Predicting Markov Volatility Switches Using Monetary Policy Variables

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Macquarie University
Asset prices variability and economic activity


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Contribution of the paper

- To assess whether monetary policy variables affect (or help to predict) big changes in the state of the economy. We look at the second moment (volatility).

- To extend a multivariate Markov switching (in the variance-covariance matrix) model by making the transition probability matrix time varying, and by allowing those probabilities to be function of the monetary variables.
References on Markov Switching Models


The model

Consider the following model for the $2 \times 1$ vector $z_t = [x_t, y_t]'$:

$$z_t = \mu + \Phi_s u_t,$$

(1)

where $\mu = [\mu_x, \mu_y]'$ and $u_t$ is a Gaussian process with zero mean and positive-definite covariance matrix $\Sigma$;

$\{s_t\}$ is modelled as a time-homogeneous Markov chain on $\{1, 2, 3, 4\}$, independent of $\{u_t\}$, with $s_t$ indicating the state that the system is in at date $t$. The time series $\{z_t\}$ (the vector of stock market return and GDP growth, respectively $x_t$ and $y_t$) satisfies therefore a four-states Markov process:

$$z_t | (s_t = s) \sim N(\mu, \Omega_{s_t}),$$

(2)

for $s = 1, 2, 3, 4$, with $\Omega_{s_t} = \Phi'_s \Sigma \Phi_s$. Accordingly, the variance-covariance matrices are:

$$\Omega = \begin{bmatrix}
\sigma_{xh}^2 & \sigma_{xh, yh} \\
\sigma_{yh, xh} & \sigma_{yh}^2
\end{bmatrix},
\begin{bmatrix}
\sigma_{xh}^2 & \sigma_{xh, yl} \\
\sigma_{yl, xh} & \sigma_{yl}^2
\end{bmatrix},
\begin{bmatrix}
\sigma_{xl}^2 & \sigma_{xl, yh} \\
\sigma_{yh, xl} & \sigma_{yh}^2
\end{bmatrix},
\begin{bmatrix}
\sigma_{xl}^2 & \sigma_{xl, yl} \\
\sigma_{yl, xl} & \sigma_{yl}^2
\end{bmatrix}$$

(3)

where the indices $h$ and $l$ refer to high and low volatility. In the general case the transition matrix will be given by a $4 \times 4$ matrix, $\Pi$ (with elements $\pi_{ij} = \Pr(s_t = i | s_{t-1} = j)$, $i, j = 1, 2, 3, 4$), where each column sums to unity and all elements are nonnegative.
We assume that each volatility follows an independent regime-shifting process:

\[
\Pi_t = \begin{pmatrix}
\pi_{xh} \pi_{yh} & \pi_{xh}(1-\pi_{yl}) & (1-\pi_{xl})\pi_{yh} & (1-\pi_{xl})(1-\pi_{yl}) \\
\pi_{xh}(1-\pi_{yh}) & \pi_{xh}\pi_{yl} & (1-\pi_{xl})(1-\pi_{yh}) & (1-\pi_{xl})\pi_{yl} \\
(1-\pi_{xh})\pi_{yh} & (1-\pi_{xh})(1-\pi_{yl}) & \pi_{xl}\pi_{yh} & \pi_{xl}(1-\pi_{yl}) \\
(1-\pi_{xh})(1-\pi_{yl}) & (1-\pi_{xh})\pi_{yl} & \pi_{xl}(1-\pi_{yh}) & \pi_{xl}\pi_{yl}
\end{pmatrix}.
\]

(4)

In order to assess the response of output/stock returns to monetary policy variables, we allow the transition probabilities to be time varying. The transition mechanism governing \((s_t)\) is defined as follows:

\[
\begin{align*}
\pi_{t}^{xh} &= \exp(\alpha_{xh} + \beta_{xh} r_{t-1} + \gamma_{xh} w_{t-1})/[1 + \exp(\alpha_{xh} + \beta_{xh} r_{t-1} + \gamma_{xh} w_{t-1})], \\
\pi_{t}^{xl} &= \exp(\alpha_{xl} + \beta_{xl} r_{t-1} + \gamma_{xl} w_{t-1})/[1 + \exp(\alpha_{xl} + \beta_{xl} r_{t-1} + \gamma_{xl} w_{t-1})], \\
\pi_{t}^{yh} &= \exp(\alpha_{yh} + \beta_{yh} r_{t-1} + \gamma_{yh} w_{t-1})/[1 + \exp(\alpha_{yh} + \beta_{yh} r_{t-1} + \gamma_{yh} w_{t-1})], \\
\pi_{t}^{yl} &= \exp(\alpha_{yl} + \beta_{yl} r_{t-1} + \gamma_{yl} w_{t-1})/[1 + \exp(\alpha_{yl} + \beta_{yl} r_{t-1} + \gamma_{yl} w_{t-1})],
\end{align*}
\]

(5)

where \(r_t\) and \(w_t\) are variables that affects the state transition probabilities (respectively the 3-months Treasury Bills and the spread between the five years bond and the 3-months Treasury Bills).
Data

- Real GDP growth
- Real Stock Market Returns
- Interest Rate
- 5 years and 3 months interest rate spread
Real GDP Growth, Real Stock Returns, and Filter Probabilities of High Volatility

Figure 1:
Table 1.

Maximum Likelihood Estimation Results$^a$

\[ z_t = \mu + \Phi_{st}u_t, \quad \text{where } z_t = [x_t, y_t]', \quad \text{and } \Omega_{st} = \Phi_{st}'\Sigma\Phi_{st} \]

\[
\Omega = \begin{bmatrix}
\sigma^2_{xh} & \sigma_{xh,yl} \\
\sigma_{yh,xh} & \sigma^2_{yh} \\
\end{bmatrix}, \quad \begin{bmatrix}
\sigma^2_{xh} & \sigma_{xh,yl} \\
\sigma_{yh,xh} & \sigma^2_{yh} \\
\end{bmatrix}, \quad \begin{bmatrix}
\sigma^2_{xl} & \sigma_{xl,yl} \\
\sigma_{yl,xl} & \sigma^2_{yl} \\
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\end{bmatrix}, \quad \begin{bmatrix}
\sigma^2_{xl} & \sigma_{xl,yl} \\
\sigma_{yl,xl} & \sigma^2_{yl} \\
\end{bmatrix}
\]

\[
\pi^x_{th} = \exp(\alpha_{xh} + \beta_{xh}r_{t-1} + \gamma_{xh}w_{t-1})/[1 + \exp(\alpha_{xh} + \beta_{xh}r_{t-1} + \gamma_{xh}w_{t-1})],
\]

\[
\pi^x_{lt} = \exp(\alpha_{xl} + \beta_{xl}r_{t-1} + \gamma_{xl}w_{t-1})/[1 + \exp(\alpha_{xl} + \beta_{xl}r_{t-1} + \gamma_{xl}w_{t-1})],
\]

\[
\pi^y_{th} = \exp(\alpha_{yh} + \beta_{yh}r_{t-1} + \gamma_{yh}w_{t-1})/[1 + \exp(\alpha_{yh} + \beta_{yh}r_{t-1} + \gamma_{yh}w_{t-1})],
\]

\[
\pi^y_{lt} = \exp(\alpha_{yl} + \beta_{yl}r_{t-1} + \gamma_{yl}w_{t-1})/[1 + \exp(\alpha_{yl} + \beta_{yl}r_{t-1} + \gamma_{yl}w_{t-1})],
\]

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log $L$ 132.0169

$^a$We report estimated parameters and asymptotic standard errors (s.e.).

$x =$ stock returns, $y =$ output growth, $r =$ interest rate, $w =$ spread between interest rates.
Figure 2:
Conclusions

- Empirical evidence on the reaction of the stock market returns and growth volatilities to changes in monetary policy

- The filter used in the paper manages to separate periods of high and low GDP growth and stock returns volatility

- Monetary variables (such as the interest rate and the spread between long and a short interest rate) have power in predicting changes in the volatility of both GDP growth and stock market returns