#### 4. Conclusions

The aim of this paper is to deepen the understanding of the different factors that affect the wealth at retirement.. We apply different models in order to analyse the dependence structure between Australian stocks and bonds. The main models we consider are based on copula functions. Copulas offer great flexibility for modeling the relationship between different financial variables. The application of copulas also yields insights with respect to nonlinear

dependence. Thus, we first investigate which copulas are most appropriate to model the dependence structure. Second, we deal with the question whether or not the dependence structure exhibits time-varying properties. The latter allows us to examine whether the relationship between the considered variables has changed over time and whether or not financial crises have had an influence on the dependence between these two assets. The usefulness of copulas is further illustrated in a retirement wealth ratio probability and distribution. We consider different portfolios combining investments in stocks and bonds and test the Gaussian as well as the Gumbel copula models against two benchmark models: a standard block bootstrap method and an EWMA block bootstrap.

The following insights emerge from these efforts. First, for the considered time series, we apply the Clayton, Gumbel, Gaussian and Student t copulas to investigate the dependence structure between Australian stocks and bonds for superannuation portfolios. To our best knowledge this is one of the first studies to apply this technique in superannuation portfolios. The Gaussian and Gumbel copulas are most appropriate, significantly outperforming both the Clayton and Student's t copulas with respect to a goodness-of-fit test for the distance between the estimated and empirical copula. Second, a significant change in the nature of

the dependence structure is found between the two assets. This is detected by the correlation parameter of the Gaussian copula, with a steep decline since the year 2,000. The parameter for the Gumbel copula is more volatile but also detects a high variation in the dependence. This confirms general results on asset returns from financial markets exhibiting different types of dependence during periods of different market conditions.

Finally, we also provide a risk analysis by using the retirement wealth ratio. The RWR analysis illustrates how likely it is to achieve determined outcomes for the different models. We find that the use of different strategies only becomes very relevant when dealing with high RWRs. In this case, if we want to maximise the probability of having a high RWR, growth assets should be preferred. Another key results is provided by the EWMA blocks bootstrap method. According to this model if returns of stocks and bonds keep behaving in the way they have in previous times, there is a need to lower expectations for wealths at retirement. In a nutshell, our results recommend copulas as an appropriate tool for describing the dependence structure between returns from Australian stocks and bonds. The application of copulas might also be particularly useful for risk management purposes and for forecasting terminal wealth.

Extensions of the conducted work could examine the impacts of related factors. Furthermore, since this study is limited to a bivariate setting, future research should extend the analysis to the multivariate case including various other asset classes next to stocks and bonds. Also the set of considered copula functions could be extended using alternative copulas models.

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