Expected Option Returns
and the Structure of Jump Risk Premia

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Abstract

Expected returns on calls with high strike prices have empirically been found to be negative in certain cases. Since a call can be considered a levered investment in the stock, this result seems puzzling. Our paper analyzes expected option returns in models with stochastic volatility and jumps. We show that the size of the jump risk premium and its decomposition into a premium for jump intensity risk, jump size risk, and jump variance risk has a significant impact on expected option returns. In particular, the empirically documented negative expected returns on out-of-the-money calls can be well explained by a risk premium for the variance of the jump size.

Keywords: Option returns, jump risk premia, volatility risk premium

JEL: G12, G13
1 Introduction and Motivation

The analysis of expected returns on options has recently moved into the focus of the attention in the literature on asset pricing and derivatives. Examples for papers in this area are Coval and Shumway (2001), Jones (2006), Ibàñez (2007), Ni (2008), Constantinides, Jackwerth, and Savov (2009), Christoffersen, Heston, and Jacobs (2010), and Bakshi, Madan, and Panayotov (2010). For a long time especially returns on out-of-the-money put options seemed rather puzzling, mostly because of their sheer size, with the usual estimates for puts on major stock market indices ranging between $-50$ and $-80$ percent per month. However, as shown recently by Broadie, Chernov, and Johannes (2009) (BCJ), these numbers are far less out of line with standard option pricing models, as they may seem at first sight. Option returns are inherently very noisy, so that models with stochastic volatility and jumps are well capable of producing expected returns which are not significantly different from the numbers obtained from empirical data.

Average returns on calls, however, seem (even more) puzzling. Empirical studies show that average returns on out-of-the-money (OTM) calls are negative. This is documented for options on single stocks (see Ni (2008)), for options on the S&P 500 and also for options on a number of broad market indices in other countries (see Bakshi, Madan, and Panayotov (2010)). These findings seem counterintuitive, since a call option is usually considered to represent a levered investment in the underlying asset and should thus, given a positive risk premium for the underlying, exhibit an even larger positive expected (excess) return. Coval and Shumway (2001) show that expected call returns are positive in any model in which the pricing kernel is a decreasing function of the stock price, which is a standard assumption in many asset pricing models.

Negative expected returns on OTM calls thus suggest that the pricing kernel is upward sloping for high stock prices. The observation that the pricing kernel is not always decreasing in the stock price has also been made in studies which extract the relative risk aversion from the risk-neutral distribution (estimated from a cross-section of option prices) and the physical distribution (estimated from a time series of the underlying).\footnote{See e.g. Jackwerth (2000), Rosenberg and Engle (2002).} Christoffersen, Heston, and Jacobs (2010) explain this behavior of the pricing kernel in the context of the GARCH option pricing model of Heston and Nandi (2000) by a negative premium for volatility risk. Bakshi, Madan, and
Panayotov (2010) rely on an equilibrium model with heterogeneous expectations where a certain proportion of the investors is very pessimistic about future stock returns. They think that the stock will have a lower return than the risk-free asset and thus short the stock. This in turn makes payoffs in states with high stock prices precious to them, so that the prices of insurance devices like OTM calls are bid up and their expected returns go down.

Our paper contributes to the literature by providing an in-depth analysis of expected option returns as a function of risk premia. In the context of a model with stochastic volatility and jumps, we consider premia for stock diffusion risk, stock jump risk, and variance risk. We focus on the impact of how the total risk compensation for the underlying (the equity premium) is divided not only between diffusive and jump risk, but also between the various components of jump risk, i.e., between the premia for jump intensity, the mean of the jump size, and its variance. We show that variations in the premia for volatility risk and, in particular, for the various elements of jump risk can generate rich patterns of expected option returns and that there are various combinations of risk premia which easily result in negative expected excess returns on calls.

One of the main messages from our analysis is that the simple intuition for expected option returns from models like Black and Scholes (1973) does not carry over to more general models with stochastic volatility and jumps. This implies that the properties of expected call returns cannot be derived by simply considering the option a levered investment into the underlying. Especially the exact composition of the total premium for jump risk matters substantially, so that ad hoc assumptions about the structure of this premium are not innocuous. Furthermore, popular assumptions about jump risk premia, based for example on the paper by Naik and Lee (1990), are not able to generate negative expected call returns, while we show that they are perfectly compatible with standard option pricing models where e.g. the variance of jump sizes carries a premium, i.e., the uncertainty about the jump size is higher under the risk neutral measure $\mathbb{Q}$ than under the physical measure $\mathbb{P}$.

We now give an overview of the main findings of our analysis. First and most importantly, negative expected returns for call with high strike prices are by no means anomalous. In widely accepted option pricing models with stochastic volatility

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2Pan (2002) and Eraker (2004) restrict the jump intensity and the jump variance to be the same under the physical and the risk-neutral measure. Broadie, Chernov, and Johannes (2007) impose this restriction on the jump intensity only.
and jumps in the stock price and in volatility (similar to the ones suggested by Bakshi, Cao, and Chen (1997), Bates (2000), and Duffie, Pan, and Singleton (2000)), this phenomenon already occurs when none of the risk factors in the model carries a premium except jump intensity risk, i.e., when the jump intensity is the only parameter with different values under the physical and the risk-neutral measure. Of course, one might argue here that this is a somewhat restrictive assumption to set all risk premia to zero except this one. However, it turns out that negative expected call returns are a rather common phenomenon in the context of this model. For example, they also occur when the model is estimated on the basis of a long time series of returns on the S&P 500 index together with a large cross-section of associated option prices.

Besides the specific issue of negative expected call returns the general research question we are tackling in this paper is the impact of different specifications of risk premia on expected option returns. Our investigation shows that the fine structure of these premia matters much more for calls than for puts, given the usual parameter scenarios known from the literature on empirical option pricing (most importantly, a negative correlation between stock returns and volatility changes and a negative average jump size in the stock price). This can also be seen from Figure 1 showing expected returns on calls and puts as a function of moneyness (defined as strike price divided by current stock price) for various times to maturity and for various versions of the model which differ in terms of the assumptions on risk premia. Expected returns on put options are negative, large in absolute values and monotonically increasing in moneyness. For calls on the other hand the picture is much richer.

When stock diffusion risk is the only priced risk factor the pattern in expected call returns is perfectly in line with the simple Black-Scholes case analyzed by Coval and Shumway (2001) in that they are uniformly positive and monotonically increasing in moneyness. This also holds when the premia for jump risk are set according to Naik and Lee (1990). If we deviate from the premia for jump intensity risk and jump size risk implied by their model and further assume e.g. that only jump intensity carries a risk premium, the results change substantially.

When we hold the total equity risk premium constant, but vary the share of the jump- and diffusion-induced parts by compensating an increase in the jump-induced premium by an appropriate decrease in the diffusion part, we find that increasing the jump component of the premium mostly leads to higher option prices and consequently lower expected returns.
Finally, we study in even more detail if the ‘fine structure’ of the jump premium matters. The total jump part of the equity premium can be decomposed into the shares of the premia (i.e., the differences between $P$ and $Q$) for jump intensity, mean jump size, and variance of the jump size. We hold the overall contribution of the jump components to the equity premium constant, then take all three possible pairs of jump-related premia and vary one of the two premia while compensating the effect of this change through variations in the other one. For example, we increase the jump intensity under the risk neutral measure, but simultaneously reduce the average jump size under $Q$.

It turns out that the structure of risk premia matters substantially, even at this deep level. For example, a decrease in the jump intensity and a more extreme average jump size under the risk-neutral measure result in expected returns on short-term calls which are positive for the whole range of moneyness levels we consider in our analysis, while in the opposite case even in the money calls exhibit negative expected returns.

The most striking result, however, is obtained for the trade-off between the variance of jumps in stock returns and either the intensity of jumps or the average size of these jumps. If we change the variance of the jump size (and offset the impact on the equity risk premium by changing one of the other two parameters), all kinds of patterns can emerge. For short-term calls a smaller variance produces expected call returns which are positive and mostly increasing in moneyness. In the opposite case, when the risk-neutral variance of the jump size is larger than under the physical measure, expected returns on short-term calls turn negative when the option is at the money and decrease strongly afterwards, so that they reach values of almost $-1$ for the right end of the moneyness spectrum.

Overall, our analysis shows that the structure of risk premia in flexible option pricing models is an important determinant for expected option returns. So when researchers think about imposing restrictions on a subset of parameters when estimating the risk-neutral measure, they should be aware of the fact that this is not without consequences for the properties of expected option returns and especially for the question whether empirically observed returns are indeed anomalous or not.

The remainder of this paper is organized as follows. In Section 2, we introduce the model setup and discuss how expected option returns can be computed in (almost) closed form. The results stated in this section are not new, they are mostly shown for the sake of completeness. Section 3 studies the decomposition of the ex-
pected option returns into the contributions of the various risk factors. Section 4 concludes.

2 Computing Expected Option Returns

2.1 Model Setup

We consider a model with stochastic volatility and common jumps in the stock price and its volatility (SVCJ model). The dynamics of the stock price $S$ and the local variance $V$ under the physical measure $\mathbb{P}$ are given by

$$
\begin{align*}
dS_t &= (r + a_t - \bar{\mu}_P)S_t dt + \sqrt{V_t}S_t dW_t^{S,\mathbb{P}} + (e^\xi - 1) S_t dN_t, \\
dV_t &= \kappa_P (\theta - V_t) dt + \sqrt{V_t} \sigma_V \left( \rho dW_t^{S,\mathbb{P}} + \sqrt{1 - \rho^2} dW_t^{V,\mathbb{P}} \right) + \Psi dN_t.
\end{align*}
$$

Equations (1) and (2) show that $W_t^{S,\mathbb{P}}$ and $W_t^{V,\mathbb{P}}$ are independent Wiener processes, $N_t$ is a Poisson process with constant intensity $\lambda_P$. The jump size $\Psi$ in the variance is exponentially distributed with expectation $\mu_V$, i.e., $\Psi \sim \exp\{\mu_V\}$. Conditional on the realized variance jump, the jump size $\xi$ in the stock return follows a normal distribution: $\xi \sim \mathcal{N}(\mu_S + \rho_j \Psi, (\sigma_S^2)^2)$, and the mean jump size in the stock price is

$$
\hat{\mu}_P = \frac{\exp\{\mu_S + (\sigma_S^2)^2\}}{1 - \rho^2 \mu_V} - 1.
$$

We denote the expected excess return on the stock at time $t$ by $a_t$.

Our SVCJ model nests several option pricing models. Setting $\lambda^P = \sigma_V = 0$ and $V_t = \theta$ gives the Black-Scholes model, $\Psi = \sigma_V = 0$ and $V_t = \theta$ gives the model with stochastic jumps (SJ) by Merton (1976). For the Heston (1993) model with stochastic volatility (SV), we set $\lambda^P = 0$, and the SVJ model developed by Bakshi, Cao, and Chen (1997) and Bates (1996) results for $\Psi = 0$.

The dynamics under the risk-neutral measure $\mathbb{Q}$ are

$$
\begin{align*}
dS_t &= (r - \hat{\mu}^Q \lambda^Q)S_t dt + \sqrt{V_t}S_t dW_t^{S,\mathbb{Q}} + (e^\xi - 1) S_t dN_t, \\
dV_t &= \kappa^Q (\theta - V_t) dt + \sqrt{V_t} \sigma_V \left( \rho dW_t^{S,\mathbb{Q}} + \sqrt{1 - \rho^2} dW_t^{V,\mathbb{Q}} \right) + \Psi dN_t.
\end{align*}
$$

The mean reversion speed and the mean-reversion level of the variance under the two measures are related by

$$
\begin{align*}
\kappa^Q &= \kappa^P + \eta_V, \\
\kappa^Q \theta^Q &= \kappa^P \theta^P.
\end{align*}
$$

5
where $\eta_V$ denotes the premium for (total) volatility diffusion risk. Thus, $\eta_V V_t$ is the compensation for the innovation $\sqrt{V_t} \sigma_V (\rho dW_t^S + \sqrt{1-\rho^2} dW_t^V)$. It can be represented as $\sigma_V (\rho \eta_S + \sqrt{1-\rho^2} \tilde{\eta}_V) V_t$, where $\tilde{\eta}_V V_t$ and $\eta_S V_t$ are the compensations for pure volatility diffusion risk $\sqrt{V_t} dW_t^V$ and $\sqrt{V_t} dW_t^S$, respectively.

The intensity of the jump process under $Q$ is denoted by $\lambda^Q$. For the jump sizes, we assume that they still follow an exponential and a normal distribution, respectively, but that all parameters of these distributions can change from $P$ to $Q$, i.e. $\Psi \sim \exp \{\mu^Q\}$ and $\xi \sim \mathcal{N} (\mu^Q_S + \rho^Q_j \Psi^Q, (\sigma^Q_S)^2)$. In most papers dealing with this model $\rho^Q_j$ is set equal to zero. This restriction is usually justified (see, e.g., Broadie, Chernov, and Johannes (2009)) on the grounds that this parameter is notoriously hard to estimate. Since we are interested in a theoretical analysis of the SVCJ model we relax this restriction by using $\rho^Q_j = 0$ only as a benchmark case and then analyzing expected option returns also for other values of this parameter.

The drift of the stock price depends on the risk-free rate $r$, on the compensator for the jump term, and on the equity risk premium $a_t$. The latter consists of the premia for stock diffusion risk, $\eta_S V_t$, and for stock jump risk, $\lambda^P \bar{\mu}^P - \lambda^Q \bar{\mu}^Q$, so that

$$a_t = \eta_S V_t + \lambda^P \bar{\mu}^P - \lambda^Q \bar{\mu}^Q. \quad (6)$$

### 2.2 Closed-Form Expressions for Expected Option Returns

Following the analysis in Broadie, Chernov, and Johannes (2009) we focus on so-called expected maturity returns $R_{0,T}$, i.e. on returns from holding an option until its maturity date $T$. With a positive probability of a zero payoff for the option at $T$, expected log returns would be equal to minus infinity. We therefore use discrete instead of continuous returns, i.e.

$$R_{0,T} \equiv E^P \left[ \frac{C_T}{C_0} - 1 \right].$$

The price of the option at $t = 0$ is given by the discounted expectation of the terminal payoff under the risk-neutral measure, so that $R_{0,T}$ can be written as

$$R_{0,T} = \frac{E^P [C_T]}{E^Q [e^{-rT} C_T]} - 1.$$

The option price $C_0$ can be calculated via Fourier transform, i.e. we need to find the characteristic function $\phi^Q(u)$ of $\ln S_T$ under the risk-neutral measure, with
\( \phi^Q(u) \equiv E^Q [e^{iu \ln S_T}] \). The expected future payoff can also be calculated via Fourier transform based on the characteristic function \( \phi^P(u) \). Both \( \phi^P(u) \) and \( \phi^Q(u) \) can be computed using standard methods, as shown in Duffie, Pan, and Singleton (2000).

Although the main object of interest is the expected return of an option when it is held until the maturity date, it is sometimes also useful to consider the local expected return of an option. Given the dynamics in Equations (1) and (2), Ito’s lemma and the fundamental partial differential equation satisfied by any contingent claim price yield

\[
E^P \left[ \frac{dC}{C} \right] = r dt + \left( \frac{\partial C}{\partial S} \cdot S \cdot \eta_S \cdot V dt + \frac{\partial C}{\partial V} \cdot \frac{1}{C} \cdot \eta_V \cdot V dt \right) + \left( \frac{E^P[C(Se^{\xi}, V + \psi)]}{C} - C \right) dt.
\]

(7)

Here, \( ER_S \) denotes the part of the total expected return which is due to the diffusive risk in the stock price, while \( ER_V \) is the analogous quantity for diffusive volatility risk. Finally, \( ER_J \) represents the jump part. Local expected returns can again be calculated in (semi-)closed form. An obvious case where the analysis of local expected returns is helpful is when they have a fixed sign for all states and all points in time, since then the expected return over the whole interval from \( t \) to \( T \) must have this sign as well.

When we analyze the composition of the jump part of the equity risk premium in detail, we will look at four components of \( ER_J \):

\[
ER_J = ER_{\lambda}^J + ER_{\mu}^J + ER_{\sigma}^J + ER_{V}^J
\]

(8)

The terms on the right-hand side represent the contribution of the premia for jump intensity (\( ER_{\lambda}^J \)), average jump size (\( ER_{\mu}^J \)), volatility of the jump size (\( ER_{\sigma}^J \)), and
Finally, volatility jumps ($ER^V$). The formal expressions for these terms are

$$ER^J_\lambda = \frac{E^P[C(Se^\xi, V)] - C}{C} (\lambda^P - \lambda^Q)$$

$$ER^J_\mu = \frac{E^P[C(Se^\xi, V)] - E^Q(\sigma^P_S)[C(Se^\xi, V)]}{\lambda^Q}$$

$$ER^J_\sigma = \frac{E^Q(\sigma^P_S)[C(Se^\xi, V)] - E^Q[C(Se^\xi, V)]}{\lambda^Q}$$

$$ER^J_V = \frac{E^P[C(Se^\xi, V + \psi)] - E^P[C(Se^\xi, V)]}{\lambda^P}$$

$$- \frac{E^Q[C(Se^\xi, V + \psi)] - E^Q[C(Se^\xi, V)]}{\lambda^Q}$$

where the expectations are taken over the joint distribution of the jump sizes $\xi$ and $\psi$. $ER^J_\lambda$ represents the part of the premium which is due to different jump intensities under $P$ and $Q$. For $\lambda^P = \lambda^Q$ it would be equal to zero. Analogously $ER^J_\mu$ is the portion of the local expected return due to a shift in the average jump size of the stock return. Note that here we need the volatility of the jump size to be the same under $P$ and $Q$, which is indicated by the notation $Q(\sigma^P_S)$ denoting the risk-neutral measure with $\sigma^Q_S = \sigma^P_S$. The term $ER^J_\sigma$ represents the impact of a change in the volatility of the stock price jump size from $P$ to $Q$, while $ER^J_V$ shows how the local expected return reacts to a premium for jumps in volatility.

### 3 Decomposing Option Returns

#### 3.1 General Approach

The main objective of our paper is to explain how different risk premia contribute to the expected returns on calls and puts with different strike prices and times to maturity. The model central to our analysis is the SVCJ model introduced in Section 2.1, since it represents one of the most flexible specifications of an option pricing model. In addition, we will also analyze the SVJ model without volatility jumps. There one can focus on the impact of the premia for pure stock price jumps, since there are by construction no additional effects caused by simultaneous jumps in volatility.

To study the impact of risk premia on expected option returns, we fix the physical measure $P$ and then vary the market prices of risk, which is equivalent
to varying the risk-neutral measure $Q$. An alternative would have been to fix the risk-neutral measure, in which case a variation in the market prices of risk would have led to a variation in the physical measure $\mathbb{P}$. Our choice is motivated by the fact that there is much less uncertainty in the estimation of $\mathbb{P}$ than there is in the estimation of $Q$. In particular, as shown, e.g., by Broadie, Chernov, and Johannes (2007), there are usually several numerically rather different parametrizations of the $Q$-measure which explain the cross-section of option prices equally well. Variation in expected option returns is thus mainly due to differences in the exact specification of the $Q$-measure (i.e., the market prices of risk) and not due to uncertainty about the dynamics of the underlying.\footnote{The results of the comparative statistics would not change if we fixed the risk-neutral measure and varied the physical measure.}

Our choice to fix the physical measure has the distinct advantage that it allows us to identify the effect of a certain parameter scenario on expected option returns by simply looking at its impact on current option prices. When option prices go up and the expected payoff under the physical measure remains unchanged, expected returns have to decrease and vice versa.

We estimate the physical measure $\mathbb{P}$ from the time series of the underlying prices. In terms of risk premia, this gives us the average total equity risk premium $\bar{a}$:

$$
\bar{a} = \eta_S E_{-\infty}^\mathbb{P}[V_t] + \lambda^\mathbb{P} \left( \frac{\exp \left\{ \mu^\mathbb{P}_S + \frac{(\sigma^\mathbb{P}_S)^2}{2} \right\} - 1}{1 - \rho^\mathbb{P}_J \mu^\mathbb{P}_V} \right) - \lambda^Q \left( \frac{\exp \left\{ \mu^Q_S + \frac{(\sigma^Q_S)^2}{2} \right\} - 1}{1 - \rho^Q_J \mu^Q_V} \right)
$$

where $E_{-\infty}$ denotes the expectation under the stationary distribution of $V$. There are, however, infinitely many possibilities to decompose a given premium into the premia for the various risk factors. First, we have to choose the relation between the premia for stock diffusion risk and stock price jump risk.\footnote{When we vary this relation, the decomposition of the equity risk premium into a constant component and a component proportional to $V$ changes.} The jump part of the equity premium in turn depends on the jump intensity under $Q$ and the expected size of stock price jumps, which again depends on the parameters of the joint distribution of the jump sizes. This structure leaves us with several degrees of freedom. If we only want to match a given jump risk premium, we can first decompose the product of jump intensity and expected jump size into its factors, and vary these while holding the product itself constant. At the next level we can choose the mean and variance of...
the log jump size subject to the restriction that we obtain the pre-specified expected jump size. In case of correlated jumps, we can also choose the average jump size in the variance and the dependence parameter. However, beyond these choices we ignore the additional degrees of freedom given by potential changes in the type of the jump size distribution.

A second type of risk premium which is of great interest in the context of option pricing is the variance risk premium. It is driven by the premium for diffusion risk and by the premium for variance jump risk. The latter is of course linked to the jump risk part of the equity risk premium, since in our SVCJ model, jumps in the stock price and in its variance happen simultaneously. They thus share the premium for jump intensity, and in case of correlated jump sizes, also the one for the jump size.

We structure our analysis of expected option returns as follows. In a first step we look at the impact of the individual risk premia one by one. Expected option returns are computed assuming an annual equity risk premium of 4%, which is assumed to be generated exclusively either by stock diffusion risk, by jump intensity risk, or by jump size risk, respectively. We also look at the impact of premia for volatility diffusion risk and jumps in volatility. This gives us an understanding of the fundamental impact of a given type of risk premium on expected option returns.

Afterwards, in the second step, we start from a given \( Q \)-measure and vary the parameters one by one. To make the different scenarios comparable, we compensate the change in the parameter of interest by a change in some other parameter, such that the total equity risk premium remains unchanged. For example, when varying the jump intensity premium we simultaneously change the premium for stock diffusion risk to keep the equity premium constant at 4%.

In this second step, our analysis has two main parts. First, we look at the pricing of diffusive versus jump risk. Second, we focus on the structure of the jump risk premium itself, i.e. on the relative importance of the premia for jump intensity, mean jump size, and variance of the jump size.

The basic parameter set is shown in Table 1. The \( P \)-parameters were estimated from the time-series of S&P 500 index prices from January 1996 to September 2008 using Markov Chain Monte Carlo (MCMC) techniques, as suggested, e.g., in Eraker, Johannes, and Polson (2003). The parameters obtained from this estimation are shown in the upper panel. The patterns in the parameters are in line with findings
in earlier studies. The correlation between index returns and volatility changes is strongly negative for both the SVJ and the SVCJ model, i.e. $\rho \ll 0$. The average jump size in the index level is negative, while that in the conditional variance is positive.

To estimate the $Q$-measure we make use of the theoretical restriction that the realized values of the index level and the conditional volatility at any time $t$ and in any state must be the same under $P$ and $Q$. Furthermore, theory dictates that only a subset of the model parameters can actually take on different values under the two measures. For the two models this is the case for all the parameters listed in the lower panel of Table 1. The different columns represent different scenarios concerning the parameters which are allowed to differ between the two measures. For the SVJ model we only consider the unrestricted case, where all free $Q$-parameters were estimated. The first column for the SVCJ model gives the analogous results (with the exception of $\rho^Q_J$, which is set to zero). Note that we do not show the value of $\kappa^Q$ explicitly, but rather report the volatility risk premium $\eta_V$. The other columns show the results of the restricted estimations, where one market price of risk at a time was constrained to equal zero. In the second column, for example, $\kappa^Q$ is restricted to be equal to $\kappa^P$. The other restrictions indicated in the table are to be interpreted accordingly.

Technically we estimate the free $Q$-parameters by minimizing the sum of squared differences between market and model implied volatilities for a sample of options on the S&P 500 over the same sample period as the one used in the MCMC estimation of the $P$-parameters. Note that for the purpose of estimating the $Q$-parameters we hold the local volatility fixed at the values obtained during the MCMC estimation of the $P$-parameters (in contrast to, e.g., Broadie, Chernov, and Johannes (2007)). The last row in Table 1 shows the root mean squared error (RMSE) in implied volatilities, evaluated at the optimal point estimate.

The parameter values in Table 1 show the potential impact of restrictions imposed during the estimation process. For example, in the SVCJ model the mean jump size in variance $\mu^Q_V$ ranges between roughly 2% (in the unrestricted case) and 14% (when the variance of the jump size in stock returns has to be the same under $P$ and $Q$). Also for this model $\mu^Q_S$, the mean jump size in the log of the stock price, is between -4% and -13%. Given the remarkably similar RMSE values obtained for the two models and the different specifications, these differences in estimated parameter values are certainly surprising and highlight the fact that the $Q$-measure is rather hard to identify.
Throughout the analysis, we always assume that jumps in the stock price are on average negative, and that the equity risk premium is positive and on average equal to 4%. For the sake of simplicity, we set the risk-free rate equal to zero. Furthermore, we set the current local variance $V$ equal to its long-run average under the physical measure.

### 3.2 Impact of Specific Risk Premia

#### 3.2.1 Stock Diffusion Risk

In this section we attribute the total equity premium of 4% exclusively to stock price diffusion risk represented by the Wiener process $W^S$ in Equation (1). Thus, $\eta_S$ is positive, and all other risk premia are set to zero. This represents an analogy to the Black-Scholes case, except that our model is of course much more flexible.

Since we are interested in the behavior of expected option returns across a wide range of moneyness levels and for different times to maturity, we chose to present the results graphically rather than in tables. Figures 2 and 3 show the results for the SVJ and SVCJ model, respectively. Expected call returns are positive and increasing in moneyness (defined as strike price divided by current stock price), while expected put returns are negative and largest in absolute value for far OTM options. These results are perfectly in line with the standard intuition that call options are levered investments in the underlying asset and thus earn a higher risk premium, while puts are negative beta assets.

For a proof, consider local expected option returns. With only stock diffusion risk being priced, the local risk premium on a call is given by $ER^S = \frac{\partial C}{\partial S} \cdot \frac{S}{C} \cdot \eta_S V$. It thus depends on the delta and on the market price for stock diffusion risk. Since the delta of a call is positive (see e.g. Bergman, Grundy, and Wiener (1996)), a positive $\eta_S$ implies that expected excess returns on calls are positive. This holds for all times to maturity, all moneyness levels, and all states, so expected excess maturity returns on calls are also positive. By a similar argument, expected excess maturity returns on puts are negative.

\[^{5}\text{This implies that expected returns and expected excess returns are equal.}\]
3.2.2 Jump Intensity

Now the 4\% equity premium is assumed to be entirely generated by the difference between the jump intensities $\lambda^P$ and $\lambda^Q$. With a negative average jump size, this implies that the intensity under the risk-neutral measure exceeds the intensity under the physical measure. It is important to note that in the SVCJ model a jump in volatility always happens simultaneously with a jump in the stock price, so that a premium for jump intensity risk in the stock price will automatically introduce a premium for volatility jump risk as well. For this reason, we first look at the SVJ model without volatility jumps.

Figure 2 shows that expected call returns in the SVJ model are still positive for the range of moneyness shown in our graphs, but they are no longer monotonic with respect to the strike price. For the shortest maturity of one month we observe decreasing expected returns beyond a moneyness of roughly 110\%. To see the intuition behind this result, note that jumps are more likely under $Q$ than under $P$. With a jump size that is on average negative, the first effect on the risk-neutral distribution of the stock price compared to the physical distribution is a shift to the left and an increase of probability mass in the left tail. Second, the jump size is stochastic, so that a higher frequency of jumps makes both the left and the right tail of the stock price distribution ‘fatter’. Both of these effects lead to high prices of puts and thus negative expected excess returns on these claims. The prices of call options decrease due to the first effect, and expected excess call returns become positive. For short-term deep OTM call options, however, the second effect works in the opposite direction, and expected excess returns on calls start to decrease in the strike price if they are very deep out of the money.

For the SVCJ model the graphs are shown in Figure 3. Again expected call returns are no longer monotonic in moneyness. In addition to this we now even observe expected returns, which are rapidly declining with increasing moneyness beyond a certain point and eventually become strongly negative. This is due to two effects. The first one is the same as the one just described for the SVJ model, namely that an increase in $\lambda^Q$ leads to higher prices of deep OTM call options and thus lower expected returns. Second, in the SVCJ model a jump in the stock price always comes with a jump in variance, so that a larger jump intensity increases the average level of variance, which again results in higher option prices and, consequently, even lower expected returns.
These results are perfectly in line with Propositions 1 and 2 of Coval and Shumway (2001). They show that the expected maturity return on an option is increasing in the strike price if the covariance between the stock price and the pricing kernel, conditional on the option ending in the money, is negative, while it is decreasing otherwise. In the SVJ model and the more general SVCJ model, the pricing kernel $m$ is given by

$$\frac{dm_t}{m_{t-}} = -rdt - \eta_S\sqrt{V_t}dW_t^S - \tilde{\eta}V_t^dW_t^V - (\lambda^Q - \lambda^P)dt + \left(\frac{q(\xi, \psi)\lambda^Q}{p(\xi, \psi)\lambda^P} - 1\right)dN_t, \quad (9)$$

where $p(\xi, \psi)$ and $q(\xi, \psi)$ denote the joint densities for the sizes of the stock price jump $\xi$ and the variance jump $\psi$ under the $\mathbb{P}$- and $\mathbb{Q}$-measure, respectively. If only jump intensity risk is priced, the pricing kernel simplifies to

$$\frac{dm_t}{m_{t-}} = -rdt - (\lambda^Q - \lambda^P)dt + \left(\frac{\lambda^Q}{\lambda^P} - 1\right)dN_t.$$

Since the pricing kernel depends on the number of jumps only, its correlation with the stock price is driven by their joint reaction to jumps. For $\lambda^Q > \lambda^P$, the pricing kernel is increasing in the number of jumps. The reaction of the stock price depends on the jump size $\xi$. Since jumps are on average downward, a decrease in the stock price due to a jump is much more likely than an increase, and the covariance between the pricing kernel and the stock price is negative. This of course also follows from the fact that the equity risk premium, which is proportional to the negative of this covariance, is positive.

The picture changes once we condition on $S_T > K$. If $K$ is very large, i.e. if the call is deep out of the money today, the conditional probability that an observed jump has been positive is much larger than the probability for the opposite event, implying a positive conditional covariance between the stock price and the pricing kernel. Consequently, the expected returns on deep OTM calls decrease in moneyness and can eventually become negative.

To assess the properties of expected returns on puts, on the other hand, we have to condition on $S_T < K$. Imposing an upper bound of $K$ on the terminal stock

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6 Appendix A.1 presents a more formal analysis of these conditional correlations in the context of the Merton (1976) jump-diffusion model. The reason for using this simpler model here is that we focus on the impact of jump risk premia, and leaving out stochastic volatility considerably simplifies the presentation of the argument.
price increases the conditional probability that an observed jump has been negative. The conditional covariance between the pricing kernel and the stock price thus stays negative, and expected put returns increase in moneyness.

### 3.2.3 Jump Size

For the case when only $\mu_S$, the average size of stock price jumps, differs between $\mathbb{P}$ and $\mathbb{Q}$ the expected maturity returns are again shown in Figure 2 for the SVJ model and in Figure 3 for the SVCJ model. Expected returns on call options are positive, but not monotonic with respect to moneyness, while the (negative) expected returns on put options are.

The findings can again be explained via the conditional correlation argument based on the analysis in Coval and Shumway (2001). The unconditional correlation between the stock price and the pricing kernel at time $T$ is negative. The behavior of expected call returns depends on the correlation conditional on $S_T > K$. For the SJ-model developed by Merton (1976), Appendix A.2 shows that this correlation is negative for small and large values of $K$, while it may be positive in between. This means that expected call returns will be increasing in moneyness for low and high moneyness levels and decreasing in moneyness in between.

For puts, we have to condition on $S_T < K$. As Appendix A.2 shows, the conditional correlation is negative for large $K$, and it does not change sign when $K$ decreases. Thus, expected put returns increase in moneyness over the whole moneyness range.

In addition, a detailed analysis of the local expected returns in Equations (7) and (8) turns out to be helpful here. In the given setting, all terms except $ER_{\mu}^J$ are now zero. For a call, the difference $E_P[C(Se^X)] - E_Q(\sigma_S^2)[C(Se^X)]$ is positive when jumps are on average more negative under $\mathbb{Q}$ than under $\mathbb{P}$ (with the jump variance unchanged). Again, details are given in Appendix A.2. This implies that local expected call returns are positive. By the same token, we get the opposite result for puts. The properties of expected returns over the interval from 0 to $T$ then follow from integrating local expected returns.

Put together, expected call returns are positive, but not necessarily monotonic in moneyness. Expected put returns, on the other hand, are negative and increasing in moneyness.
3.2.4 Volatility Risk

Now we look at the premium for volatility risk. In both the SVJ and the SVCJ model, volatility diffusion risk can be priced. In the latter, there can also be premia for the size and the intensity of jumps in variance, which happen simultaneously with jumps in the stock price.

Just like the premia discussed above, the risk premium on volatility is important for the characteristics of expected option returns. In contrast to those, however, changing the volatility risk premium has no impact on the total equity premium, which is again equal to 4% as before.

Compared to the physical measure, a positive risk premium \( \eta_V \) implies a larger risk-neutral mean-reversion speed \( \kappa^Q \) and a lower mean-reversion level \( \theta^Q \) in the SVJ model, as can be seen from Equations (4) and (5). These two effects combined lead to a lower average volatility under \( Q \). Since option prices are increasing in \( Q \)-volatility, this implies lower prices and higher expected returns. As an alternative explanation note that options exhibit a positive exposure to volatility risk so that a positive risk premium automatically increases expected returns.

This logic also goes through in the SVCJ model. Along similar lines one can show that in the SVCJ model larger or more frequent jumps in variance imply larger option prices and lower expected returns, although especially for puts the effect is rather weak.

Finally, in the SVCJ model the jump size in the stock price is correlated with the jump size in variance. A large and positive value for the parameter \( \rho^Q_J \) increases the skewness of the risk-neutral distribution. Prices of OTM puts decrease, while prices of OTM calls increase. Consequently, expected returns on OTM puts increase in the dependence parameter \( \rho^Q_J \), while expected returns on (deep) OTM calls increase. Altogether, the impact on call returns is much larger than the impact on put returns.

3.3 Jump versus Diffusion Risk

The following analysis is conducted in a model where all risk factors carry a premium, so that the parameters of the base case correspond to the first column for the SVCJ model in Table 1.\(^7\) The main goal then is to see to what degree it actually

\(^7\)As a robustness check we also investigated the case where initially only stock diffusion risk is priced, and the results remain basically unchanged.
matters how a given total equity risk premium is distributed across the premia for jump and diffusion components. As stated in the introduction the key motivation for this analysis is the fact that in the course of the estimation of the $Q$-measure certain risk premia are sometimes set equal to zero (most likely to facilitate the parameter estimation), and that these restrictions are basically considered immaterial. However, as we will show below, they are not. In fact, quite the opposite is true. If we vary the composition of the total premium in certain ways, the empirical 'anomalies' of negative expected call returns are almost a 'natural' outcome, which is perfectly compatible with a widely-accepted option pricing model like SVCJ.

3.3.1 Jump Intensity versus Stock Diffusion

We first vary the risk-neutral jump intensity $\lambda^Q$ around its point estimate of 0.5017. To keep the overall equity premium constant at 4% we change $\eta_S$, the premium on stock price diffusion risk, accordingly. We apply the same approach to the subsequent variations of the mean jump size and the variance of the jump size under $Q$.

The main result to be taken from Figure 4 is that the larger $\lambda^Q$ (and the smaller thus $\eta_S$), the lower expected returns on calls and puts. Again, expected returns on calls can eventually become negative for a high moneyness. The intuition behind this result is that an increase in $\lambda^Q$ (together with on average negative jump sizes for the stock price) moves probability mass into the left tail of the risk-neutral distribution of the stock price. Since the jump size is stochastic, a larger $\lambda^Q$ also leads to a higher variance of the stock price, increasing the mass in both tails of the distribution. The resulting decrease in the mean of the distribution is offset by shifting the probability distribution slightly to the right, which can be attributed to the change in $\eta_S$. Due to the thicker tails, the prices of put and call options increase in $\lambda^Q$ and are highest for the case of a high $\lambda^Q$ combined with a a small or even negative $\eta^Q$. The expected returns on options then decrease in this parameter.

3.3.2 Jump Size versus Stock Diffusion

Figure 5 shows that making the jump size on average more negative yields lower expected returns for options with low and medium moneyness, regardless of their time to maturity. Returns on OTM calls, on the other hand, increase.

With a more negative average jump size, the risk-neutral distribution becomes
more left-skewed. The prices of put options with low strikes (and thus low moneyness) increase, and analogously, the prices of call options with higher strikes will decrease. Again the shift of the risk-neutral distribution to the right to keep the expected stock return equal to \( r \) has a much lower overall impact. In summary, the higher prices of options with low moneyness lead to a decrease in their expected returns, and analogously, the lower prices of options with higher strikes make their expected returns go up.

### 3.3.3 Jump Variance versus Stock Diffusion

The effect of a variation in the variance of the jump size under \( \mathbb{Q} \) is shown in Figure 6. Analogous to Equation (3) which gives the mean jump size under \( \mathbb{P} \), a higher variance of stock price jumps under \( \mathbb{Q} \) also implies a higher mean jump size under this measure. From Equation (6), we see that this would imply a lower equity risk premium, which is offset by an increase in \( \eta_S \).

The higher jump variance results in a larger kurtosis of the risk-neutral distribution. The prices of OTM calls and puts go up, and their expected returns go down. The premium for the variance of jumps can even lead to negative expected returns on calls. This effect becomes weaker with increasing time to maturity.

### 3.4 Structure of the Jump Risk Premium

In this section we take the analysis yet one level deeper. Instead of varying the contribution of diffusive and jump components to the total equity premium as in Section 3.3 we now hold the diffusive part and the total jump part fixed, but vary the exact structure of the latter. So the quantity \( \lambda^\mathbb{P} E^\mathbb{P} [e^{\xi} - 1] - \lambda^\mathbb{Q} E^\mathbb{Q} [e^{\xi} - 1] \) will remain constant, but we will vary the contributions of the terms related to the jump intensity, the average jump size, and the variance of the jump size.

First we trade off against each other the premia for jump intensity and average jump size. A higher \( \mathbb{Q} \) intensity will be compensated by a less negative mean jump size. In the SV CJ model, jumps in the stock price happen simultaneously with jumps in its variance, so that a premium for the jump intensity premium impacts expected option returns both via its impact onto the pricing of stock jump risk and variance jump risk. For this reason, we first analyze the SVJ model to isolate the impact of premia for stock jump risk.
In the SVJ model, an increase in the jump intensity under the risk-neutral measure is accompanied by a less negative average jump size under $Q$ to keep the jump part of the equity risk premium unchanged. The overall effect of these two changes is that the total variance of the stock price under the risk-neutral measure decreases, which would lead to a drop in prices and an increase in expected excess returns. Secondly, the larger jump intensity under $Q$ thickens the tails of the risk-neutral distribution, while the increase in the average jump size shifts probability mass from the left tail to the right tail. Stated differently, the kurtosis of the risk-neutral distribution increases, while its skewness, which arises (partly) due to a non-zero average jump size, is reduced. The more symmetric distribution leads to lower prices of OTM puts and higher prices of (deep) OTM calls compared to the base case. Expected returns on OTM puts thus increase, while expected returns on OTM calls decrease. For smaller jump intensities (and thus extreme jump sizes under the risk-neutral measure), the first effect of an overall lower variance dominates for ATM options, whose expected excess returns increase. For larger jump intensities, the second effect dominates, and the expected returns on OTM and ATM calls decrease.

In the SVCJ model, an increase in the jump intensity has a second effect in that it also increases the frequency of variance jumps and thus the average level of variance under the risk-neutral measure. The resulting premium for variance risk decreases expected option returns. Figure 7 shows that for low intensities, the expected returns on ATM options increase in the jump intensity due to the effect described for the SVJ model. For an already large intensity, however, the more symmetric distribution and the decrease in the risk-neutral expectation of the variance dominates. Expected returns decrease again, and expected returns on calls eventually become negative for a high moneyness.

Second, we analyze the trade-off between a premium for the variance of the jump size in stock returns and the jump intensity. A higher variance under the risk-neutral measure makes the average jump size in the stock price less negative and thus requires a higher (and not a lower) $Q$-jump intensity to keep the total equity risk premium constant. The direct implication is that the prices of options increase, and their returns consequently decrease. This also holds true in the SVCJ model where the jump intensity additionally impacts expected option returns via the variance risk premium. For a higher jump intensity, the average variance under $Q$ increases, which also increases the prices of options and thus lowers expected
returns. The graphs in Figure 8 confirm this intuition and make it obvious that the SVCJ model has no problem to produce negative average call returns. For puts the specification does not matter that much. Their expected returns are basically always negative and very large in absolute value.

Third, we consider the trade-off between the premia for the variance and for the average size of stock price jumps. The result of this exercise is shown in Figure 9. In our model a higher variance of jumps necessitates a more negative mean jump size, if the average jump size in the stock price and thus the equity premium should remain unchanged. The prices of OTM options (puts and calls) increase due to this higher variance. Expected options returns thus decrease in general in the risk-neutral variance of jumps. For OTM calls this effect is so strong that their expected returns can actually become negative. For puts the effect on expected returns is much weaker, because they are already close to their theoretical lower bound of $-1$.

Overall, these results show that the exact composition of the jump part of the equity risk premium has a significant impact on expected option returns. Restricting one or more of the premia on jump risk to zero can change the properties of option returns fundamentally, up to the point where expected returns on calls become negative.

4 Conclusion

The main motivation for our analysis was the empirical observation that expected returns on call options with high strike prices are frequently found to be negative. Intuitively, calls should have positive expected returns increasing in the strike price (and thus also in moneyness, as we define it) since a call represents a levered investment in the stock and should therefore, given a positive risk premium on the underlying, exhibit an even higher expected (excess) return.

However, this intuition need not be true in models with more risk factors than just stock diffusion risk. In general expected returns are given by the sum across all risk factors of the product of the exposure to the given factor, multiplied by the associated risk premium. The approach we have taken in this paper was to start from an SVCJ model for which the parameters under the physical and the risk-neutral measure were estimated using state-of-the-art techniques and then to vary certain market prices of risk under the constraint that the total equity premium remained
constant. In a first step the distribution of this premium across diffusive and jump risks was varied. Afterwards we kept the contributions of these two general types of risk factors to the equity premium fixed and performed a detailed sensitivity analysis for the components of the jump risk premium related to the intensity, to the mean and to the variance of the jump size, and to volatility jumps.

The contribution of our paper is two-fold. First, on a more general level, we study the impact of risk premia on expected option returns in detail. In this part of the paper we show that there is basically no innocuous assumption about the structure of risk premia in the SVCJ model. Both the distribution across jump and diffusion risk and the fine structure of jump risk premia have a substantial impact on expected option returns, at least for calls. Expected put returns turn out to be much less sensitive to these specification issues. They are basically always negative, large in absolute value, and monotonic in moneyness. For calls the monotonicity in moneyness which holds in a simple Black-Scholes framework vanishes in most of the scenarios we consider in our investigation. More importantly, and this is the second major contribution of our paper, they can even change sign from positive to negative. The case which shows most impressively how sensitive expected call returns are to the fine structure of the jump risk premium, i.e., to the distribution of the total jump risk premium across its components, is when we vary the share of the premia for the mean of the jump size in the stock price and its variance in the total jump risk premium.

These results have important implications for empirical research on option pricing. One avenue for future research is certainly the parameter estimation in complex models like SVCJ. Researchers should be aware of the drastic consequences that seemingly less important restrictions on parameters and risk premia can have on the properties of a model. Furthermore, the literature on expected option returns, including our paper, has almost exclusively focused on maturity returns, i.e. on returns when the option is held until maturity. It would be interesting to study also the properties of option returns when the investment is liquidated before the option maturity date. In this case the price of the option at the date of the sale depends on both the stock price and on the conditional volatility (and probably on even more state variables), which could cause interesting effects on expected returns. Finally, we have shown that the fine structure of the premium for jump risk has quite a huge impact onto the shape of the pricing kernel and can go a long way in explaining seemingly puzzling observations. This offers a potential role for equilibrium mod-
els in an economy with jump or disaster risk to explain the negative average call returns.
A Expected Option Returns in the Merton (1976) Jump-Diffusion Model

To analyze the impact of jump risk, we look at the Merton (1976) model. The dynamics of the stock price are

\[ dS_t = (r + \eta S_t - \bar{\mu}^Q S_t)dt + \sigma S_t dW^{S,P}_t + (e^\xi - 1) S_t dN_t, \]

yielding

\[ S_T = S_0 e^{(r + \eta S_0 - 0.5 \sigma^2 - \bar{\mu}^Q T + \sigma W^{S,P}_T + \int_0^T \xi_t dN_t).} \tag{10} \]

The pricing kernel evolves as

\[ dm_t = m_t \left\{ - r dt - \eta S_t dW^{S,P}_t - (\lambda^Q - \lambda^P) dt + \left( \frac{q(\xi, \psi) \lambda^Q}{p(\xi, \psi) \lambda^P} - 1 \right) dN_t \right\}, \]

so that, with \( m_0 = 1 \),

\[ m_T = \exp \left\{ -(r + \lambda^Q - \lambda^P + 0.5 \sigma^2 \eta^2) T - \eta S_T W^{S,P}_T + \int_0^T \ln \frac{q(\xi_t, \psi_t) \lambda^Q}{p(\xi_t, \psi_t) \lambda^P} dN_t \right\}. \]

Since we focus on jump risk premia, we set the premium for stock diffusion risk equal to zero, i.e. \( \eta = 0 \), so that the pricing kernel does not depend on the Wiener process. Since \( W^{S,P}, N \) and \( \xi \) are independent, the sign of the correlation between the stock price and \( m \) is driven by the joint dependence on jumps.

A.1 Only Jump Intensity is Priced

We assume that there is a premium for the jump intensity only, i.e. \( \lambda^Q \neq \lambda^P \). Together with a positive equity premium and on average negative jumps, this implies \( \lambda^Q > \lambda^P \).

The pricing kernel and the stock price at time \( T \) are then given as

\[ m_T = e^{-(r + \lambda^Q - \lambda^P) T + \int_0^T \ln \frac{\lambda^Q}{\lambda^P} dN_t} \]

and

\[ S_T = S_0 e^{(r - 0.5 \sigma^2 - \bar{\mu}^Q T + \sigma W^{S,P}_T + \int_0^T \xi_t dN_t).} \tag{11} \]

The stochastic discount factor increases in the number of jumps. For the stock price, there is no such unique relation, since the jump size \( \xi \) can be positive or negative. With jumps that are on average downward jumps, however, the stock price tends to
decrease in the number of jumps, and the (unconditional) correlation between the stock price and the pricing kernel is negative. This is in line with the positive equity risk premium.

Next, we condition on the event \( S_T > K \). Using (11), this implies the condition

\[
\sigma W_{T}^{S,}\bar{p} + \int_{0}^{T} \xi_t dN_t > \ln K - \ln S_0 - (r - 0.5\sigma^2 - \bar{\mu}^Q\lambda^Q)T.
\]

For large \( K \), this holds true if the realization of the Wiener process is 'large enough' or jumps are positive and 'sufficiently large'. For positive jumps, however, the terminal stock price \( S_T \) is increasing in the Poisson process \( N \), implying a positive conditional covariance between \( m_T \) and \( S_T \).

For put returns, we have to condition on \( S_T < K \). By the same line of argument, the stock price tends to decrease in the realization of the Poisson process, implying a negative conditional correlation.

### A.2 Only Jump Size is Priced

Now we assume that there is only a premium for the jump size which, with a positive equity risk premium and on average negative jumps implies \( \mu_Q^S < \mu_P^S < 0 \). In this case

\[
m_T = e^{-rT + \int_{0}^{T} \ln \frac{q(\xi_t, \psi_t)}{p(\xi_t, \psi_t)} dN_t}.
\]

For the log ratio of the density functions, one obtains

\[
\ln \frac{q(\xi, \psi)}{p(\xi, \psi)} = \frac{1}{\sigma_S^2} (\mu_Q^S - \mu_P^S) (\xi - 0.5(\mu_Q^S + \mu_P^S)).
\]

With \( \mu_Q^S < \mu_P^S \), the term \( \ln \frac{q(\xi, \psi)}{p(\xi, \psi)} \) is positive for jumps smaller than \( 0.5(\mu_Q^S + \mu_P^S) \), which have a higher probability under \( Q \), and negative for jumps greater than \( 0.5(\mu_Q^S + \mu_P^S) \), which are less likely under \( Q \). The stochastic discount factor thus increases (decreases) for small (large) jumps in the stock price.

The terminal stock price from Equation (11) basically behaves the other way round. It decreases for negative jumps and increases for positive jumps. Putting the results together, the joint behavior of the pricing kernel and the stock price due to a jump \( \xi \) depends on which of three intervals \( \xi \) is in. For \( \xi < 0.5(\mu_Q^S + \mu_P^S) < 0 \), the pricing kernel increases and the stock price decreases, while for \( 0.5(\mu_Q^S + \mu_P^S) < \xi < 0 \), both the pricing kernel and the stock price decrease due to jumps. Finally, for \( \xi > 0 \), the pricing kernel decreases and the stock price increases.

With no restriction on the terminal stock price, the unconditional covariance between the pricing kernel and the stock price is negative, which follows from the
positive equity risk premium. In this case, the left and the right interval where the pricing kernel and the stock price conditional a jump event move into opposite directions dominate the middle range.

If the stock price is restricted to be greater than some positive number $K$, the left and the middle interval becomes less and less important for increasing $K$. For very large values of $K$, the conditional correlation is thus negative. When the middle interval is wide enough, the conditional correlation can turn positive for intermediate values of $K$. The width of the middle interval is the larger the larger in absolute value $\mu_P^S$ and $\mu_Q^S$. So, expected call returns will first increase, then decrease and then increase again in moneyness.

Here, it is again instructive to look at local expected returns. Under the assumptions above, only $ER_{\mu}^I$ with

$$ER_{\mu}^I = \frac{E^P[C(S e^{\xi}, V)] - E^Q(\sigma_P^x)[C(S e^{\xi}, V)]}{C} \lambda^Q$$

is different from zero. According to our model assumptions, the jump size $\xi$ in the log stock price follows a normal distribution with means $\mu_P^S$ and $\mu_Q^S$ under the measures $P$ and $Q(\sigma_P^S)$, respectively. This gives

$$ER_{\mu}^I = \frac{1}{C} \int_{-\infty}^{\infty} \left[ C(S e^{\mu_P^S + \sigma_P^x x}, V) - C(S e^{\mu_Q^S + \sigma_P^x x}, V) \right] n(x) dx \lambda^Q,$$

where $n$ is the density function of the standard normal distribution. The assumption $\mu_Q^S < \mu_P^S < 0$ then implies

$$S e^{\mu_P^S + \sigma_P^x x} > S e^{\mu_Q^S + \sigma_P^x x}$$

for all $x \in \mathbb{R}$. Since the call price is an increasing function of the stock price, it holds that

$$C(S e^{\mu_P^S + \sigma_P^x x}, V) > C(S e^{\mu_Q^S + \sigma_P^x x}, V).$$

Taking expectations results in

$$E^P[C(S e^{\xi}, V)] - E^Q(\sigma_P^x)[C(S e^{\xi}, V)] > 0.$$

The premium for jump size risk is thus positive for calls and, analogously, negative for puts.
A.3 Trade-Off Between Premia for Jump Size and Jump Intensity Risk

We assume that only jump size risk and jump intensity risk are priced, and that the equity risk premium is positive. The dynamics of the log pricing kernel are

\[
\begin{align*}
  d \ln m_t &= -r dt - (\lambda^Q - \lambda^P) dt + \ln \frac{q(\xi_t, \psi_t)\lambda^Q}{p(\xi_t, \psi_t)\lambda^P} dN_t \\
  &= -r dt - (\lambda^Q - \lambda^P) dt + \left[ \frac{\mu^Q_S - \mu^P_S}{\sigma^2_S} \xi_t - \frac{(\mu^Q_S - \mu^P_S)(\mu^Q_S + \mu^P_S)}{2\sigma^2_S} + \ln \frac{\lambda^Q}{\lambda^P} \right] dN_t,
\end{align*}
\]

where we have set \( \sigma^Q_S = \sigma^P_S \equiv \sigma_S \).

When analyzing the trade-off between the two risk premia, we will also come across the special case of the Naik and Lee (1990) equilibrium model, represented by the parametrization

\[
\begin{align*}
  \mu^Q_S &= \mu^P_S - \gamma \sigma^2_S \\
  \lambda^Q &= e^{-\gamma \mu^P_S + 0.5 \gamma^2 \sigma^2_S \lambda^P}. \tag{12} \tag{13}
\end{align*}
\]

where \( \gamma \) is the relative risk aversion of the representative agent. Since the jump part of the equity risk premium is monotonically increasing in \( \gamma \), there is one value of \( \gamma \) for every combination of \( \mu^P_S \) and \( \lambda^P \) which yields an equity risk premium of 4%.

Using (12) and (13), the dynamics of the log pricing kernel become

\[
\begin{align*}
  d \ln m_t &= -r dt - (\lambda^Q - \lambda^P) dt - \gamma \xi_t dN_t.
\end{align*}
\]

Together with Equation (10), this implies for the special Naik and Lee (1990) case that the conditional correlation between the pricing kernel \( m_T \) and the terminal stock price is negative across the whole range of \( S_T \).

Similar to the previous analyses, we now consider different combinations of the jump intensity and the average jump size under the risk-neutral measure which all imply an equity risk premium of 4%. We start with the case where only jump size risk is priced (i.e., \( \mu^Q_S < \mu^P_S \) and \( \lambda^Q = \lambda^P \)). Here, the total variance of the stock price is larger and the distribution is more left-skewed under \( \mathbb{Q} \) than under \( \mathbb{P} \). The pricing kernel is decreasing in the stock price for small and large stock prices, while it may be increasing for intermediate stock prices. Expected call returns are thus increasing in the strike price for small and large moneyness levels, while they may decrease in the strike price for intermediate moneyness levels.

If we now increase \( \lambda^Q \) and offset this by a less and less negative average jump size \( \mu^Q_S \), we will at some point reach the scenario described by Equations (12) and (13). As shown above, the conditional correlation between the pricing kernel and the stock price is then negative across the whole range of stock prices. Thus, expected call and put returns are uniformly increasing in the strike price.
Increasing the jump intensity even further brings us to the case where only jump intensity risk is priced (i.e., $\mu^Q_S = \mu^P_S$ and $\lambda^Q > \lambda^P$). The increase in the jump intensity shifts probability mass to the tails. At the same time, the total variance of future stock prices goes down. This lowers the prices of ATM options and increases their expected returns. Furthermore, it leads to higher prices and lower expected returns for OTM calls. For a sufficiently high moneyness, expected call returns decrease in the strike price and will eventually become negative, which shows that they are definitely lower than in the case representing the Naik and Lee (1990) specification.

If we increase the jump intensity further, we are in a situation where $\mu^P_S < \mu^Q_S < 0$ and $\lambda^Q > \lambda^P$. Then, the smile becomes more and more symmetric, and the U-shaped pattern of the pricing kernel becomes even more pronounced. Compared to the previous case, the moneyness range with negative expected call returns starts even closer to the point $K = S_0$.

Summing up, increasing the jump intensity and simultaneously making the average jump size less negative leads to higher expected returns on options with a low moneyness and decreases the expected returns on options with a high moneyness. The larger the intensity, the lower the strike price above which expected call returns become negative.
References


Table 1: Parameters under $\mathbb{P}$ and $\mathbb{Q}$ measures

<table>
<thead>
<tr>
<th></th>
<th>SVJ</th>
<th>SVCJ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SVJ</td>
<td>SVCJ</td>
</tr>
<tr>
<td>$r$</td>
<td>0.0360</td>
<td>0.0360</td>
</tr>
<tr>
<td>$\kappa^P$</td>
<td>3.1334</td>
<td>4.3814</td>
</tr>
<tr>
<td>$\theta^P$</td>
<td>0.0301</td>
<td>0.0248</td>
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<tr>
<td>$\sigma_V$</td>
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<td>0.2948</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.6712</td>
<td>-0.6487</td>
</tr>
<tr>
<td>$\lambda^P$</td>
<td>1.2859</td>
<td>1.5698</td>
</tr>
<tr>
<td>$\mu_S^P$</td>
<td>-0.0388</td>
<td>-0.0307</td>
</tr>
<tr>
<td>$\sigma_S^P$</td>
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<td>0.0229</td>
</tr>
<tr>
<td>$\mu_V^P$</td>
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<td>0.0137</td>
</tr>
<tr>
<td>$\rho_{QJ}^P$</td>
<td>—</td>
<td>-0.1757</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters under $\mathbb{Q}$ (with Restrictions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_S$</td>
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<tr>
<td>$\eta_V$</td>
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<tr>
<td>$\lambda^Q$</td>
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<tr>
<td>$\mu_S^Q$</td>
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</tr>
<tr>
<td>$\mu_V^Q$</td>
</tr>
<tr>
<td>$\rho_{QJ}^Q$</td>
</tr>
</tbody>
</table>

RMSE | 3.71% | 3.72% | 3.80% | 4.46% | 4.18% | 4.27% | 3.80%

The table shows the parameters of the SVJ and SVCJ model from Section 2.1 under the physical ($\mathbb{P}$) and the risk-neutral measure ($\mathbb{Q}$) measure. All parameters are given as annual decimals. The $\mathbb{P}$-parameters were estimated from the time-series of S&P 500 index prices from January 1996 to September 2008 using an MCMC estimation (see Rodrigues and Schlag (2009)). The parameters obtained from this estimation are shown in the upper panel. The $\mathbb{Q}$-parameters are estimated by minimizing the sum of squared differences between market and model implied volatilities based on the realized index values and on the conditional volatilities from the first-step estimation levels. The sample consists of options on the S&P 500 index over the same sample period which was used to estimate the parameters under the $\mathbb{P}$ measure. The last row shows the root mean squared error (RMSE) in implied volatilities, evaluated at the optimal point estimate.

For the SVJ model we only estimate the unrestricted version where all parameters (for which this is theoretically possible) are allowed to differ between the two measures. The first column for the SVCJ model shows the results for the unrestricted case (with the exception of $\rho_{QJ}^Q$, which is restricted to equal zero). The other columns show the results of the restricted estimations, where one market price of risk at a time was set equal to zero. In the second column, for example, $\kappa^Q$ is restricted to be equal to $\kappa^P$. The other restrictions indicated in the table are to be interpreted accordingly. Note that we do not show the value of $\kappa^Q$ explicitly, but rather report the volatility risk premium.
The graphs show expected call and put returns in the SVCJ model as a function of moneyness (defined as strike price divided by current stock price) for a maturity of 1 month, 3 and 6 months. The $P$- and $Q$-parameters are shown in Table 1, where we set the risk-free rate equal to zero. ‘No restriction’ refers to the case where the risk-neutral measure is estimated without any restrictions on the market prices of risk. ‘$\lambda^Q = \lambda^P$’ refers to the restriction that the jump intensity has to be the same under $P$ and $Q$. The other cases are to be interpreted analogously.
The graphs show expected call and put returns in the SVCJ model as a function of moneyness (defined as strike price divided by current stock price) for a maturity of 1 month, 3 and 6 months. The $\mathbb{P}$-parameters are shown in Table 1, where we set the risk-free rate equal to zero. If only stock diffusion risk is priced, all risk premia are set to zero except $\eta_S$, i.e., $\eta_V = 0$, $\lambda^Q = \lambda^P$, and $q(\xi, \psi) = p(\xi, \psi)$. If only the jump intensity is priced, all risk premia are set to zero except the premium on jump intensity $\lambda^P - \lambda^Q$, i.e., $\eta_S = \eta_V = 0$ and $q(\xi, \psi) = p(\xi, \psi)$. If only the jump size is priced, all risk premia are set to zero except the one for the jump size in the stock price ($\mu^Q_S < \mu^P_S$). In all three cases, the premia are set such that the total equity risk premium is equal to 4%.
The graphs show expected call and put returns in the SVCJ model as a function of moneyness (defined as strike price divided by current stock price) for a maturity of 1 month, 3 and 6 months. The $\mathbb{P}$-parameters are shown in Table 1, where we set the risk-free rate equal to zero. If only stock diffusion risk is priced, all risk premia are set to zero except $\eta_S$, i.e., $\eta_V = 0$, $\lambda_Q = \lambda^p$, and $q(\xi, \psi) = p(\xi, \psi)$. If only the jump intensity is priced, all risk premia are set to zero except the premium on jump intensity $\lambda^p - \lambda^Q$, i.e., $\eta_S = \eta_V = 0$ and $q(\xi, \psi) = p(\xi, \psi)$. If only the jump size is priced, all risk premia are set to zero except the one for the jump size in the stock price ($\mu_Q^S < \mu^p_S$). In all three cases, the premia are set such that the total equity risk premium is equal to 4\%.
The graphs show expected call and put returns in the SVCJ model as a function of moneyness (defined as strike price divided by current stock price) for a maturity of 1 month, 3 and 6 months. The \( P \)-parameters and the \( Q \)-parameters for the base case are shown in the first column for the SVCJ model in Table 1, where we set the risk-free rate equal to zero. The jump intensity under the \( Q \)-measure, \( \lambda^Q \), is varied, and the stock diffusion risk premium \( \eta_S \) is set such that the total equity risk premium is equal to 4%.
Figure 5: Expected Option Returns for Different Combinations of $\mu_S^Q$ and $\eta_S$

The graphs show expected call and put returns in the SVCJ model as a function of moneyness (defined as strike price divided by current stock price) for a maturity of 1 month, 3 and 6 months. The $\mathbb{P}$-parameters and the $\mathbb{Q}$-parameters for the base case are shown in the first column for the SVCJ model in Table 1, where we set the risk-free rate equal to zero. The jump size under the $\mathbb{Q}$-measure, $\mu_S^Q$, is varied, and the stock diffusion risk premium $\eta_S$ is set such that the total equity risk premium is equal to 4%.
The graphs show expected call and put returns in the SVCJ model as a function of moneyness (defined as strike price divided by current stock price) for a maturity of 1 month, 3 and 6 months. The $P$-parameters and the $Q$-parameters for the base case are shown in the first column for the SVCJ model in Table 1, where we set the risk-free rate equal to zero. The variance of the jump size under the $Q$-measure, $\sigma^Q_S$, is varied, and the stock diffusion risk premium $\eta_S$ is set such that the total equity risk premium is equal to 4%.
The graphs show expected call and put returns in the SVCJ model as a function of moneyness (defined as strike price divided by current stock price) for a maturity of 1 month, 3 and 6 months. The $\mathbb{P}$-parameters and the $\mathbb{Q}$-parameters for the base case are shown in the first column for the SVCJ model in Table 1, where we set the risk-free rate equal to zero. The jump intensity under the $\mathbb{Q}$-measure, $\mu_Q^S$, is varied, and the risk-neutral average jump size, $\lambda_Q^S$, is set such that the total equity risk premium is equal to 4%.
The graphs show expected call and put returns in the SVCJ model as a function of moneyness (defined as strike price divided by current stock price) for a maturity of 1 month, 3 and 6 months. The \( P \)-parameters and the \( Q \)-parameters for the base case are shown in the first column for the SVCJ model in Table 1, where we set the risk-free rate equal to zero. The intensity of jumps under the \( Q \)-measure, \( \lambda^Q \), is varied, and the risk-neutral volatility of the jump size, \( \sigma_S^Q \), is set such that the total equity risk premium is equal to 4\%.
Figure 9: Expected Option Returns for Different Combinations of $\mu^Q_S$ and $\sigma^Q_S$

The graphs show expected call and put returns in the SVCJ model as a function of moneyness (defined as strike price divided by current stock price) for a maturity of 1 month, 3 and 6 months. The $\mathbb{P}$-parameters and the $\mathbb{Q}$-parameters for the base case are shown in the first column for the SVCJ model in Table 1, where we set the risk-free rate equal to zero. The average jump size under the $\mathbb{Q}$-measure, $\mu^Q_S$, is varied, and the risk-neutral volatility of the jump size, $\sigma^Q_S$ is set such that the total equity risk premium is equal to 4%.