Exporting Collusion under Capacity Constraints:
an Anti-Competitive Effect of Market Integration*

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Abstract
This paper examines the welfare effects of interconnecting two (network) markets that were previously separated. In each market a different set of capacity-constrained firms operate. Firms engage in a supergame and collude whenever it is rational for them to do so.

We find that, under certain parametric restrictions, interconnection of the two markets may reduce total welfare. The collusive horizon may extend from a single market to the overall integrated market. In such case, interconnection can be viewed as "exporting" collusion, rather than competition.

Keywords: collusion, capacity constraints, market integration
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1 Introduction

Market integration has been increasingly recognized as a source of welfare by economists as well as policy makers. The ongoing debate on interconnection in network industries stresses its benefits - in particular in terms of cost reduction (see Boffa and Pingali (2006) for an empirical assessment of the efficiency effects of interconnection), through exploitation of economies of scale, and of greater competition\(^1\). Our paper questions the validity of this conventional view, and finds conditions under which openness and interconnection harm competition, and generate welfare reduction.

Our results hinge on the fact that better interconnection - both through an increase in the physical possibility of trading and/or an improvement of the compatibility between systems - may bring about not only greater competition, but also greater collusion. This may happen when the productive capacity of competing firms is limited. In this case, when each firm’s individual capacity is sufficiently small relative to the market size, each firm’s deviation profit from the collusive agreement\(^2\) is limited by the capacity constraint. As a consequence, in addition to the discount factor, it is the relation between aggregate capacity and the market size, and not the number of firms alone (as it is the case in the standard setting without capacity constraints), that determines whether or not the monopoly outcome can emerge as an equilibrium of the collusive game.

To illustrate the mechanism through which integration reduces welfare, consider two markets, A and B. In each of them, there exists a level of aggregate capacity below which collusion at the monopoly price is sustainable. When the markets are integrated, both overall capacity (which is the sum of capacities in the two markets, A and B), and the threshold of aggregate capacity below which collusion at the monopoly price is sustainable (which is the sum of the two thresholds in the two markets, A and B), increase. Further suppose, before interconnection, that capacity in market A is low relative to the market size, and is below the above mentioned threshold for market A (monopoly is then sustainable). On the other hand, capacity in market B is high relative to the market size, and is above the threshold for market B (hence, monopoly is not sustainable). Moreover, after interconnection, the overall capacity is lower than the threshold for the integrated market. As a result, the monopoly outcome will

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\(^1\)Integration has been central in the European debate both in goods markets, where the Single European Act has tried to eliminate remaining barriers to trade, as well as in public utility sectors, where in particular telecommunications and energy markets have been tackled by several Directives. More recently, the Lisbon declaration points out how increasing the capacity of transporting electricity among EU member states represents a priority goal and a key to increasing competition and system security. Something similar holds in the transport sector, where building a system of Trans-European Transport Networks (TEN-T) is considered, among other things, “a key element for the creation of the Internal Market” both with reference to the goods transported through the system and to the competition among railway companies. The key words are interconnection (which refers to the elimination of bottlenecks, the physical constraints to the capacity of networks) and interoperability (the compatibility among networks). Quite clearly, increasing the physical capacity of passing from one network to the other is not of much use if the two networks are incompatible.

\(^2\)Deviation occurs through undercutting the rivals.
prevail in the integrated market. In other words, monopoly has been "exported" from market A to the overall integrated market. To grasp the intuition for the result, one can think of firms located in market B to dedicate, after interconnection, a portion of their capacity to serve customers of market A. Indeed, in market A the monopoly outcome may emerge even with an increase in capacity dedicated to it, while in market B collusion at the monopoly price is sustainable only if part of the capacity of firm located in market B is "diverted" to market A.

Our paper is related to the following three strands of literature.

In the international trade literature, the positive effect of market integration on competition is well known. The analysis of the same phenomenon with imperfect competition (see e.g. Markusen, 1981) has confirmed the pro-competitive effect of trade. However no analysis of the effects of integration on collusion in the presence of capacity constraints has been carried out. Our paper attempts to bridge this gap.

The second strand of literature is the one on collusion and capacity constraints (in a single market). Brock and Scheinkman (1985) point out the role of aggregate capacity in determining the threat after a deviation. When aggregate capacity is sufficiently low, and no firm is essential in producing the competitive outcome, Bertrand equilibrium involves positive profit (see Kreps and Scheinkman (1983)). In such situations, the threat posed by deviating from the collusive equilibrium pattern becomes less effective, making collusion harder to achieve. For this reason, Brock and Scheinkman find that, for a fixed capacity per firm, changes in the number of firms have a non-monotone effect on the best enforceable cartel price.

Compte, Jenny and Rey (2002), examine the impact of asymmetric capacity constraints on the sustainability of collusion. They show that asymmetry may hinder collusion, thus contributing to welfare enhancement. A similar result was obtained by Penard (1997). The relevance of capacity constraints in collusion is confirmed by Fabra (2006), by building on Haltiwanger and Harrington (1991). She shows that when capacity constraints are very tight collusion may be easier in periods of low demand.

Finally, our paper is related to Bernheim and Whinston’s (1990) multimarket contacts paper. They find that multimarket contacts may make collusion easier to sustain, by transferring collusion across markets. Our paper substantially differs from theirs. The main difference is that their result is driven by the fact that the same firms operate in more than one market, while ours does not require multiproduct firms.

The paper is organized as follows. The next section lays down a motivating example which illustrates the contribution of our paper. Section 3 illustrates the basic model and derives equilibrium prices with separated and integrated
markets. Section 4 contains the result in general form, while the final section contains some concluding remarks.

2 A motivating example

Consider two separate countries, each with the same linear demand function \( Q_j = a - p_j \), where \( Q \) is quantity and \( p \) is price in country \( j \). All firms have zero marginal cost and can produce up to capacity \( k = \frac{a}{3} \).

Firms operate with an infinite time horizon and discount future profits at a factor\(^4 \) \( \delta = \frac{3}{4} \), common to all firms.

In the first country, four firms operate. These firms may form a cartel, and, by assumption, select the “best” (in the sense, the aggregate profit-maximizing) equilibrium pair (price and quantity) in a supergame supported by a Bertrand-Nash reversion trigger strategy.

The IR constraint, when firm \( i \) is capacity constrained (and the collusive output exceeds individual capacity \( k \)), requires the profit from a small \(( \varepsilon \rightarrow 0)\) deviation from the cartel price\(^5 \) to be smaller than the profit from remaining in the cartel and producing \( q_i \). It differs from the IR constraint of a standard collusive supergame, in that capacity constraints here bind the deviation output, thereby limiting the deviation profit (at a level of \( p^c k \)). This suggests that the collusion may be easier to sustain when firms are capacity constrained.

Formally, the sustainability of the cartel requires the following individual rationality (IR) constraint to hold for each of the four firms:

\[
\frac{p^c q_i(p^c)}{1 - \delta} \geq k p^c \quad \implies \quad q_i(p^c) \geq k = \frac{a}{3} \quad (1)
\]

\[
q_i \geq (1 - \delta) \frac{a}{3} = \frac{a}{12} \quad (2)
\]

Therefore, a firm will find it optimal to remain within the cartel only if its production is not lower than \( \frac{a}{12} \).

Therefore, the minimum aggregate sustainable output in the first country, \( \frac{a}{3} \), is lower than the monopoly output \( \frac{a}{2} \). Hence, the monopoly outcome can be sustained as an equilibrium of the collusive supergame, and will indeed be sustained, being the “best” (in the sense of aggregate profit maximizing) equilibrium\(^6 \).

In the second country, 7 identical firms are located. The IR constraint for each individual firm is (2) - the same as in the first country. However, as the

\(^4\) The example holds whenever \( \delta \in (\frac{8}{11}, \frac{11}{14}) \).

\(^5\) Notice that the capacity of the firms is such that none of the firm is essential to produce the competitive output. Hence, the static Bertrand equilibrium prescribes zero profit, as will be explained in greater detail later.

\(^6\) Notice that - as proved by Brock and Scheinkman (1985) - with capacity constraints the collusive price depends on capacity.

Also, observe that any other equilibrium price/output pair would not be "better" than the monopoly outcome, as it would yield a lower aggregate profit.
number of firms in the two markets differ, the minimum aggregate sustainable output in the second market is \((1 - \delta) \frac{7}{2}a = \frac{7}{2}a\) - higher than the monopoly output \(\frac{3}{2}\) and lower than the competitive output \(a\). Therefore, in the second country, the monopoly outcome is not an equilibrium of the game. The minimum aggregate sustainable output is in this case also the supergame equilibrium output that maximizes aggregate profit, on which firms coordinate. It follows that the optimal collusive output in the second country exceeds monopoly output, and the equilibrium price in this country is lower than monopoly price.

Suppose now that, in order to strengthen competition, the two countries decide to remove all barriers separating the two markets. They then create an integrated market. The new aggregate demand function is thus: \(Q = 2(a - p)\). The 11 firms face an infinite horizon and collude whenever rational. The individual rationality constraint for the sustainability of the eleven-firm cartel is again given by \((2)\).

The minimum aggregate sustainable output in the integrated market is then \((1 - \delta) \frac{11}{2}a = \frac{11}{2}a\), which is lower than the monopoly output \(a\).

Therefore, in the interconnected market the monopoly output, and hence the monopoly price \(p_{\text{mon}} = \frac{a}{2}\), can be sustained.

The creation of an integrated market - far from bringing about more competition - has “exported” the monopoly outcome from the first market into the second one. Loosely speaking, the reason is that from the viewpoint of the country where seven firms operate, the increase in market size due to integration is more important than the increase in total capacity due to the additional firms, which are located abroad, but which can now sell at home. The seven firms located in the second market are now (i.e., after integration) able to divert part of their capacity to serve the first market, where monopoly price prevails. As a result, the decrease in the capacity utilized to serve the second market allows them to sustain a monopoly outcome there. At the same time, the first market (where only four firms are located) is able to absorb the extra capacity, while preserving the monopoly outcome. Indeed, the new capacity available for the first market (resulting from the sum of the capacity of the four firms and the portion of capacity originated from the second market), is still lower than the threshold below which the monopoly outcome may emerge as a supergame equilibrium.

How general is this result? Is it pathological, or should it raise a genuine concern in markets - such as electricity generation or railway transport - where capacity constraints are often binding in the equilibrium? We will show in the next sections conditions under which this result may be obtained. This helps us in understanding how the interplay of market size, number of firms and capacity levels can determine such an outcome.
3 The model

A good is produced in two countries, labeled A and B. In country j a given number of firms\(^7\) \(N_j \geq 2\) operates (with \(N_A + N_B = N\)). The demand curve for market \(j = A, B\) at time \(t\) is\(^8\) \(Q_{j,t}(p_{j,t})\). The reservation prices in the two markets are identical; this assumption is useful to assume away the possibility of multi-market price discrimination by a decision maker who maximizes the combined profit in the two markets. If the good can be freely traded between the two markets (i.e., if they are perfectly interconnected), in each period the price is \(p_{ic}\) and total demand \((Q_{ic} = Q_A + Q_B)\) is \(Q(p_{ic})\). Its inverse\(^9\) is given by: \((Q_A + Q_B)^{-1}(p_{ic})\).

Firms are capacity constrained. All firms have the same capacity\(^10\) \(k\). Firm \(i\) produces output \(q_i\) at a constant marginal cost up to capacity and cannot produce beyond it.

\[
C(q_i) = \begin{cases} \quad c q_i & \text{if } q_i \leq k \\ \infty & \text{if } q_i > k \end{cases}
\quad (3)
\]

Competition takes place in prices over an infinite number of periods. If in any period rationing occurs, it follows the efficient rationing rule, proposed by Levitan and Shubik (1972). For ease of exposition, we assume throughout the paper that no firm is essential for producing the competitive output both in A and in B. That is,

\[
(N_j - 1) k > Q_j(c)
\quad (4)
\]

This assumption, identified as the “non-essentiality condition”, or NEC, ensures that no firm is essential for producing the competitive quantity. This is necessary for the Bertrand equilibrium price in the static game \(p^b\) to be set at \(p^b = c\). Hence, under (4), Bertrand profit, and as a consequence the deviation profit, is null. This greatly simplifies computations (for a version of the paper when NEC assumption is relaxed, see Boffa, (2006)).

We compare two different scenarios. In the first one, markets are separated, while in the second one A and B are interconnected into a single market. Price discrimination is ruled out by assumption. In each market (indices omitted), the flow of profit of each firm \(i\) starting at a given \(t_0\) depends on a demand function \(Q_j(p_t)\), a constant marginal cost (normalized to be zero for all firms), and the vector of prices charged in all future periods, \(p_i = [p_{i,t_0}, p_{i,t_0+1}, ..., p_{i,\infty}]\).

Firms adopt a standard Bertrand - Nash reversion trigger strategy. For each \(t\), \(p_{i,t} = p^c\) if for all firms \(p_{i,t-1} = p^c\), where \(i = 1, ..., N_j\) in the \(j\) market, and \(i =

\footnotetext{\(7\)Although in the example we have considered a market with one firm, here we prefer to avoid this extreme case, concentrating on cases where no firm is essential to cover demand. As we will see later, and as the example above shows, this assumption does not affect our result.}

\footnotetext{\(8\)We will usually omit the time index \(t\) to keep the notation less cumbersome.}

\footnotetext{\(9\)The demand functions are decreasing and concave, and they satisfy the conditions that make consumer surplus an adequate welfare measure.}

\footnotetext{\(10\)This is a simplifying assumption adopted for ease of exposition. The main result of the paper obtains even dispensing of that assumption (see Boffa, 2006).}
1,..., N in the interconnected market. Otherwise, i.e. if \( p_{i,t-1} < p^c \) for some \( i \), \( p_{i,t} = p^b = c \), which obviously represents a credible punishment.

### 3.1 Equilibrium analysis: separated markets

We focus on characterizing equilibria in an oligopoly supergame when markets are separate and firms are capacity constrained. Capacity constraints may limit the profit achievable by each firm in the collusive agreement, but especially they make deviation less attractive, as a firm cannot serve the whole market. Hence, they widen the set of discount factors for which a cartel is sustainable.

Moreover, unlike the case of collusion with unlimited capacity, when firms are small the feasibility of collusion depends on the collusive price firms coordinate on. Hence, under capacity constraints, it may well happen that collusion at the monopoly price is not feasible, while a cartel coordinating at a lower price \( c < p^c < p^{mon} \) is. To see this, consider that for \( p^c \) to be an equilibrium price, the following has to hold:

\[
q_i (p^c) (p^c - c) \geq (p^c - c) \min (k, Q_i (p^c))
\]

(5)

where the optimal deviation price is arbitrarily close to the collusive price. Suppose that \( p^c \leq p^{mon} \) and that the collusive price is such that \( k < \sum_i q_i^c (p^c) \). In this case, the optimal deviation output\(^\text{11}\) will be \( q_i = k \), and (5) can be written as

\[
q_i (p^c) \geq k
\]

(6)

This condition depends on the collusive price, and in particular it holds more easily when \( p^c \) is low and thus \( q_i^c \) is large.

Let us now focus on the features of equilibrium prices in greater depth. Analogously to the standard unconstrained case, it remains true that, as long as firms have an aggregate capacity sufficient to supply the monopoly quantity, the collusive price cannot exceed monopoly price. Given that we maintain assumption (4), we prove the statement only for such set of values of \( k \).

**Proposition 1** Under the NEC condition, \( p^c > p^{mon} \) is never an aggregate profit maximizing equilibrium.

**Proof.** We first prove that if \( p^c = p' > p^{mon} \) is sustainable, then also \( p^c = p^{mon} \) is sustainable. From the IR constraint,

\[
q_i^c (p' - c) \geq (p' - c) \min (k, q (p^*))
\]

(7)

where \( p^* \) is the optimal deviation price. Notice that we must have \( p^* \geq p^{mon} \). Suppose not, i.e. assume \( p^* = p'' < p^{mon} \). In this case, firm \( i \) would profit

\(^{11}\)Notice that in this case we will have rationing, with \( k_i \) units sold at \( p^c - \epsilon \) and others sold at \( p^c \). This argument, which focuses on the incentive of firm \( i \), does not depend on the rationing rule.
from deviating to \( p^* = p^{\text{mon}} \), as it clearly results from the observation of the deviation profit expressions \( \pi_i^{\text{dev}} = (p^* - c) \min (k, q(p^*)) \). If \( k \geq q(p^{\text{mon}}) \), then \( \pi_i^{\text{dev}} = (p^{\text{mon}} - c) q(p^{\text{mon}}) > (p'' - c) \min (q(p''), k) \); if \( q(p') < k < q(p^{\text{mon}}) \), then \( \pi_i^{\text{dev}} = (p(k) - c) k > (p'' - c) k \); if \( k \leq q(p') \), then \( \pi_i^{\text{dev}} = (p' - c) k > (p'' - c) k \).

Hence, when \( k \geq q(p^{\text{mon}}) \), colluding at the monopoly price \( p^c = p^{\text{mon}} \) increases the collusive profit while leaving the deviation profit unchanged. When \( q(p') < k < q(p^{\text{mon}}) \), colluding at the monopoly price \( p^c = p^{\text{mon}} \) increases the collusive profit more than the deviation profit. Finally, when \( k \leq q(p') \), colluding at the monopoly price \( p^c = p^{\text{mon}} \) is sustainable (moreover, in the first two cases, \( p^c = p^{\text{mon}} \) is sustainable for a larger set of discount factors than \( p^c = p' > p^{\text{mon}} \)).

When many prices are sustainable, firms coordinate by assumption on the price that maximizes aggregate profit, which in this case is \( p^{\text{mon}} \). Hence, when both \( p' > p^{\text{mon}} \) and \( p^{\text{mon}} \) are sustainable, \( p^{\text{mon}} \) will be the equilibrium collusive price.

Q.E.D. ■

We can now characterize the equilibrium price in the oligopoly supergame in market \( j \), denoted by \( p^c_j \), in the following:

**Proposition 2** Under (4), the aggregate profit maximizing equilibrium price of the supergame is:

\[
p^c_j = \begin{cases} 
p^{\text{mon}}_j & \text{if } N_j \leq \frac{\max \left( \frac{Q_j(p^{\text{mon}})}{k}, 1 \right)}{(1-\delta)k}, \\
p_{(N_jk(1-\delta)) \in (c, p^{\text{mon}})} & \text{if } \frac{\max \left( \frac{Q_j(p^{\text{mon}})}{k}, 1 \right)}{(1-\delta)k} \leq N_j \leq \frac{\max \left( \frac{Q_j(c)}{k}, 1 \right)}{(1-\delta)k}, \\
c & \text{if } N_j \geq \frac{\max \left( \frac{Q_j(c)}{k}, 1 \right)}{(1-\delta)k}. \end{cases}
\]

(8)

See the Appendix for the Proof.

Some comments are in order. Starting from the bottom of (8), as usual a very high number of firms (relative to the discount factor) leads to a supergame equilibrium which is a mere repetition of static Bertrand outcomes.

As the number of competitors decreases, output decreases and equilibrium price increases until the monopoly level is reached. Firms will find it easier to sustain a monopolistic cartel when the following holds:

- when capacity constraints do not matter, the number of firms is sufficiently small, in that each individual firm is able to supply the monopoly output (as in the standard case of collusion);
- when capacity constraints matter, in that they do not allow any individual firm to supply the monopolistic output, aggregate capacity is sufficiently low.
Moreover, collusion arises at an intermediate price (between monopoly and competition) if and only if the three following conditions hold simultaneously:

- firms are relatively numerous (so that, were they to have unlimited capacity, they would not be able to sustain a cartel);
- capacity constraints are such that each individual firm cannot produce the competitive outcome (this condition is represented by $k < Q_j(c)$, which holds in (8)), when $p_{mon} > p^c > c$; \(^{12}\)
- aggregate capacity is above discounted monopoly, but below the discounted competition level (otherwise, they would find it rational to produce the Bertrand output).

4 **Integration and welfare reduction**

Having discussed the equilibrium in an isolated market, we now turn to consider the effects of integrating the two markets. Our goal consists in showing that interconnection may, for certain values of the parameters, reduce welfare.

We begin by characterizing the outcome of the dynamic game when the two markets are integrated. In this case, we only have one price, $p_{ic}$.

Under our usual assumption that condition (4) holds in both markets\(^ {13}\), the collusive price in the supergame played by $N$ firms in the interconnected market is the following:\(^ {14}\)

$$p_{ic} = \begin{cases} 
  p_{mon} & \text{if } N \leq \frac{\max\left(\frac{(Q_A+Q_B)(p_{mon}^c)}{k}, 1\right)}{\left(1-\delta\right)} \\
  p\left(Nk(1-\delta)\right) & \text{if } \frac{\max\left(\frac{(Q_A+Q_B)(p_{mon}^c)}{k}, 1\right)}{\left(1-\delta\right)} < N \leq \frac{\max\left(\frac{(Q_A+Q_B)(p_{mon}^c)}{k}, 1\right)}{\left(1-\delta\right)k} \\
  c & \text{if } N > \frac{\max\left(\frac{(Q_A+Q_B)(p_{mon}^c)}{k}, 1\right)}{\left(1-\delta\right)} 
\end{cases}$$

(9)

As for the relationship between integrated (collusive) price and the (collusive) prices in the separated markets, we can establish the following result.

**Proposition 3** If the NEC condition (4) holds, $p_{ic} \leq \max(p_A^c, p_B^c)$, i.e., the price in the interconnected market has to be lower or equal to the higher of the prices in the two nodes.

The proof is reported in the Appendix.

The Proposition shows that the price cannot strictly increase in both countries. However, this does not exclude the event that the price increases in one

\(^{12}\)If this condition did not hold, we would revert back to the standard case of no capacity constraints.

\(^{13}\)So that in each market the only Bertrand-Nash equilibrium outcome has $p = c$.

\(^{14}\)The proof of this statement would be a trivial replica of the proof of Proposition 2 and is thus omitted.
country, while remaining constant in the other one. We explore this occurrence in the next section.

Total surplus in market $j$ is $TS_j$, the sum of consumer surplus $CS_j$ and aggregate profit $\Pi_j$. Hence, total surplus is:

$$TS_A = \int_{v=p_j}^{\rho^{res}} Q_j(v) dv - cQ_j(p_j)$$

where $\rho^{res}$ denotes the reservation price, and analogously in the $B$ market. In the interconnected market,

$$TS_{ic} = \int_{v=p_{ic}}^{\rho^{res}} (Q_A + Q_B)(v) dv - c(Q_A + Q_B)p_{ic}$$

Interconnection lowers total welfare if and only if:

$$TS_A + TS_B > TS_{ic}$$

We now investigate under what conditions this may happen.

**Remark 4** As long as $p_A \neq p_B$, a sufficient condition for $TS_{ic} < TS_A + TS_B$ is that $p_{ic} \geq \max(p_B, p_A)$

This is very intuitive, in that under this condition welfare is not increased in either market. Obviously, given Proposition 3, the only relevant case is the one where $p_{ic} = \max(p_B, p_A)$. However, could such a price be the outcome of integration?

In what follows, we provide sufficient conditions under which interconnection increases the price in one market, while remaining same in the other one. Export of collusion results from the interplay between collusive output and aggregate capacity. If the relationship were linear, collusive output in the integrated market would simply equal the sum of the collusive outputs in the non-integrated markets, and the price would be an average. However, the non-linearity broadens the set of possible outcomes, and makes it possible that price in the interconnected market equals the higher of the two prices.

**Exporting collusion** The ability to collude mainly depends on two countervailing factors. First, it depends on the number of firms, which increases with interconnection. This makes collusion harder in the interconnected market (pro-competitive effect), and tends to raise welfare.

Second, it depends on the relationship between aggregate capacity and the size of the market both with separation and with integration. Aggregate capacity determines the minimum output (i.e., the maximum price) that can be sustained in the collusive agreement, when the capacity constraint does not allow any single firm to produce by itself the monopoly output (possibly anti-competitive effect).

The reason why this happens may be better understood by the example provided in Section 2. Intuitively, interconnection is being exported from the
market with the smallest number of firms to the integrated market. The following proposition characterizes a set of sufficient conditions under which collusion is exported.

**Proposition 5** Suppose \( p^c_A = p^{mon}_A \) and \( p^c_B < p^{mon}_B \). A sufficient condition for interconnection to reduce overall welfare is:

\[
N \in \left( \frac{Q_A(c) + Q_B(c) + k}{k}, \max \left( \frac{(Q_A + Q_B)(p^c_A)}{k}, 1 \right) \right)
\]

and there are always parameters configurations, such that this interval is not empty.

**Proof.** For \( p^c_A = p^{mon}_A \) we must have \( \frac{Q_A(c) + k}{k} \leq N_A \leq \max \left( \frac{Q_A(p^{mon}_A)}{k}, 1 \right) \), while for \( p^c_B \) to be lower than monopoly price \( p^{mon}_B \) we must have \( N_B > \max \left( \frac{Q_B(p^{mon}_B)}{k}, 1 \right) \). If (10) is met, then \( p^c_A = p^{mon}_A \), which by the previous Remark implies that interconnection has reduced welfare. The only thing to prove is that the set of parameters for which these prices are equilibrium prices is non-empty.

Assume \( k < \min \left( \frac{Q_A(p^{mon}_A)}{k}, \frac{Q_B(p^{mon}_B)}{k} \right) \). It has to be \( \frac{Q_A(c) + k}{k} \leq N_A \leq \frac{Q_A(p^{mon}_A)}{k} \), \( \frac{Q_B(p^{mon}_B)}{k} > N_B > \frac{Q_B(c)}{k} \). For there to be a set of values for which the (10) holds true, one needs

\[
\frac{Q(c) + k}{k} \leq Q_A \left( \frac{p^{mon}_A}{k(1 - \delta)} \right)
\]

which becomes

\[
k \leq \frac{Q_A(p^{mon}_A)}{(1 - \delta)} - Q(c)
\]

which certainly holds for \( p^c_A = p^{mon}_A \) to be true.

Q.E.D.

The implication of this result is that we always have a non empty set of parameters, such that if two markets are interconnected, firms may end up coordinating on the higher of the two previous prices. In this way, the high price country exports (very effective) collusion into the country where collusion was relatively less damaging.

To provide the intuition for this result, notice that the outcome of the game depends on the relationship between aggregate capacity and market size\(^{15}\). In this environment, there may exist situations in which the two isolated markets sustain different outcomes. In market A aggregate capacity is smaller, and as a result monopoly pricing emerges. Notice that it may be the case that a

\(^{15}\)Notice that, given our assumption on symmetric capacity, heterogeneity in aggregate capacity is generated by changes in the number of firms.
small increase in the number of firms operating in market A does not affect this outcome: in other words, even with (little) extra capacity, monopoly would prevail. Market B has a larger number of firms, with too much available capacity to be able to sustain collusion at monopoly price.

After integration, we may think of firms operating in the market B to split into two groups. One serves customers located in market B, and this group is composed of exactly the number of firms that allows the monopolistic outcome to prevail in this market. The other group serves customers in market A, thus providing this market with extra capacity, but not enough to thwart the emergence of the monopolistic outcome in the market A. We can view interconnection as a way to shift capacity from one group of customers to the other in order to “better” collude.

Obviously, the set of parameters which leads to this result may or may not be very large, but there always exists a set of values which makes this result possible.

5 Conclusions

While most of the established literature claims that market interconnection leads to greater competition, our results suggests that it may instead foster collusion, leading to an overall welfare reduction.

Limits on capacity increase the collusive potential of a market, as, in a repeated game, they may bind the deviation profit. This happens precisely when the aggregate output produced in the cartel could not be produced by a single individual firm, since it exceeds its capacity. In such a situation, the collusive quantity is a (non-linear) function of aggregate capacity. When aggregate capacity is below a certain threshold, whose value depends on the demand function, the cartel may be run as a monopoly; for values of aggregate capacity above the threshold, the collusive price decreases as aggregate capacity increases, until the price, for a sufficiently high level of aggregate capacity, equals the competitive one. From that level of aggregate capacity on, collusion is not feasible, and the prevailing price is the competitive one. After integration, both aggregate capacity and the threshold increase. It may happen that, while before interconnection only the capacity one market was below the threshold, after interconnection the overall capacity (the sum of the capacity in the two individual markets) is below the overall threshold (given by the sum of the two threshold). Hence, welfare decreases as a result of interconnection.

More rigorously, outcome in the supergame results from the interplay between collusive output, market demand, and aggregate capacity. If the relation were linear, collusive output in the integrated market would simply equal the sum of collusive output in the two disintegrated markets, and the price would be an average. However, the non-linearity broadens the set of possible outcomes; namely, it might occur that the price in the interconnected market simply equals the price in the higher of the two separate markets.

Although most of the analysis is carried out under the assumption of sym-
metric capacity, it is easy to see that nothing substantial would change, if we relaxed such an assumption by introducing the possibility of differences in firms’ capacity\textsuperscript{16}. The paper has focused on capacity constraints, and not on increasing cost functions for expositional simplicity. However, as increasing cost functions possess most of the qualitative properties of capacity constraints, most of the results hold even in the case of continuously increasing cost functions.

Moreover, given our interest in showing that the effect of market integration on welfare is not unambiguous, we have only provided sufficient conditions for welfare reduction (due to a diffusion of collusion after market integration). The fact that there is always a set of parameters which may satisfy such conditions is a striking feature of our result. A possible extension should take into account that our paper considers two extreme situations, total separation versus full integration, neglecting the intermediate scenario of partial integration, in which the maximum flow of goods from one node to the other is limited, yet positive. In that case, it could be possible to examine the interplay of productive capacity constraints and transmission capacity constraint which may lead to a similar result.

The findings may be regarded as quite surprising, since, in most of the ongoing policy debates, it is often taken for granted that the interconnection of different nodes - for example in the electricity or in the railways sectors - has beneficial competitive effects. The paper, on the contrary, calls for a case-by-case analysis of the competitive effects of integration. In particular, an evaluation of the market microstructures in the two relevant regions may be necessary in order to evaluate the welfare impact of the interconnection.

References


\textsuperscript{16}If, in a context of asymmetric capacity, no firm’s capacity exceeds the monopoly output of each market, the results of the paper extend in a straightforward way; otherwise, a more complicated analysis.


6 Appendix

**Proof of Proposition 2** A collusive agreement in market $j$ (the index $j$ is omitted wherever possible) is sustainable if and only if

$$q_i (p_j^c) \geq (1-\delta) \min (k, N_j q_i (p_j^c))$$

Assume first

$$\min (k, N_j q_i (p_j^c)) = k$$

i.e., no firm has sufficient capacity to produce the whole collusive equilibrium output. Under this assumption, a collusive agreement requires

$$q_i (p_j^c) \geq (1-\delta) k$$

and the equilibrium price which maximizes aggregate profit is then $p_j^c$ if $k < Q_j (p_j^c)$ where:

$$p_j^c = \begin{cases} p_j^{mon} & \text{if } (1-\delta) N_j k \leq Q_j (p_j^{mon}) \text{ and } k \leq Q_j (p_j^{mon}) \\ p_j ((1-\delta) N_j k) & \text{if } Q_j (p_j^{mon}) \leq (1-\delta) N_j k \leq Q_j (c) \text{ and } k \leq (1-\delta) N_j k \\ c & \text{if } Q_j (c) \leq (1-\delta) N_j k \text{ and } k \leq (1-\delta) N_j k \end{cases}$$

Assume now

$$\min (k, N_j q_i (p_j^c)) = N_j q_i (p_j^c)$$

i.e., each individual firm has sufficient capacity to produce the entire collusive equilibrium outcome. In such case, we revert back to the standard IR constraint.
for collusion when firms have unlimited capacity which entails (if $k \geq Q^c$):

$$p_j^c = \begin{cases} 
  c, & \text{if } \delta \geq \frac{N_j}{N_j - 1} \\
  p_j^{mon}, & \text{if } \delta \leq \frac{N_j}{N_j - 1}
\end{cases}$$ \hspace{1cm} (14)

Combining (12) and (14), one obtains the aggregate profit maximizing equilibrium of the supergame. We start by conditions under which a cartel coordinating on monopoly price can be sustained:

$$p_j^c = p_j^{mon} \text{ if } i \text{ OR } ii \text{ OR } iii \text{ holds : } \begin{cases} 
  i. N_j \leq \frac{1}{(1-\delta)} \text{ and } k \leq Q_j (p_j^{mon}) \\
  ii. N_j \leq \frac{Q_j (p_j^{mon})}{(1-\delta)k} \text{ and } N_j \geq \frac{1}{(1-\delta)} \\
  iii. N_j \leq \frac{1}{(1-\delta)} \text{ and } k \geq Q_j (p_j^{mon})
\end{cases}$$ \hspace{1cm} (15)

This may be rewritten as:

$$p_j^c = p_j^{mon} \text{ if } i \text{ OR } ii \text{ holds : } \begin{cases} 
  i. N_j \leq \frac{1}{(1-\delta)} \\
  ii. \frac{1}{(1-\delta)} \leq N_j \leq \frac{Q_j (p_j^{mon})}{(1-\delta)k}
\end{cases}$$

$$p_j^c = p_j^{mon} \text{ if } N_j \leq \frac{\max \left( \frac{Q_j (p_j^{mon})}{k}, 1 \right)}{(1-\delta)}$$

Now, by combining (12) and (14), we examine conditions under which an intermediate price between monopoly and competition emerges as the aggregate profit maximizing supergame equilibrium:

$$p_j^c = p_j \left( (1-\delta) N_j k \right) \text{ if } \frac{Q_j (p_j^{mon})}{(1-\delta)k} \leq N_j \leq \frac{Q_j (c)}{(1-\delta)k} \text{ and } N_j \geq \frac{1}{(1-\delta)}$$ \hspace{1cm} (16)

This may be rewritten as:

$$p_j^c = p_j \left( (1-\delta) N_j k \right) \text{ if } \frac{\max \left( 1, \frac{Q_j (p_j^{mon})}{k} \right)}{(1-\delta)} \leq N_j \leq \frac{\max \left( 1, \frac{Q_j (c)}{k} \right)}{(1-\delta)k}$$ \hspace{1cm} (17)

Finally, again by combining (12) and (14), we check under what conditions collusion cannot be sustained, and competitive price is prevailing:

$$p_j^c = c \text{ if } i. OR ii. \text{ holds : } \begin{cases} 
  i. \frac{Q_j (c)}{k(1-\delta)} \leq N_j \text{ and } \frac{1}{(1-\delta)} \leq N_j \\
  ii. N_j \geq \frac{1}{(1-\delta)} \text{ and } \frac{1}{(1-\delta)} \geq N_j
\end{cases}$$ \hspace{1cm} (18)

Notice that the collusion at an intermediate price with $\delta \geq \frac{N_j-1}{N_j}$ and $k \geq Q_j^c$, in spite of being an equilibrium, is never part of an aggregate profit maximizing equilibrium. Indeed, when $\delta \geq \frac{N_j-1}{N_j}$ and $k \geq Q_j^c$ firms can sustain a cartel at a monopoly price, and this maximizes their profit.
Rewriting (18), we obtain:

\[ p^c_j = c \text{ if } N_j \geq \frac{\max \left( \frac{Q_j(c)}{k}, 1 \right)}{(1 - \delta)} \]

Q.E.D.

**Proof of Proposition 3**  Suppose not, i.e. that \( p^c_{ic} > \max (p^c_A, p^c_B) \). For that to be true, \( p^c_{ic} > c \), hence either \( p^c_{ic} = p^\text{mon}_{ic} \) or \( p^\text{mon}_{ic} > p^c_{ic} > c \). Assume first \( p^c_{ic} = p^\text{mon}_{ic} \). Then, it has to be that \( p^c_A, p^c_B < p^c_{ic} = p^\text{mon}_A = p^\text{mon}_B \).

\[ p^c_A, p^c_B < p^\text{mon}_{ic} = p^\text{mon}_A = p^\text{mon}_B \]

requires:

\[ N \geq \frac{\max \left( \frac{Q_j(p^\text{mon}_{ic})}{k}, 1 \right)}{(1 - \delta)}, \quad j = A, B \]  \hspace{1cm} (19)

while \( p^c_{ic} = p^\text{mon}_{ic} \) requires

\[ N \leq \frac{\max \left( \frac{(Q_A+Q_B)(p^\text{mon}_{ic})}{k}, 1 \right)}{(1 - \delta)} \]  \hspace{1cm} (20)

By (19), it follows that \( N = N_A + N_B > \max \left( \frac{Q_A(p^\text{mon}_{ic})}{k}, 1 \right) + \max \left( \frac{Q_B(p^\text{mon}_{ic})}{k}, 1 \right) \geq \max \left( \frac{(Q_A+Q_B)(p^\text{mon}_{ic})}{k}, 1 \right) \), since \((Q_A + Q_B)(p^\text{mon}_{ic}) = Q_A p^\text{mon}_A + Q_B p^\text{mon}_B\), given the assumption on common reservation price across the two separate markets.

Hence, \( N > \max \left( \frac{(Q_A+Q_B)(p^\text{mon}_{ic})}{k}, 1 \right) \), in contradiction with (20).

Assume now \( p^\text{mon}_{ic} > p^c_{ic} > c \). The requirement that \( p^c_{ic} > \max (p^c_A, p^c_B) \)

\[ p^c_{ic} > (p^c_A, p^c_B), \]

\[ p^\text{mon}_{ic} > p^c_{ic} > c \]

requires

\[ p^c_{ic} = p(Nk(1 - \delta)) \text{ if } \max \left( \frac{(Q_A+Q_B)(p^\text{mon}_{ic})}{k}, 1 \right) < N \leq \frac{\max \left( \frac{(Q_A+Q_B)(p^\text{mon}_{ic})}{k}, 1 \right)}{(1 - \delta) k} \]

while \( p^c_{ic} > (p^c_A, p^c_B) \) requires

\[ p^c_A = \begin{cases} 
  p(N_Ak(1 - \delta)) & \text{if } \max \left( \frac{Q_A(p^\text{mon}_{ic})}{k}, 1 \right) \leq N_A \leq \frac{\max \left( \frac{Q_A(c)}{k}, 1 \right)}{(1 - \delta)} \\
  c & \text{if } N_A \geq \frac{\max \left( \frac{Q_A(c)}{k}, 1 \right)}{(1 - \delta)} 
\end{cases} \]

\[ p^c_B = \begin{cases} 
  p(N_Bk(1 - \delta)) & \text{if } \max \left( \frac{Q_B(p^\text{mon}_{ic})}{k}, 1 \right) \leq N_B \leq \frac{\max \left( \frac{Q_B(c)}{k}, 1 \right)}{(1 - \delta)} \\
  c & \text{if } N_B \geq \frac{\max \left( \frac{Q_B(c)}{k}, 1 \right)}{(1 - \delta)} 
\end{cases} \]

\(^{18}\)Notice that, given our assumption on equal reservation prices in markets A and B (and as a consequence in the interconnected market), \( p^\text{mon}_A = p^\text{mon}_B = p^\text{mon}_{ic} \).
Assuming \( \max_{(1-\delta)} \frac{Q_A(c)}{1} \leq N_A \leq \max_{(1-\delta)} \frac{Q_A(c)}{1} \) and \( \max_{(1-\delta)} \frac{Q_B(c)}{1} \leq N_B \leq \max_{(1-\delta)} \frac{Q_B(c)}{1} \), then

\[
q_{ic}^c = (N_A + N_B) k (1 - \delta) = q_A(p_A^c) + q_B(p_B^c) = N_A k (1 - \delta) + N_B k (1 - \delta)
\]

Hence, if \( p_A^c = p_B^c \), then \( p_{ic}^c = p_A^c = p_B^c \); if \( p_A^c \neq p_B^c \), then \( p_{ic}^c \) is at an intermediate level between \( p_A^c \) and \( p_B^c \), so \( p_{ic}^c \leq \max \{p_A^c; p_B^c\} \). This is a contradiction with \( p_{ic}^c > \max \{p_A^c; p_B^c\} \). Assuming the supergame equilibrium entails the competitive outcome in one of the markets (say, without loss of generality, market A), then \( N_A \geq \max_{(1-\delta)} \frac{Q_A(c)}{1} \), then

\[
q_{ic}^c = (N_A + N_B) k (1 - \delta) \leq N_B k (1 - \delta) + Q_A(c)
\]

In this case, one of the separate market (say, without loss of generality, A) has \( p_A^c = c \). Since \( p_{ic}^c > c \), then it has to be that \( p_{ic}^c < p_B^c \). This is a contradiction with \( p_{ic}^c > \max \{p_A^c; p_B^c\} \). Q.E.D