The public private partnership paradox

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ABSTRACT

A public private partnership (PPP) is a contractual arrangement between government and the private sector, usually for the delivery of a piece of social infrastructure or a social service. Over the past 10 years, PPP activity around the globe amounts to many billions of dollars. The key features of a PPP arrangement are (a) that government will make a series of cash payments to the private sector, usually over a long "concession" period in excess of 20 years; and (b) that the risk (particularly the systematic risk) of the project is shared between the government and private sector. Governments must determine whether the payments to be made under the PPP (given their amount and risk) represent value for money relative to the cash flows (and risk) that would be involved with traditional or alternative government procurement options. The standard valuation framework based on the Capital Asset Pricing Model (CAPM) suggests that alternative streams of cash flows should be discounted to present value at a rate reflecting their systematic risk. In the context of PPPs, it has been argued that the standard framework produces a paradox whereby government appears to be made *better off* by taking on *more* systematic risk. This has led to a range of approaches being applied in practice, none of which are consistent with the standard CAPM valuation approach. In this paper, we demonstrate that the proposed approaches suffer from internal inconsistencies and produce illogical outcomes in some cases. We also show that there is no problem with current accepted theory, and that the apparent paradox is not the result of a deficiency in the current theory, but rather is caused by its misapplication in practice. In particular, we show that the systematic risk of cash flows is frequently mis-estimated, and the correction of this error solves the apparent paradox. In this regard, we show that our results are consistent with the substantial 1970s and 1980s literature on the discounting of cash outflows – a literature that was apparently ignored when PPP evaluation frameworks were developed.

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ABSTRACT

A public private partnership (PPP) is a contractual arrangement between government and the private sector, usually for the delivery of a piece of social infrastructure or a social service. Over the past 10 years, PPP activity around the globe amounts to many billions of dollars. The key features of a PPP arrangement are (a) that government will make a series of cash payments to the private sector, usually over a long "concession" period in excess of 20 years; and (b) that the risk (particularly the systematic risk) of the project is shared between the government and private sector. Governments must determine whether the payments to be made under the PPP (given their amount and risk) represent value for money relative to the cash flows (and risk) that would be involved with traditional or alternative government procurement options. The standard valuation framework based on the Capital Asset Pricing Model (CAPM) suggests that alternative streams of cash flows should be discounted to present value at a rate reflecting their systematic risk. In the context of PPPs, it has been argued that the standard framework produces a paradox whereby government appears to be made *better off* by taking on *more* systematic risk. This has led to a range of approaches being applied in practice, none of which are consistent with the standard CAPM valuation approach. In this paper, we demonstrate that the proposed approaches suffer from internal inconsistencies and produce illogical outcomes in some cases. We also show that there is no problem with current accepted theory, and that the apparent paradox is not the result of a deficiency in the current theory, but rather is caused by its misapplication in practice. In particular, we show that the systematic risk of cash flows is frequently mis-estimated, and the correction of this error solves the apparent paradox. In this regard, we show that our results are consistent with the substantial 1970s literature on the discounting of cash outflows – a literature that was apparently ignored when PPP evaluation frameworks were developed.

1. Introduction

In recent years, many national and provincial governments have entered into *public private partnerships* (PPPs) with the private sector. A PPP is a contractual arrangement between government and the private sector, usually for the delivery of a piece of social infrastructure or a social service. Generally, the term PPP is used when the private sector also finances the project. For example, the contract may be for the design, construction, finance and operation of a hospital, toll road or rail link. It might also be for the provision of school classrooms or pathology services over a fixed period of 20 years. These contractual arrangements between government and the private sector have different names in different jurisdictions and each contract has its own unique characteristics, but there are a number of features that are common in such agreements.

First, the contract usually involves a private sector consortium that includes one or more banks (involved in financing and structuring), engineering and construction firms (involved in design and construction), and an operations firm (who is responsible for operations, billing or revenue collection, and maintenance).

Second, the contract usually requires the government to make a series of payments to the private sector consortium. It is common for the private sector consortium to own the asset for a concession period, of say 30 years, at which time the infrastructure is generally transferred to government. Consequently, this is sometimes seen as a form of off-balance sheet financing for government – even though government is contractually committed to make the series of agreed payments over the concession period.

Third, the contract usually involves some sharing of risk between government and the private sector. It is common for the private sector consortium to bear some form of demand risk, for example, uncertainty over the volume of traffic that uses a new toll road, or construction risk, such as the risk that a sub-contractor will fail and will have to be replaced causing delay. Government will always bear at least residual delivery risk – if the consortium fails and is unable to deliver on its contractual commitments, government will likely have to step in to ensure the delivery of the particular piece of infrastructure or social service.

Some recent examples of the many billions of dollars of public assets that are subject to PPP arrangements are as follows:

- The Canada Line in Vancouver, Canada. The British Columbia provincial government has contracted with a private consortium for the design, construction, finance and operation of a section of the SkyTrain network. The government has committed $435 million and the private sector consortium has a 35-year concession period.

- Military headquarters in Canberra, Australia. The private sector was contracted to design, construct, finance, maintain, and provide certain IT and catering services over a 28-year concession period. In return, government pays $40 million per year.

- The Karolinska Solna hospital in Stockholm, Sweden. The Stockholm County Council has commissioned a private sector consortium to design, construct, finance, and operate the hospital for a concession period of 25 years.

- The Chicago Skyway in Chicago, USA. A consortium of Australian and Spanish companies have a 99-year concession period during which they must maintain the assets and are able to charge tolls.

- A large number of schools and hospitals in the UK have been structured as PPPs. Some hospitals have been designed and constructed by private sector consortiums in return for government subsidies. Some school PPPs have involved land being sold to developers with school grounds then being leased back to government.

The central focus of this paper is on the financial evaluation of PPP arrangements. The process begins with government determining that the particular piece of infrastructure or social service should be delivered. This decision is made as a matter of public policy and is not modelled in this paper. Rather, in this paper we examine the question of how the project should be procured.

Broadly, there are two procurement methods – government financed procurement and a PPP arrangement with the private sector. Government financed procurement involves (on balance sheet) government financing of the project. This includes traditional government procurement, where government effectively bears the majority of the risks (and generally all of the systematic risk) of the project. It also includes alternative government financed procurement options including alliance contracting, design and construction contracts together with either short or long term operating agreements where the risks of the project can be allocated between government and its counterparties. The selected government financed procurement option is known as the "public sector comparator" or PSC.

A PPP requires government to commit to make a series of future cash flows to the private sector consortium, and generally involves a greater transfer of risk by the government to the private sector. It is the role of government to determine which of the two options, PSC or PPP, provides the best value for money to taxpayers.

It is the risk-sharing aspect of PPPs that is of central importance to proper financial evaluation. In particular, the task confronting government is to evaluate the costs and the risks of the PPP against the PSC. Corporate finance and valuation theory, based on the Capital Asset Pricing Model (CAPM), would suggest that this evaluation can be performed by first setting out the expected cash flows of each alternative and then discounting them at a rate that properly reflects their systematic risk. That is, the techniques that have been adopted as standard in other areas of valuation practice, should apply equally well to the evaluation of PPPs.

By contrast, a range of different practices for evaluating PPPs are used by different national and provincial governments around the world, and none of these are consistent with corporate finance and valuation theory, or with the standard valuation practice that is used in commercial settings.

The reason that is usually given for the ad hoc or "modified" approaches that are adopted in practice is that there is some sort of paradox when evaluating PPPs against the PSC. Specifically, it is argued that standard corporate finance and valuation theory must be rejected (or at least modified) because it implies that government can be made worse off by transferring systematic risk to the private sector. Since this cannot be true, it is argued, the standard theory and practice cannot be applied to the evaluation of PPPs.

In Section 2, we set out the standard valuation theory and practice and show by way of example where the paradox is thought to arise. Section 3 then sets out the approaches that various jurisdictions have used to "solve" the paradox and to evaluate PPPs against standard government procurement. For example, the UK government has determined that all cash flows (whether under a PPP or government procurement, and regardless of the systematic risk of the cash flow) are to be discounted at the same constant social time preference discount rate. By contrast, the approach adopted in Australia and Canada has been to retain as many features of the accepted corporate finance and valuation theory as possible, but to make adjustments or modifications to it to avoid the apparent paradox discussed in Section 2.

In Section 4 we set out a number of problems with the approaches that have been proposed, and that are currently being used in practice. We show that while the modification of the accepted theory appears to solve the apparent paradox, it has a number of side effects. In particular, it causes a number of internal inconsistencies and produces illogical outcomes in some cases.

In Section 5, we set out our proposed approach. Our proposed approach is perfectly consistent with accepted theory and involves discounting expected future cash flows at a rate that properly reflects their risk. We show that there is no problem with current accepted theory, and that the apparent paradox is not the result of a deficiency in the current theory, but rather is caused by its misapplication in practice. In particular, we show that the systematic risk of the PSC and PPP cash flows is frequently mis-estimated, and the correction of this error solves the apparent paradox. Moreover, our proposed approach does not have the unintended consequences or side effects of the approaches that are currently used in practice.

Section 6 shows that our proposed results are consistent with the 1970s and 1980s literature on the discounting of cash outflows. Indeed a "paradox" that is closely related to the one examined in this paper was identified and solved almost thirty years ago and many of the results from that literature are very useful in the PPP context. The designers of current PPP evaluation frameworks were apparently unaware that a round wheel had already been invented and set out in the finance literature, and have designed square ones instead.

Section 7 contains our conclusions, and the formal mathematical derivations that underpin our proposed approach are set out in detail in the Appendix.

2. Do we need a unique framework for assessing PPPs?

Focus on discount rates

When comparing a PPP with traditional government procurement, there are two aspects to consider: (1) the cash flows relating to each alternative; and (2) the discount rate that should be applied to each set of cash flows. In relation to the cash flows, a number of papers report that cost overruns and time delays are more common for government projects. For example, Malone (2005, p.422) concludes that "[t]here is a recognition that large public sector infrastructure projects have historically been delivered with large time and cost overruns." In this paper, we set aside issues relating to cost management efficiencies and cash flows more generally and focus on the discount rate that should be applied to a series of expected cash flows.

Standard valuation practice

Standard discounted cash flow (DCF) valuation practice is to take a set of expected cash flows and to discount them back to present value using a discount rate that properly reflects the risk of those cash flows. Where there are two or more mutually exclusive alternatives, the standard approach is applied to each alternative and the alternatives can be ranked in terms of their present values. The different alternatives will almost certainly involve different expected cash flows and if the risk of the cash flows differs between alternatives different discount rates would be used, commensurate with their risk.

In this section we set out the reasons that have been used to justify the need for a unique approach for evaluating PPPs. In essence, the proposed reasons are based on the notion that standard DCF valuation practice works well for all other projects, but leads to implausible outcomes, unintended consequences and a paradox when applied to PPPs. In this section we review the proposed reasons.

Discounted cash flow valuation

Traditionally, finance practitioners have used DCF analysis, grounded in corporate finance and valuation theory, to value project proposals. This methodology requires the estimation of the project's expected cash flow stream which is then discounted to present value using a risk-adjusted discount rate, computed as:

$$\text{Value} = \frac{E[CF_1]}{(1+r_p)^1} + \ldots + \frac{E[CF_n]}{(1+r_p)^n} = \sum_{t=1}^{n} \frac{E[CF_t]}{(1+r_p)^t}$$

where:

$E[CF_t]$ is the expected cash flow of the project in year *t*; and

r_p is the project's required return on capital.

The project's required return on capital is usually derived from the CAPM in which the required return is the sum of the risk-free rate of interest and compensation for bearing systematic risk. Systematic risk, also referred to as market risk or non-diversifiable risk, is the risk associated with the performance of the overall economy. It can be contrasted with company-specific risk, which is associated with factors uncorrelated with overall economic events, and is also referred to as diversifiable risk. The theory underlying the CAPM is that market prices incorporate only compensation for systematic risk because investors are able to eliminate diversifiable risk from their portfolios by holding a broad collection of assets. Expressed as an equation, the required return on the project is computed as:

$$r_p = r_f + \beta_a \times MRP$$

where:

r_f is the risk-free rate of interest;

β_a is the asset beta, which is a measure of the systematic risk of the project;[1] and

MRP is the market risk premium, which is the expected return on the market portfolio of all risky assets relative to the risk-free rate.

Under standard DCF valuation, the expected cash flows of a project are discounted at a rate that reflects the risk of those cash flows. The sum of the present values of the cash flows is called the Net Present Value (NPV). The NPV is an estimate of the value of the project as a lump sum in today's dollars that is equivalent (in value) to the future cash flow stream that the project is expected to generate.

Negative NPV projects

Whereas private sector commercial projects tend to have positive NPVs (that is, they create value for owners) PPP projects most commonly have negative NPVs. That is, the present value of the cash flows is negative, and the project can only proceed if it is subsidised by a government who decides that it should proceed for reasons other than the stand-alone financial viability of the project. That is, in these cases the government must either:

- Undertake the project itself and bear all the associated risks and costs; or

- Invite a private sector party to undertake the project via a PPP with the government providing some compensation to offset the losses that would otherwise be incurred by the private investor.

From the government's perspective, the most beneficial proposal is the one which results in the lowest negative NPV and thus requires the lowest present value commitment by government.[2]

Anomalous outcomes for negative NPV projects?

The primary argument for the need for a special valuation approach for evaluating PPPs is that the standard DCF approach leads to a paradox (implausible outcomes and unintended consequences) when applied to negative NPV projects. Since PPPs tend to have negative NPVs, the argument is that a special evaluation approach is required. The reasoning behind this argument can be best explained via a simple example.

Suppose that the project is to build a new rail link with a construction cost of $100 million. Over the 5-year concession period, the allowed passenger charge is insufficient to cover

[1] In statistical terms, asset beta is the covariance of expected returns of a particular project with returns on the market portfolio of all risky assets, divided by the variance of returns on the market portfolio, that is, $\beta_a = COV(r_a, r_m)/\sigma_m^2$.

[2] Throughout this section, we assume that an identical project is delivered by the PSC and all PPP bids so they can be ranked on present value cost alone. That is, we deal with one complexity and one issue at a time.

operating expenses, with a loss of $40 million per year anticipated.[3] That is, the project's expected cash flows, from the perspective of government, are as follows:

Year	0	1	2	3	4	5
Cash flow	-100	-40	-40	-40	-40	-40

Assuming that the risk-free rate is 4%, the *MRP* is 6% and the asset beta is 0.5 (perhaps inferred from an analysis of listed firms), the project's required return on capital is 7% (perhaps derived from the CAPM). By applying this discount rate to the above cash flows, the NPV of the project is –$264 million. That is, in order to construct the rail link and operate it for five years, government would expect to pay the series of cash flows set out above, and this series of cash flows has an equivalent lump sum present value of –$264 million.

Now suppose that something can be done to alter the project risks, so that the expected cash flows remain unchanged, but the risk to government is increased. That is, the operating losses are still expected to be $40 million per year, but the range of possible outcomes is greatly increased – previously the operating losses might be a little above or a little below $40 million, but now they might be greatly above or below $40 million. Now government is exposed to more risk, but all other things are equal. The new riskier project is clearly inferior to the otherwise identical, but much less risky, project.

However, if we assume that the systematic risk of the project increases such that the asset beta is now equal to 1.0 (that is, the relevant measure of risk doubles), the riskier project with the same expected cash flows would have an NPV of –$252 million (applying a discount rate of 10%, commensurate with the increase in risk).

If the government were to make its investment decision purely on the basis of which project had the lowest negative NPV, the riskier project would be favoured. However, this outcome appears to be inconsistent with the intuitive view that the government should prefer the project that is less risky, but otherwise identical in all respects. The reason for this inconsistency is that a higher discount rate results in a lower net present value for the future negative cash flows. This outcome is seen as a paradox that results from a deficiency in the accepted theory, at least insofar as it applies to the evaluation of PPPs.

To avoid this paradox and apparently incongruous results, different governments have adopted different methodologies to rank PPP proposals and to compare them with PSC alternatives – as set out in the following section. In subsequent discussion, we demonstrate that the apparent paradox presented above does not present an issue to be solved and that standard valuation theory does in fact generate sensible project evaluations for negative NPV projects.

3. Approaches used in practice

In this section we set out the approaches that various governments have used to "solve" the apparent paradox and to evaluate PPPs against standard government procurement. For example, the UK government has determined that all cash flows, whether under a PPP or government procurement and regardless of the systematic risk of the cash flows, are to be discounted at the same constant social time preference discount rate. By contrast, the

[3] Of course, PPPs tend to involve construction periods of 2-3 years followed by a concession period of 20 years or more. We have kept this example as simple as possible to illustrate the point.

approach adopted in Australia and Canada (British Columbia) has been to retain as many features of the accepted corporate finance and valuation theory as possible, but to make adjustments or modifications to it to avoid the apparent paradox discussed in Section 2.

It is generally recognised that these modifications to the standard framework are ad hoc and not supported by theory – they are designed only to avoid the apparent paradox. For example, Partnerships Victoria (2003, p.23), from the Treasury of the Australian state of Victoria, argues that its modified approach will preserve the correct ranking among alternatives even though the actual final numbers derived "will have no direct meaning."

US approach

The United States does not currently have a unified approach or framework for evaluating PPPs. To date, PPP deals have been less common in the US than in other jurisdictions, although there are indications that US interest in this structure is now growing strongly. For example, a recent report from the US Department of Transport (2009) concluded that "PPPs are an effective strategy for delivering highway projects" and that "potential PPP projects must be analyzed and structured thoughtfully." The key recommendation of the report was that "U.S. implementation include convening workshops, developing training guidelines, establishing an expert task group, developing a research strategy, and publishing principles and guideline documents on PPP topics." In this regard, the report (p. 2) singles out the proper assessment and evaluation of value-for-money and risk transfer as being particularly important, and reviews the jurisdictions and frameworks that are set out below in considerable detail.

The US Office of Management and Budget (1992) has published a set of guidelines for how government agencies should compare the costs and benefits of proposed programs. These guidelines propose that cash flows from proposed alternatives should be compared by discounting to present value using a government bond rate.

In summary, the assessment of PPPs is only now becoming an important issue in the United States, and the approaches and frameworks that are used in other jurisdictions appear to be of interest to government agencies and policy-makers.

Single discount rate approach

The UK Treasury recommends that all PPP-type projects should be evaluated using a single discount rate. Under this approach, all expected cash flows are discounted at the same rate regardless of whether they are made under a PPP or PSC arrangement. Consequently, no assessment of the risk of a particular series of cash flows is required – the same discount rate will be applied regardless.

The UK approach is set out in the UK Treasury's "Green Book." The single discount rate applied to all projects is based on an estimate of the social time preference rate.

> This Annex shows how the discount rate of 3.5 per cent real is derived and the circumstances in which it should be applied.

> Social Time Preference is defined as the value society attaches to present, as opposed to future, consumption. The Social Time Preference Rate (STPR) is a rate used for discounting future benefits and costs, and is based on comparisons of utility across different points in time or different generations. This guidance recommends that the STPR be used as the standard real discount rate.[4]

This approach technically avoids the paradox of government being made better off by taking on more risk. Under the single discount rate approach, the present value of a particular set of cash flows is completely independent of the risk of those cash flows. Consequently, an increase or decrease in systematic risk will have no impact at all on the estimated present value of a set of cash flows.

This approach is inconsistent with the notion that the present value of a future cash flow depends on its systematic risk. Moreover, it leads to its own set of illogical and unintended consequences. Suppose for example that the proposed PPP had expected cash flows that, year-by-year, were slightly lower than the expected cash flows under the PSC, but that the PPP cash flows involved dramatically higher risk to government. The single discount rate approach would rank the PPP ahead of the PSC, even though this would clearly be a detrimental outcome for taxpayers.

Private sector discount rate corporate finance approach

In some jurisdictions, the private sector cost of capital is used as the appropriate discount rate for all procurement decisions. Under this approach, the private sector cost of capital is estimated as it would be in a standard corporate finance setting. This involves the estimation of the systematic risk (that is, beta) for the particular type of project and the use of the CAPM to estimate the discount rate. This same discount rate, reflective of the usual overall systematic risk of that kind of project, is applied to all cash flow streams under both PSC and PPP alternatives. For example, the British Columbia Government has used a market-based discount rate[5], being an estimate of the private sector weighted average cost of capital for a project of a similar type, for the evaluation of PPP proposals and the PSC.

This approach is also inconsistent with the standard valuation framework that is based on the CAPM, which requires that all cash flows be discounted at a rate that properly reflects their systematic risk. A single discount rate applied to all alternatives does not reflect the amount of systematic risk that may have been transferred to the private sector under a PPP arrangement.

Systematic risk transfer corporate finance approach

The approach that is closest to being consistent with the standard CAPM valuation framework is the approach that is currently adopted in Australia. This approach focuses on capturing differential allocation of systematic risk as the driver of different discount rates between project options. Under this approach, there is an attempt to recognise the sharing of systematic risk between the parties. Under different arrangements, different parties will bear

[4] HM Treasury in The Green Book: Appraisal and Evaluation in Central Government, Annex 6: Discount Rate, page 97.

[5] *British Columbia Ministry of Transportation: Project Report: Achieving Value for Money William R. Bennett Bridge Project,* September 2005, p.8. "For the comparison, both proposals and the PSC used a market-based discount rate of eight per cent, which is an estimate of the private sector weighted average cost of capital for a project of this type."

different amounts of systematic risk. As systematic risk varies, so too should the discount rate that is applied. The systematic risk transfer approach follows a step-by-step process to quantifying the amount of systematic risk involved in a project and the way it is divided among the parties.

In December 2008, Infrastructure Australia released the *National Public Private Partnership Guidelines*: *Discount Rate Methodology Guidance* (*"The Australia Guidelines"*). The *Australian Guidelines* set out an approach for evaluating PPPs that is designed to overcome the perceived limitations of traditional valuation approaches in this setting. They are based on the framework that had been previously proposed by the state governments of Victoria and New South Wales.[6]

The vast majority of PPP arrangements involve social infrastructure projects that require some form of subsidy from government to make them viable for the private sector. The *Australian Guidelines* refer to such cases as "net cost projects," defined to be projects where the sum of the cash flows (from the perspective of government) is negative. For a net cost project:

- The PSC cash flows are discounted at the risk free rate; and
- The PPP cash flows are discounted at a rate that reflects the amount of systematic risk transferred to the private sector.

The *Australian Guidelines* apply different discount rates to net cost projects (where the sum of the cash flows is negative from the perspective of government) and a net revenue project (where the sum of the cash flows is positive from the perspective of government). The Guidelines assume that in relation to net revenue projects structured as PPPs government will bear no systematic risk. Consequently, there is no guidance for net revenue projects structured as PPPs that involve government bearing some risk. In this regard, the *Guidelines* (p.62) state that:

> The PPP bids will include either a payment to, or from the state independent of the actual future revenue experience and are thus devoid of systematic and project risk, from the government perspective and hence should discounted by the risk free rate.

A net revenue project operated solely by government is simply a standard project. Cash flows in this case should be discounted at the usual CAPM-based project rate. The discount rates to be applied under the *Australian Guidelines* are summarised in Table 1 below, in which assumptions are incorporated as to the level of the risk-free rate (5%) and the project's systematic risk premium (3%).

[6] In Victoria, these guidelines are set out in the Partnerships Victoria Technical Note of July 2003, *Use of Discount Rates in the Partnerships Victoria Process*. In New South Wales, the relevant guidelines are found in the New South Wales Government technical paper of February 2007, *Determination of Appropriate Discount Rates for the Evaluation of Private Financing Proposals*.

Table 1. The discount rate approach in the *Guidelines*

	PSC	PPP		
		Govt. bears all risk	**Govt. transfers two thirds of systematic risk**	**Govt. bears no risk**
Net Cost Project (Sum of cash flows is negative)	Risk-free rate (5%)	Risk-free rate (5%)	Risk-free rate plus two thirds of systematic risk premium (7%)	Project rate (8%)
Net Revenue Project (Sum of cash flows is positive)	Project rate (8%)	n/a	n/a	Risk-free rate (5%)

To illustrate the application of the *Australian Guidelines*, we consider an example based on the following data:

- The risk-free rate is 5% p.a. This would be estimated as the yield on long-term government bonds;

- The total systematic risk of the project is 3%. This is estimated within a CAPM framework where the beta is estimated with reference to a set of exchange-listed firms (for example, beta estimate of 0.5) and the market risk premium is estimated with reference to historical stock and government bond returns or from equity prices and earnings forecasts (for example, MRP of 6%). This step is performed in the same way as for any standard CAPM estimate of a required return;

- Under the proposed PPP, two thirds of the systematic risk is transferred to the private sector, making the PPP discount rate 7% (that is, 5% plus two-thirds of 3%) under the *Australian Guidelines;* and

- The cash flows under the PPP and PSC alternatives are as set out in Table 2 below. Under the PSC there is a three-year construction phase followed by a 30-year operations phase. Under the PPP, government makes annual concession payments over the 30-year operations phase of the project.

Table 2. Net cash flows from government

Year	0	1	2	3	4	5	…	33
PSC cash flow		-100	-100	-100	-10	-10	…	-10
PPP cash flow					-35	-35	…	-35

Under the *Australian Guidelines,* the PSC cash flows are discounted at the risk-free rate (5%) and the PPP cash flows are discounted at a rate that reflects the systematic risk transferred to the private sector (7%). The present value calculations set out in Table 3 below indicate that the PPP alternative would be preferred in this case.

Table 3. NPV calculations as per the *Australian Guidelines*

Year	**0**	**1**	**2**	**3**	**4**	**5**	**…**	**33**
PSC cash flow		-100	-100	-100	-10	-10	…	-10
NPV @ 5%	-405							
PPP cash flow					-35	-35	…	-35
NPV @ 7%	-355							

4. Problems with current approaches

Overview

In the previous section, we discussed how the UK and British Columbia approaches apply a constant discount rate to all proposals regardless of the amount of systematic risk that must be borne by government. The UK approach is to use a 3.5% social time preference discount rate, whereas the British Columbia approach is to use a discount rate based on CAPM, but where there is no recognition of any sharing of systematic risk between the parties.

These approaches are inconsistent with the standard valuation framework that is based on the CAPM (or indeed on any factor pricing model in which there is a positive relationship between expected returns and the relevant measure of risk), which requires that all cash flows be discounted at a rate that properly reflects their risk. A single discount rate applied to all alternatives does not reflect the amount of systematic risk that may have been transferred to the private sector under a PPP arrangement.

No advantage from government cost of capital

It is sometimes argued that government procurement has a natural advantage since the government's cost of funds, as measured by the government bond rate, will be lower than the private sector cost of capital. Such arguments have been rejected in favour of the view that the appropriate discount rate to be applied to a series of future cash flows depends upon the systematic or factor risk of those future cash flows, and that the identity of the owner of those cash flows is irrelevant.

For example, Brealey, Cooper and Habib (1997, pp.23-24) note that "The UK government applies the same discount rate of 6%[7] across the vast majority of public-sector projects. Yet it is widely accepted that the discount rate should vary with a project's exposure to factor risk" and that "when assets are exchanged between the public and private sectors, spurious apparent value may be created by the use of an inappropriate discount rate." They conclude that "the cost of capital is the same in the public and private sectors."

Along the same lines, Klein (1997, p.30) concludes that "the apparent cheapness of sovereign funds reflects the fact that the taxpayers, who effectively provide credit insurance to the sovereign, are not remunerated for the contingent liability they assume. If they were to be

[7] Now 3.5%.

remunerated properly, then the advantage of sovereign finance would – almost by definition – disappear."

Attempts to align discount rates to systematic risk

Consistent with the existing literature, we reject approaches that apply the same discount rate to cash flow streams that have demonstrably different systematic risks. Consequently, our focus in the remainder of this section is on the Australian approach, which seeks to incorporate the effect of systematic risk transfer and to apply a discount rate that reflects the systematic risk of the relevant cash flows.

The approach set out in the *Australian Guidelines* is designed to rank alternatives in terms of their value for money to government. As mentioned earlier, Partnerships Victoria has recognised that the value estimate obtained for each alternative has "no direct meaning" and cannot be interpreted as the actual present value of the cash flows to government under that alternative. However, the focus is on the *ranking* of alternatives rather than the direct economic or financial interpretation of each value estimate.

In the remainder of this section, we set out a number of shortcomings of the Australian approach. We also note that the Australian framework formed much of the basis of the recent US Department of Transport (2009) report in relation to PPPs. We show that:

- A guaranteed cash flow from government is effectively a government bond and should be valued accordingly, but the Australian approach does not do so;
- Perfectly offsetting cash flows provide no net benefit to government, but the Australian approach can lead to different conclusions; and
- In some circumstances, the Australian approach can lead to the conclusion that government is made worse off by an unambiguous improvement in the cash flows.

The alternative approach that we propose in the following section does not suffer from any of these problems, is based on the proper application of standard valuation techniques, and provides valuation estimates that are directly interpretable as the present value of a particular stream of cash flows to government.

Guaranteed cash flows from government should be valued as a government bond

Consider a PPP under which government agrees to make a fixed payment of exactly $40 million per year for five years to the private sector partner, ignoring any abatement mechanism.[8] Under this arrangement, government is bearing no risk because the payments from government are fixed. It is the private sector partner that bears all of the demand and inflation risk. Consequently, the *Australian Guidelines* would require that these cash flows be discounted at the project rate, 7% in our earlier example, giving a present value of $164 million.

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[8] That is, to make the relevant point here we consider the case where government will make a completely fixed set of cash flows. In practice, abatement mechanisms may be common and result in the cash flow stream from government varying. At this juncture, we seek to make a simple point relating to the valuation of a fixed set of cash flows from government.

However, this series of cash flows is effectively a government bond – it is a series of fixed payments to be made by government at specified times. There are five "coupon" payments of $40 million each. Because this is a government bond, it must be valued as a government bond. The CAPM requires that the cash flows must be discounted at the relevant government bond rate, 4% in our earlier example, giving a present value of $178 million.

That is, government could raise $178 million by issuing bonds that were backed by the promise to make a payment of $40 million per year for five years, and would record this amount as a liability in its accounts – that being the present value of its commitment to make these future cash flows. But if exactly the same set of cash flows is to be made in a PPP context, the *Australian Guidelines* require them to be discounted at 7%, in which case the present value is only $164 million. This clearly understates the value of the government's commitment to make the future cash flows, which can be an important consideration to the extent that this figure flows through to government accounts.

Moreover, the Australian approach can also lead to government being "arbitraged" by the private sector to the extent the private sector can repackage the cash flows for a profit. Suppose a government did follow the *Australian Guidelines* and estimated the present value of this negative $40 million annuity to be –$164 million. If, in return, the project offered services or infrastructure (that is, social benefits or economic externalities) that the government valued at $165 million and which cost the private sector $165 million to supply, this would appear to be a good arrangement from the perspective of government.[9] However, to properly consider whether or not this is a favourable deal for government, consider what would happen if the proposal proceeds. The private partner could immediately securitize the guaranteed payment series (of $40 million per year) from government and sell it off as (effectively) a government bond. The market would pay $178 million for these fixed payments from government, valuing the stream of fixed payments as a government bond as set out above, and the private sector partner would supply the infrastructure or services for $165 million and would pocket the difference.

Davis (2005, p.441) makes a similar point. He notes that when government makes a guaranteed series of cash flows "the private owner has a risk-free cash flow stream promised by government" and that the present value of such guaranteed cash flows should be calculated using "the government bond rate."

Perfectly offsetting cash flow streams provide no net benefit

Consider two local councils that sit adjacent to one another and are otherwise identical in all respects. Each spends $40 per year on the maintenance of parks and gardens. Now suppose that each council contracts with the other to provide the maintenance service for $40 per year over five years. That is, Council A will maintain the parks and gardens of Council B and vice versa. Clearly, neither council is made better or worse off by this arrangement – they both continue to pay $40 per year and they continue to have their parks and gardens maintained.

[9] For example, the $165 million might be the PSC estimate for providing the same service or infrastructure, or it might simply be government's estimate of the value of the project broadly defined to include social benefits and so on. The point here is that government is, or believes that it is, receiving more than $164 million in benefits from the project.

The arrangement by which Council A contracts with an external provider (Council B) to provide a service at a prescribed standard for a fixed period of time is in nature a PPP. Since Council A is bearing no risk (the payment of $40 per year is fixed and does not vary with demand, inflation or any other economic variable) the *Australian Guidelines* require that the cash flows must be discounted at the project rate, 7% in our earlier example. In this case, the present value of the cash flows, from the perspective of Council A, is –$164.

Next, the arrangement by which Council A agrees to receive $40 per year in exchange for performing maintenance services for Council B is a net revenue project from the perspective of Council A. The *Australian Guidelines* require these cash flows to be discounted at the risk free rate, 4% in our earlier example, giving a present value of +$178.

That is, the approach set out in the *Australian Guidelines* suggests that Council A is made $14 better off by this arrangement. Moreover, Council B will perform the same exercise from its perspective and will conclude that it is also $14 better off. And this approach will also conclude that both Councils can increase the amount of value they create by charging each other a higher amount each year. In reality, of course, such an arrangement creates no value at all beyond having the respective parks and gardens maintained at the cost of $40 per year.

In such a transparent case as this example, it is likely that common sense would prevent the *Australian Guidelines* from being applied mechanically and that a more sensible conclusion would be reached. But the example does illustrate that there is an inconsistency in the way that cash inflows and cash outflows are evaluated. In a large state government, for example, there may be a diverse mix of net cost and net revenue projects across a range of portfolios. The *Australian Guidelines* would indicate a positive net benefit even though, from a whole of government perspective, there are cash inflows and outflows that cancel each other out.

Inconsistent evaluation of net revenue and net cost projects

The *Australian Guidelines* require that, for the PSC, cash flows for net cost projects are discounted at the risk-free rate whereas cash flows for net revenue projects are discounted at the project rate. This arbitrary distinction can lead to projects with similar cash flows and similar risks having significantly different NPVs.

Consider the following project cash flows from the perspective of government under the PSC. In this case the project loses money and requires net cash outflows early in its life, but there is some fee for service and demand is expected to grow over the life of the project to the extent that net cash flows in later years are expected to be positive.

Year	**1**	**2**	**3**	**4**	**5**
Cash flow	-75	-40	-5	30	87

The sum of these cash flows is –$3 million, meaning this project is deemed to be a net cost project. As such, the *Australian Guidelines* require that these cash flows be discounted at the risk-free rate (4%) to give a present value of –$16.4 million.

Now suppose that the expected cash flows remain the same except that the last cash flow at Time 5 is increased to +$93 million from +$87 million. This unambiguously improves the project and should make it more attractive in any financial evaluation.

Year	1	2	3	4	5
Cash flow	-75	-40	-5	30	93

The sum of the expected cash flows is now +$3 million, meaning this project is deemed to be a net revenue project. As such, the *Guidelines* require that the cash flows be discounted at the project rate (7%), giving a present value of –$19.9 million.

That is, under the *Australian Guidelines* an unambiguous improvement in the expected cash flows appears to have made government worse off. This stems from the fact that there is a discontinuity in the discount rate that is applied to net cost versus net revenue projects – the discount rate jumps immediately from 4% to 7% at a specific point. The cash flow series in the two cases set out above are all but identical and must have immaterially different systematic risk profiles – yet they are discounted at materially different rates under the *Australian Guidelines*.

Conclusion

The current Australian approach to assessing PPPs as set out in the *Guidelines* can lead to counter-intuitive results. In the following section, we propose a new approach that:

- provides the correct ranking in terms of value for money to government;
- provides outputs that are economically meaningful in that they measure the true present value of government's liability to make a future series of cash outflows;
- evaluates the merits of the PPP proposal from the perspective of the government by determining the present value *to the government* of cash flows paid *by the government;*
- produces robust and reliable results that are consistent with the CAPM framework that is the basis of the analysis; and
- is simple, easy to understand and easy to implement.

5. Using standard DCF valuation to evaluate PPPs

Overview – sensible and meaningful valuation outcomes

Our proposal is to apply standard DCF valuation techniques to determine the appropriate discount rate. The alternative approaches used in different jurisdictions were originally motivated by a belief that the standard valuation approach did not work in the PPP setting in that it produced perverse outcomes under which bearing higher risk seemed to make government better off.[10] In our view, the problem does not lie in an inherent flaw in standard valuation techniques, but in the application of the techniques by analysts in the PPP setting. That is, the problem is not with the technique itself, but with its misapplication. We illustrate

[10] There are also some other motivations for approaches used in other jurisdictions. For example, the UK use of a social time preference discount rate is also consistent with a view that this is the appropriate rate to be applied to all social infrastructure regardless of the risk associated with cash flows.

how the standard valuation techniques that are applied in all other settings can be properly implemented in the PPP setting.

Perspective of government

The starting point for our approach is that the analysis should be performed from the perspective of government. Government has decided, for community benefit and social policy reasons, that a particular project should be undertaken. The goal is to maximise the value for money in relation to the delivery of the project. For ease of exposition, suppose that there are two ways of achieving an identical project outcome – a government procurement (PSC) option and a PPP option. Since both options produce the same result, the least cost alternative should be preferred. Thus, we have two alternative series of cash flows to be made by government. Each needs to be discounted back to present value using a discount rate that properly reflects the systematic risk.

The cash flows are to be made *by government* and the analysis is being performed to estimate the present value of the cash flows *to government*, so the relevant discount rate is one that reflects the systematic risk of those cash flows *to government*. If the cash flows to be made by government involve little or no systematic risk to government, that should be reflected in the discount rate applied to them. Conversely, if the cash flows to be made by government involve substantial systematic risk to government, that should also be reflected in the discount rate applied to them.

By contrast, the focus of the Australian approach is on the perspective of the private sector partner. Indeed the discount rate that is applied to PPP cash flows depends on the amount of systematic risk that is transferred to the private sector, rather than on the systematic risk that is retained by government. We demonstrate below that this is the starting point for error and inconsistency in the proposed application of valuation principles to the PPP setting.

Negative betas?

The next point to note is that systematic risk or beta, from the perspective of government, can be (and often is) negative for the sorts of cases that are likely to be examined. Beta is usually estimated by examining a number of exchange-listed comparable firms. For each of the comparable firms, beta is estimated using some form of regression of stock returns on broad market returns. Listed companies tend to have positive betas because their stock returns are positively correlated with broad market returns – individual stock prices tend to go up (on average) when the market is up and down when the market is down.

The issue of whether regression analysis of historical returns generates reliable beta estimates for estimating expected returns is a separate but related issue. A limitation of the use of comparable listed firms for estimating the systematic risk of a PPP is that these listed firms, by construction, will be different to PPPs. If the project under consideration for a PPP were a positive NPV project, more than likely a private sector firm would be prepared to pay the government for the opportunity to undertake this project. In contrast, the issue addressed here only arises because projects under consideration are predominantly negative NPV projects. Hence, an implicit assumption of using comparable listed firms for estimating project beta is that the *magnitude* of the systematic risks of cash flows will be the same for the listed firms and the PPP, but that the *sign* of expected cash flows is different and/or the PPP could be structured in a manner whereby the private sector participants bears *positive* or *negative* systematic risk.

An alternative technique is to directly estimate the project's beta from scenario analysis on the project itself, specifically asking, "What would be the expected cash flows in states where the market return was above or below expectations?" However, for the purposes of this paper, we focus only on the application of beta estimates in the valuation of PPPs, not the technique in which those estimates are made.

The source of the positive correlation between stock returns and market returns lies in the fact that a company's cash flows tend to be better than expected when the economy is expanding and the market is up and worse than expected when the economy is contracting and the market is down. That is, $\text{cov}(CF_t, r_{m,t}) > 0$.

The corporate finance literature has long recognised the concept of a cash flow beta *(b)*[11] – the systematic risk of a particular cash flow – defined as:

$$b = \frac{\text{cov}(CF_t, r_{m,t})}{\text{var}(r_m)}.$$

The relationship between the cash flow beta and the standard returns beta *(β)* is:

$$b = \beta \times PV[CF_t].$$

Consequently:

$$\beta = \frac{\text{cov}(CF_t, r_{m,t})}{\text{var}(r_m) PV[CF_t]}.$$

Note here that the sign of the standard returns beta depends on the sign of the expected cash flow, $PV[CF_t]$. In particular, consider a set of comparable listed firms. It is likely that the cash flows from these firms are higher than expected when the market is up and vice versa, so that $\text{cov}(CF_t, r_{m,t}) > 0$. It is also likely that future cash flows to the firm are expected to be positive (if the firm is to remain solvent), so that $PV[CF_t] > 0$. Since $\text{var}(r_m) > 0$ by definition, this all implies that $\beta > 0$ so the firm has a positive beta and (under the CAPM) a required return on equity that exceeds the risk-free rate.[12]

But now consider the government provision of a social infrastructure or service project. In this case, government is making a series of cash outflows, so that $PV[CF_t] < 0$. If it remains the case that this project generates cash flows that are better than expected when the market is

[11] See for example, Brealey, Myers and Allen, 8th ed. (2006), p.227, and the derivations on the accompanying web site at www.mhhe.com/mba8e.

[12] We note that throughout this analysis we consider the extent to which systematic risk is driven by the correlation between cash flows and market returns (as a proxy for aggregate wealth). Correlation between stock and market returns can also be driven by changes in the market price of risk. See Campbell and Mei (1993) and Davis (2005). That issue is the topic of a separate paper.

up and worse than expected when the market is down, as is the case with the listed comparables, we have $\text{cov}(CF_t, r_{m,t}) > 0$. The implication then is that $\beta < 0$, in which case the appropriate discount rate is *less* than the risk-free rate.

To see this by way of a simple example, consider the cash flow from a listed comparable that is expected to be \$10. Suppose that this cash flow will be \$1 better than expected if the market is up (\$11) and \$1 worse than expected if the market is down (\$9), and that there is a 50/50 chance of an up or down market. In this case, note that the return is +10% when the market is up and –10% when the market is down, so there is a positive relationship between returns for the company and returns on the market and a positive returns beta:

$$\frac{11-10}{10} = +10\%; \ \frac{9-10}{10} = -10\%.$$

Now consider the government provision of a comparable project that is not economically viable in its own right. In this case, the project does not generate surplus cash flows, but rather requires a subsidy from government. Consider the case where the cash flow to be made by government is expected to be –\$10. Again suppose that, as for the listed comparable, this cash flow will be \$1 better than expected if the market is up (–\$9) and \$1 worse than expected if the market is down (–\$11). In this case, note that the return is –10% when the market is up and +10% when the market is down so there is a *negative* relationship between returns for the company and returns on the market and a negative returns beta:

$$\frac{(-9)-(-10)}{(-10)} = -10\%; \ \frac{(-11)-(-10)}{(-10)} = +10\%.$$

Example of application of our proposed approach

We continue the example from the previous section in which the risk-free rate is 5%, market risk premium is estimated at 6%, beta is estimated at 0.5, and the proposed PPP would involve government transferring two thirds of the systematic risk to the private sector. Recall that the expected cash outflows under the PSC are \$100 per year for three years followed by \$10 per year for 30 years.

In this case, government bears all of the systematic risk of the project under the PSC because there is no other party to share any of that risk. If those PSC cash flows were completely free of risk, the appropriate discount rate would be the risk-free rate (5%) and the present value would be –\$405. However, government is bearing detrimental systematic risk under the PSC, in which case the net "cost" of this project to government must be more than –\$405.

Suppose it is the case that $\text{cov}(CF_t, r_{m,t}) > 0$ for the listed comparables and the PSC. In this case, the sign on the beta estimate is reversed and becomes –0.5 and the premium for systematic risk is –3%, as set out above. Consequently, the appropriate discount rate is 2% (5% – 3%). The present value of the PSC cash flows at a discount rate of 2% is –\$499. That is, the present value of the PSC cash flows, if they were risk free, is –\$405. But they are not risk free, and the value of that risk (from the perspective of government) is –\$94. This is summarised in Table 4 below.

Table 4. Evaluation of PSC cash flows under proposed approach

Year	0	1	2	3	4	5	…	33
PSC cash flow		-100	-100	-100	-10	-10	…	-10
NPV @ 5%	-405							
NPV @ 2%	-499							
Dollar value of systematic risk	-94							

Under the PPP, a different series of cash flows is required from the perspective of government. Recall that these expected cash outflows are $35 per year for 30 years, beginning in the fourth year. If those cash flows were completely free of risk, the appropriate discount rate would be the risk-free rate, which produces a present value of –$465.

However, under the PPP government bears one third of the detrimental systematic risk and consequently an adjustment must be made for one third of the systematic risk premium. Again, since the cash flows are negative in this case but $\mathrm{cov}(CF_t, r_{m,t}) > 0$, the adjustment for systematic risk must also be negative. The resulting discount rate is 4% (5% – 1%) and the present value of the PPP cash flows is –$538. That is, the present value of the PPP cash flows, if they were risk free, is –$465. But they are not risk free, and the value of that risk (from the perspective of government) is –$73. This is summarised in Table 5 below.

Table 5. Evaluation of PPP cash flows under proposed approach

Year	0	1	2	3	4	5	…	33
PPP cash flow					-35	-35	…	-35
NPV @ 5%	-465							
NPV @ 4%	-538							
Dollar value of systematic risk	-73							

Table 6 below sets out a summary of the calculations under our proposed approach.

Table 6. Summary of proposed estimation method

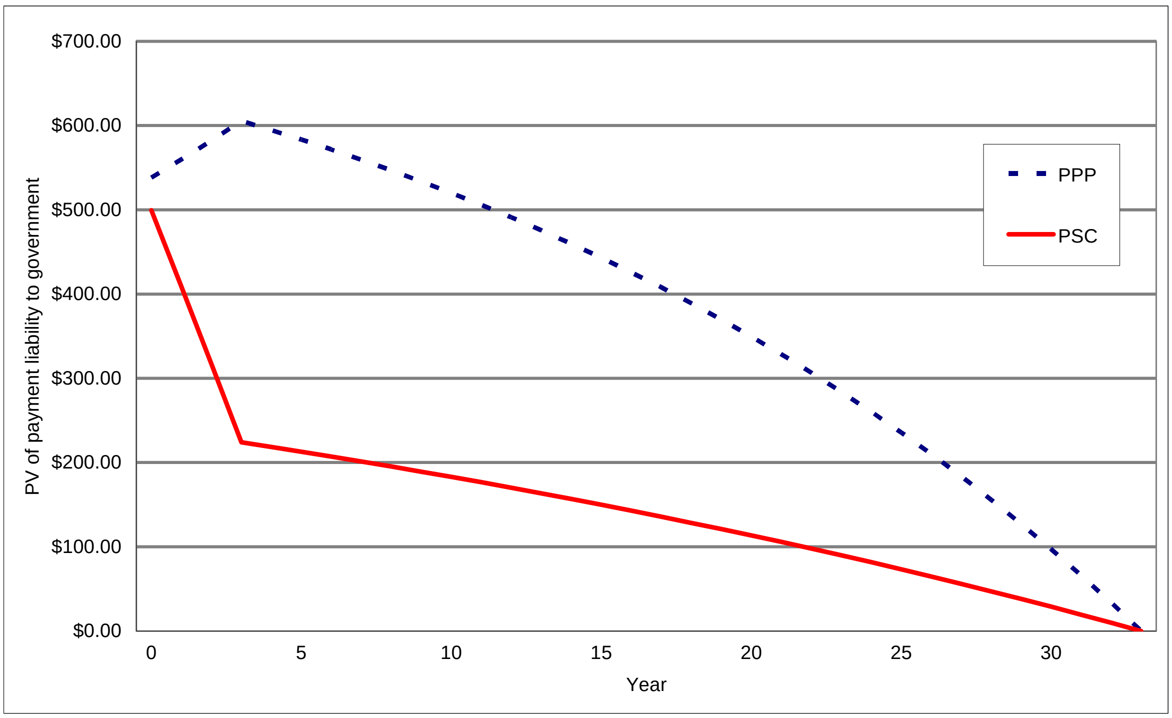
	Present value of cash flows at risk free rate	Dollar value of systematic risk	Net position
PSC	-405	-94	-499
PPP	-465	-73	-538
Difference	-60	+21	-39

First, note that government is bearing $94 of systematic risk under the PSC but only $73 of systematic risk under the PPP.[13] The cost of systematic risk is lower under the PPP because some of it has been transferred to the private sector. Even though under the PPP two thirds of the systematic risk is transferred to the private sector, the reduction in the cost of systematic risk is relatively small ($94 to $73). This is because the present value of the payment liability to government is considerably higher under the PPP for a considerably longer period of time.

[13] More formally, "the cost to government of bearing systematic risk" has a present value of $94 under the PSC and $73 under the PPP.

This is set out in Figure 1 below which shows the present value of the payment liability to government under each alternative. In each case, the present value of the payment liability to government is initially set at the NPV from Table 6 above. Each year, the value of the payment liability is inflated by the relevant rate (5% for PSC and 7% for PPP) and reduced by the amount of any payment from government as set out in Table 2 above. This is essentially an amortisation schedule where the balance grows with "interest" and is reduced by "payments." Although the premium for systematic risk is reduced from 3% to 1%, it is applied to a substantially higher payment liability over the life of the project.

Figure 1. Present value of payment liability to government



Next, we note that the present value of the cash flows at the risk-free rate is substantially higher (that is, more negative) under the PPP. These figures represent the present value of the two cash flow streams putting aside any consideration of risk – only the time value of money is taken into account, with the effects of systematic risk being separately accounted for in the second column of Table 6 above. Table 6 shows that the risk-free present value of the cash flows is $60 higher under the PPP.

In summary, by adopting the PPP rather than PSC, government reduces the cost of bearing systematic risk by $21. However, for this benefit government must pay the private sector a present value of $60 in terms of additional cash flows. This leaves government *worse* off by $39. That is, the additional magnitude of the cash flows (in present value terms) under the PPP more than offsets the value of systematic risk transferred.

By contrast, the *Australian Guidelines* approach suggests that government is made $50 *better* off under the PPP (see the calculations in Section 2 above). Under the Australian approach, the difference in the present value of the cash flow streams is still $60, as set out in the first column of Table 6 above. However, under the *Australian Guidelines* approach, the value of systematic risk transferred is estimated to be $110, which more than offsets the $60 difference in the risk-free present value of cash flows.

Beneficial risks and detrimental risks

Thus far our analysis has assumed that cash flows will be better than expected when the market is up and worse than expected when the market is down, so that $\text{cov}(CF_t, r_{m,t}) > 0$ for the set of listed comparables and the project being evaluated. A PPP arrangement involves a contract between government and the private sector under which government agrees to make a series of payments to induce the private sector into providing a piece of social infrastructure or a service that would not be economically viable but for the government subsidy. These contracts can take a number of different forms and the form of the contract can affect the relationship between cash flows and market returns so that $\text{cov}(CF_t, r_{m,t})$ is no longer positive therefore no longer exposes the project to detrimental market risk. This, in turn, will affect the sign of the returns beta that is used in the CAPM to estimate an appropriate discount rate.

Systematic risk is generally considered to be "detrimental" because it increases the uncertainty of payment/receipt of the cash flows, and therefore will require a premium in the form of higher expected returns. An asset with a positive beta (or positive systematic risk) tends to generate higher than average returns when the economy is strong and the market is up and lower than average returns when the economy is weak and the market is down. This is viewed unfavourably by investors – this market price of this asset tends fall when the price of other assets falls and the investor is in greatest need for some positive returns, and its price rises when the investor already has plenty of return from other assets. This is a detrimental systematic risk and investors will require higher average returns to attract them to such an asset.

But systematic risk is not always positive and therefore detrimental. An investment that pays off more when returns in the broader market are declining but less when returns in the broader market are increasing is a valuable hedge asset (that is, a beneficial risk) for which the market will pay a positive price. Such an asset has a negative beta and requires a negative risk premium – that is, the market will require a return below the risk free rate for such an asset that "insures" against market movements.

In a PPP context, systematic risks borne by the government can be beneficial or detrimental depending on how the deal is structured. Alternatively, it is possible to eliminate the systematic risk to be borne by government either by transferring it to the private sector or by writing a contract that involves government making a fixed series of payments that do not vary with general economic conditions.[14] In any case, the key question that must be answered is, from the government's perspective, what is the risk of the cash flows that the government is required to make?

Consider the following three payment structures under a PPP (ignoring any abatement mechanism):

[14] This assumes that the contract remains materially the same over the contract term and the private sector operator fulfils its obligations under the contract.

- Case 1: The government pays the private sector a fixed amount each year;
- Case 2: The government payment is based on demand for a service and this demand is positively related to general economic conditions; and
- Case 3: The government effectively immunises private sector losses.

In Case 1, the government agrees to make a fixed payment (for example $40 million) each year to subsidise the project irrespective of the impact of external factors on the actual cash flows of the project. These fixed payments involve no risk at all to the government and should therefore be discounted at the risk-free rate.

In Case 2, government may agree to pay a subsidy of $2 per car that uses a toll road or per passenger who uses a new rail link. Suppose that the volume of traffic is positively correlated with economic growth. In this case, the volume of traffic (and therefore the government payment) will be higher when the economy is expanding and lower during a recession. Suppose this results in the following payment structure:

- –$44 million in an economic expansion when the broad market is up; and
- –$36 million in an economic recession when the broad market is down.

This payment structure is equivalent to a guaranteed fixed payment of $40 million per year, plus a hedge contract of:

- –$4 million in an economic expansion when the broad market is up; and
- +$4 million in an economic recession when the broad market is down.

In the case of a recession when the government's tax receipts are expected to fall and welfare payments are expected to increase, the government will be paying less to the private operator (that is, the hedge contract has a positive pay-off of $4 million in a recession). In other words, when the government's other assets are falling (or its liabilities are rising), the value of this hedge asset is increasing. This payoff structure is a beneficial risk for which the government would be willing to pay.

That is, this series of uncertain cash flows is better, from the perspective of government, than the fixed payment of $40 million every year. Under the fixed payment option, the government is bound to pay $40 million whether its tax receipts are high or low. Under the alternative, the government is required to make a lower payment when tax receipts are down and a higher payment when tax receipts are up. This is a hedge asset for government, which reduces the variability of its budget bottom line. Consequently, it should be preferred to the fixed payment option.

Davis (2005, p. 443) also considers this type of situation and concludes that, "Here confusion abounds. It is often argued that a lower discount rate (such as the risk-free rate) should be used for risky expected future cash outflows for the PSC, because a higher rate would (supposedly counter-intuitively) give a lower present value figure. This is the approach commonly advocated (Partnerships Victoria 2003). This argument completely misses a key message of portfolio theory. Suppose that risky future cash flows outflows are positively correlated[15] with some indicator of economic activity (such as the stock market index). The commitment to make those risky cash flows, if combined with the holding of assets whose value (cash inflow) is also positively correlated with the stock market index, reduces the risk of the overall portfolio position. That diversification benefit is correctly captured if cash outflows with higher systematic risk are discounted at higher discount rates."

In Case 3, the government agrees to effectively immunise the private sector operator for losses incurred over the period. These losses, and hence the government payment, will be higher in a recession as traffic volume and toll revenue is lower. Suppose this results in the following payment structure:

- –$36 million in an economic expansion when the broad market is up; and
- –$44 million in an economic recession when the broad market is down.

This payment structure is equivalent to a guaranteed fixed payment of $40 million per year, plus a risky cash flow of:

- +$4 million in an economic expansion when the broad market is up; and
- –$4 million in an economic recession when the broad market is down.

In the case of a recession when the government's tax receipts are expected to fall and welfare payments are expected to rise, the government will be paying more to the private operator (that is, the government must pay an additional $4 million in a recession). When the government's other assets are falling (or its liabilities are rising), the government will incur an additional cost to subsidise the private sector operator for their additional losses. This payoff structure is a detrimental risk and the government should be prepared to pay to have it removed.

Measuring systematic risk: Conclusions

When estimating systematic risk, there are two key considerations relating to the sign of the risk, that is, whether the relevant beta should be positive or negative:

- Whether the cash flow will be higher than expected when the market is up and lower than expected when the market is down or vice versa; and
- Whether the expected cash flow is positive or negative.

Both of these considerations are vital to the proper estimation of systematic risk and the proper implementation of standard valuation techniques. Our approach summarises these considerations in Table 7 below.

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15 By this, Davis means that the absolute value of the cash outflow is higher when economic activity is high and lower when economic activity is low.

Table 7. Sign applied to beta under our proposed approach

		Sign of expected cash flow	
		Positive	**Negative**
Relationship between cash flow and market return	**Detrimental systematic risk:** $\text{cov}(CF_t, r_{m,t}) > 0$	+	–
	Beneficial systematic risk: $\text{cov}(CF_t, r_{m,t}) < 0$	–	+

Of most interest in the PPP setting will be the negative cash flows (from the perspective of government) that are required for social infrastructure projects (or any project that is not economically viable in its own right and requires a contribution from government). This is the right-hand column of the table above.

If the cash flows to be made by government are likely to be *better (lower)* than expected during economic expansions and *worse (higher)* than expected during recessions, $\text{cov}(CF_t, r_{m,t}) > 0$ and the systematic risk is detrimental as government will have to make a larger cash payment in circumstances when its revenues are already under pressure. If the expected cash flows are negative, the sign applied to the beta estimate must be reversed and made negative. That is, the cash flows from the perspective of government are:

- similar to those of exchange listed comparable firms in that $\text{cov}(CF_t, r_{m,t}) > 0$, but
- different from those of listed comparables in that $PV[CF_t] < 0$,

This requires the sign applied to beta to be reversed and made negative.

If the cash flows to be made by government are likely to be *worse (higher)* than expected during economic expansions, $\text{cov}(CF_t, r_{m,t}) < 0$ and the systematic risk is beneficial as government will have to make a smaller cash payment in circumstances when its revenues are under pressure. In this case, the cash flows from the perspective of government are:

- different from those of exchange listed comparable firms in that $\text{cov}(CF_t, r_{m,t}) < 0$, and
- different from those of listed comparables in that $PV[CF_t] < 0$.

This requires no sign change to be applied to the estimate of beta – the two differences effectively cancel each other out.

In summary, the initial beta estimate and resulting systematic risk premium for the project will be computed with reference to a set of listed comparable firms. The projects that make up these firms tend to lie in the top left cell of Table 7 above. The expected cash flows for these projects tend to be positive (consistent with the firm continuing to exist) and the cash flows tend to be better than expected when the market is up and worse than expected when the market is down (consistent with the beta estimates for these firms being positive). If, for a set of PPP or PSC cash flows, one or other of these features is reversed, the sign of the beta estimate (and systematic risk premium) must also be reversed. If both of these features are reversed no sign change is required (of course there are actually two sign changes, but these cancel each other out).

6. Consistency with literature on discounting cash outflows

A substantial literature on the discounting of cash outflows developed in the 1970s and continued into the 1980s. This literature began with the identification of a "paradox" whereby an increase in the risk of a cash outflow would seem to require a higher discount rate, which would reduce its present value – thus the firm is (paradoxically) made better off by increasing its risk. For example, Beedles (1978) considers an expected cash outflow of –$6,600 that is the outcome of a 50/50 chance of –$6,200 and –$7,000 and discounts this at a rate of 9%, reflecting its risk. He then notes that if risk is increased to a 50/50 chance of –$5,200 or –$8,000 a higher discount rate (he suggests 11%) would be appropriate and the present value of the cash outflow is reduced by the increase in risk. Beedles (p.174) concludes that, "Such a result is paradoxical since the income stream's value has *increased* with increase risk." He concludes that the risk-adjusted discount rate should not be used to discount cash outflows (or what he refers to as "negative benefits").

This sort of example sparked a number of papers that concluded that a higher discount rate is exactly what is required and that the firm is indeed made better off by the increase in risk in Beedles' example. Suppose the 50/50 outcomes in the Beedles example correspond to states of decreasing and increasing aggregate wealth respectively. When risk increases in Beedles' example, the cash outflow is $1,000 higher ($8,000 – $7,000) in the high-wealth state and $1,000 lower ($5,200 – $6,200) in the low-wealth state. That is, relative to the first case, the firm is $1,000 better off when the economy is performing poorly and $1,000 worse off when the economy is performing well. A number of papers make the point that in any asset pricing model based on risk aversion, including the CAPM, this makes the firm better off. For example, Miles and Choi (1979) consider the Beedles example using the Value Additivity Principle – suppose the risky cash flow was being made by one firm to another. It would be wrong for an increase in risk to cause the receiving firm to *increase* the discount rate (reducing the absolute value of the cash flow) but for the paying firm to *reduce* the discount rate (increasing the absolute value of the cash flow). They note that such a differential would lead to arbitrage opportunities. Miles and Choi (p.1098) conclude that "the valuation process must be the same for cash inflows and cash outflows. If risk-adjusted discounting correctly values inflows, then it will also correctly value outflows."

Lewellen (1977, p.1332) summarises the argument as "there is something at least vaguely disturbing about the associated 'write down' of the present value of cash outlays, for risk. Surely, the complaint runs, if cash outflows are highly uncertain, an extra penalty on project present value – via a lower discount rate on risky expected outlays, conceivably – would be the appropriate computational response." He shows that a *higher* discount rate should be applied to a risky cash outflow that is larger than expected when the market return is high. He provides an example where the cash outflow is –$500, –$400 or –$300 in states where the market return is 20%, 10% and 0% respectively. In this case, $\text{cov}(CF_t, r_{m,t}) < 0$ and the expected cash flow is negative. Lewellen concludes that a positive beta is required in this case and the appropriate discount rate is higher than the risk-free rate – the risk in this case is valuable to the firm in that an additional $100 outlay is required in the state when the market return is high and the cash outflow is reduced by $100 in the state where the market return is low. Consequently, a smaller negative present value is entirely appropriate. Lewellen further notes that if $\text{cov}(CF_t, r_{m,t}) > 0$, a negative beta and a discount rate lower than the risk-free rate would be appropriate.[16]

More recently, Ariel (1998, p.19) reaches the same conclusion – that it is incorrect that "cash outflows (costs) should be discounted at progressively lower [risk adjusted discount rates] as they become progressively more risky." Yet this is exactly what the *Guidelines* require. Ariel concludes that for inflows and outflows equally, the appropriate discount rate is obtained by properly measuring the systematic risk of the cash flow and then using the CAPM in the standard way. He also notes that "troublesome errors frequently arise from misapplication of otherwise sound principles."

Berry and Dyson (1980, 1983) set out a table that summarises the circumstances in which the appropriate discount rate is higher or lower than the risk free rate.[17] They note that this depends on (a) whether the sign of the cash flow is positive or negative, and (b) whether the covariance between the cash flow and market return is positive or negative. We reproduce this in Table 8 below.

Table 8. Sign applied to beta under our proposed approach

	$\text{cov}(CF_t, r_{m,t}) > 0$	$\text{cov}(CF_t, r_{m,t}) < 0$
Cash inflow	$r_p > r_f$	$r_p < r_f$
Cash outflow	$r_p < r_f$	$r_p > r_f$

Hull (1986) also notes that the appropriate beta depends upon the sign of the cash flow and the correlation between the cash flow and the return on the market.

7. Conclusions

Every year governments around the globe enter into PPP arrangements with a combined value of many billions of dollars. To date, there has been no academic literature on the financial evaluation of PPPs relative to traditional government procurement methods. In this paper, we examine a range of approaches that have been adopted by different national and provincial governments for the financial evaluation of PPPs. We show that the approaches that are currently adopted are inconsistent with the standard CAPM/discounted cash flow approach to valuation that is used in other settings. The current approaches produce illogical outcomes in some settings and are likely to mis-rank alternatives.

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[16] In this regard, consider Lewellen's example in which the cash outflow is –$500, –$400 or –$300 in states where the market return is 20%, 10% and 0% respectively. This cash flow is negatively correlated with market returns in that an outcome worse than expected occurs in the state when the market return is better than expected. Note that the correlation between the absolute value of the cash outflow and the market return is positive. When interpreting these results it is important not to confuse these two concepts.

[17] Booth (1983) notes that these arguments apply beyond the CAPM to any state-preference type of asset pricing model in which investors are risk averse.

We show that there is no problem with the standard CAPM-based valuation approach, and that the alleged paradox is not the result of a deficiency in the current theory, but rather is caused by its misapplication in practice. In particular, we show that the systematic risk of cash flows is frequently mis-estimated, and the correction of this error solves the apparent paradox.

The key contribution of this paper is in the proper understanding of how systematic risk (beta) should be estimated. Beta estimates are based on an analysis of comparable firms listed on a stock exchange. These listed comparables have two key properties:

- Expected cash flows are positive (if they were not, the firm would not be economically viable and would not be listed); and

- On average, cash flows are higher than expected when the market is up and lower than expected when the market is down. This implies that stock returns are higher than average during economic expansions when the stock market is up and lower than expected during recessions when the stock market is down (which is why beta estimates for these firms are positive).

We show that (other things equal) if one of these properties is reversed, the sign on the beta estimate must be reversed and made negative. If both of these properties is reversed, there is a cancelling effect and no change is required to the sign of beta.

With properly signed estimates of beta, the standard CAPM-based valuation framework can be applied to the PPP setting and produces correct rankings and economically meaningful output. Specifically, the output of this process is a direct estimate of the present value of the liability from the perspective of government.

Appendix: Derivation of beta estimation framework

Context

In this appendix we set out the mathematical derivation of our proposed beta estimation framework. The goal of the appendix is to establish that our proposed framework is mathematically rigorous and consistent with finance and valuation theory and that it produces economically sensible and meaningful results. We begin by considering a one-period example in which there is a single cash flow to be made one period from now. We also consider all projects to be 100% equity financed so that the asset beta is the relevant measure of systematic risk and there is no need to consider re-levering of equity betas and so on. Indeed, for the remainder of this appendix we refer to the asset beta as simply "beta."

Definition of beta

In a CAPM context, beta is formally defined as:

$$\beta = \frac{\operatorname{cov}(r_p, r_m)}{\operatorname{var}(r_m)}$$

where $\operatorname{cov}(r_p, r_m)$ is the covariance between the return on the project (r_p) and the return on the market (r_m, usually proxied by the return on a broad market index such as the All Ordinaries Index) and $\operatorname{var}(r_m)$ is the variance of the returns on the market portfolio.

Definition of returns

For the one-period examples in this appendix we define the present time to be Time 0 and the end of the period (when the cash flow is to occur) as Time 1.

In general, the return on an asset over the period can be written as:

$$r = \frac{P_1}{P_0} - 1$$

where P_0 is the price of the asset today and P_1 is the price of the asset at the end of the period. For example, if the price of an asset increases from 100 to 110 over the period, the return is 10%.

In the case at hand, we will be considering the payment of a single cash flow at the end of the period. This payment will be made by government to the private partner and will be denoted by CF_1. The present value of this cash flow (at Time 0, the beginning of the period) is denoted $PV[CF_1]$ – the present value of the cash flow that will be made at Time 1, the end of the period.

The return in relation to this cash flow is therefore:

$$r = \frac{CF_1}{PV[CF_1]} - 1$$

Suppose the present value of the cash flow is 100 and the actual cash flow made at the end of the period is 110. This would represent a 10% return over the period.

Definition of the present value of a cash flow

We have not yet discussed how to compute the present value of a cash flow. Standard finance and valuation practice is to estimate the present value of a cash flow by discounting the expected cash flow using an expected return:

$$PV[CF_1] = \frac{E[CF_1]}{1 + E[r_p]} - 1$$

Here, we recognise that CF_1 is risky in the sense that the amount of the payment that will occur at Time 1 is unknown at Time 0. That is, the payment to be made at Time 1 might be contingent on the number of cars that use a rail link or the number of patients that use a hospital over a period. So the amount of the payment is uncertain. If there is a 50/50 chance of that payment being 100 or 90, the expected cash flow is 100. It is this expected cash flow that is discounted back to present value.

Definition of beta in terms of cash flows

We substitute in expressions for the return and present value of cash flows to write the definition of beta in terms of cash flows as follows:

$$\beta = \frac{\mathrm{cov}(r_p, r_m)}{\mathrm{var}(r_m)}$$

$$= \frac{\mathrm{cov}\left(\left(\frac{CF_1}{E[CF_1]/(1 + E[r_p])} - 1\right), r_m\right)}{\mathrm{var}(r_m)}$$

Since $E[r_p]$ and 1 are both constants, we have:

$$\beta = \frac{(1 + E[r_p])}{\mathrm{var}(r_m)} \mathrm{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right)$$

That is, beta (or systematic risk) for a particular cash flow depends on the covariance between the return on the market (r_m) and the actual cash flow relative to its expected value $\left(\frac{CF_1}{E[CF_1]}\right)$.

Aside: Reconciliation with cash flow betas

The cash flow beta is defined as:

$$b = \frac{\text{cov}(CF_1, r_m)}{\text{var}(r_m)}.$$

The relationship between the cash flow beta and the standard returns beta is:

$$b = \beta \times PV[CF_1].$$

Consequently:

$$\beta = \frac{\text{cov}(CF_1, r_m)}{\text{var}(r_m) PV[CF_1]}$$

and using the definition of $PV[CF_1]$ from above yields:

$$\beta = \frac{\text{cov}(CF_1, r_m)(1 + r_p)}{\text{var}(r_m) E[CF_1]}$$

which can be rearranged as:

$$\beta = \frac{(1 + E[r_p])}{\text{var}(r_m)} \text{cov}\left(\frac{CF_1}{E[CF_1]}, r_m \right).$$

That is, our derivation above is consistent with the standard definition of a cash flow beta and the well-established relationship between cash flow and returns betas. The cash flow beta, and the relationships set out above are standard results in finance that are central to valuation using the certainty equivalent approach.[18]

Further derivations in relation to beta

We have noted above that:

$$\beta = \frac{(1 + E[r_p])}{\text{var}(r_m)} \text{cov}\left(\frac{CF_1}{E[CF_1]}, r_m \right)$$

It is most common to use the CAPM to estimate the expected (or required) return on a project:

$$E[r_p] = r_f + \beta \times MRP$$

[18] See for example, Brealey, Myers and Allen, 8th ed. (2006), p. 227, and the derivations on the accompanying web site at www.mhhe.com/mba8e.

where r_f is the risk free rate of interest and MRP is the market risk premium.

But this means that beta appears on both sides of the equation above – on the left hand side and on the right hand side as a component of $E\lfloor r_p \rfloor$. Substituting in for $E\lfloor r_p \rfloor$ and rearranging the expression yields:

$$\beta = \frac{\operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right)(1+r_f)}{\operatorname{var}(r_m) - MRP \times \operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right)}.$$

This is an unwieldy expression, but it does not have to be evaluated at all when implementing our proposed framework. We only report the derivation in this appendix to demonstrate that beta can be written in terms of the key covariance term and a number of other basic quantities that can be estimated in standard ways. That is, there are well-established techniques for estimating the risk free rate (r_f), market risk premium (MRP) and market volatility ($\operatorname{var}(r_m)$). All of these terms are estimated in precisely the same way whether it is a PPP or any other project being analysed. This leaves only the key covariance term to be estimated, which is the focus of this appendix. An understanding of how this covariance term works, and how it drives beta, is fundamental to an understanding of our proposed approach and the problems with the approach set out in the *Guidelines*.

We also note that the derivation above is consistent with Hull (1986), who expresses a similar result in terms of returns as:

$$r = r_f + \frac{\frac{MRP}{\operatorname{var}(r_m)}\operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right)(1+r_f)}{1 - \frac{MRP}{\operatorname{var}(r_m)}\operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right)}.$$

Definition of variance

The statistical definition of variance, illustrated in terms of the market return, is:

$$\operatorname{var}(r_m) = E[r_m - E[r_m]]^2$$

Suppose, for example, that the uncertainty around the market return is such that there is a 50/50 chance of the return being +30% or –10% over the period.

In this case, the expected return on the market is:

$$E[r_m] = 0.5 \times (+30\%) + 0.5 \times (-10\%) = 10\%$$

The variance of the return on the market in this case is:

$$\operatorname{var}(r_m) = 0.5\times(30\% - 10\%)^2 + 0.5\times(-10\% - 10\%)^2 = 0.04\,.$$

Definition of covariance

The statistical definition of covariance, illustrated in terms relevant to the case at hand, is:

$$\operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right) = E\left[\left[\frac{CF_1}{E[CF_1]} - E\left[\frac{CF_1}{E[CF_1]}\right]\right][r_m - E[r_m]]\right]$$

Since:

$$E\left[\frac{CF_1}{E[CF_1]}\right] = \frac{E[CF_1]}{E[CF_1]} = 1$$

we have:

$$\operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right) = E\left[\left[\frac{CF_1}{E[CF_1]} - 1\right][r_m - E[r_m]]\right].$$

Extending our earlier example, suppose that a cash flow will be 110 if the market is up and the economy is doing well and 90 if the market is down and the economy is doing poorly. In this case the expected cash flow is:

$$E[CF_1] = 0.5\times 110 + 0.5\times 90 = 100$$

and the covariance will be:

$$\operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right) = 0.5(1.1-1)(0.3-0.1) + 0.5(0.9-1)(-0.1-0.1) = 0.02.$$

Calculation of beta

In extending this example further, suppose that the risk free rate is 4% and the market risk premium is 6%. Note that this is consistent with the expected return on the market portfolio being 10%, as in our example above.

We can compute the beta for this case as:

$$\beta = \frac{\operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right)(1+r_f)}{\operatorname{var}(r_m) - MRP \times \operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right)} = \frac{0.02(1+0.04)}{0.04 - 0.06 \times 0.02} = 0.54\,.$$

Using the CAPM to estimate the required return yields:

$$E[r_p] = 4\% + 0.54 \times 6\% = 7.2\%\,.$$

Negative cash flow, better than expected when the market is down

In this case, we extend our example from above but consider a negative cash flow. Suppose that government has agreed to pay a subsidy to the private partner and that the cash flow (from the perspective of government) will be –44 if the market is up and the economy is doing well, or –36 if the market is down and the economy is doing poorly. For example, government may be paying an amount for each service provided by the private sector partner, and demand for the service might be positively related to the state of the economy. We continue to assume that there is a 50/50 chance of the market return being +30% or –10% over the period.

In this case we have:

$$E[CF_1] = 0.5 \times (-44) + 0.5 \times (-36) = -40$$

and the outcome (from the perspective of government) is worse than expected if the market is up and better than expected if the market is down.

Note that this is the reverse of the standard case for a profitable project with positive cash flows, where cash flow outcomes tend to be better than expected if the market is up and worse than expected if the market is down. Intuition might lead one to conclude that it would be appropriate in these circumstances to use the opposite beta that would be applied to a comparable, but profitable, project. However, this is not correct. The sign of the key covariance term, and consequently beta, is altered by the fact that the expected cash flow is negative. In particular, we have:

$$\operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right) = 0.5(1.1-1)(0.3-0.1) + 0.5(0.9-1)(-0.1-0.1) = 0.02.$$

We can compute the beta for this case as:

$$\beta = \frac{\operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right)(1+r_f)}{\operatorname{var}(r_m) - MRP \times \operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right)} = \frac{0.02(1+0.04)}{0.04 - 0.06 \times (0.02)} = 0.54\,.$$

Using the CAPM to estimate the required return yields:

$$E[r_p] = 4\% + 0.54 \times 6\% = 7.2\%\,.$$

Negative cash flow, guaranteed

Next consider the case where government is to make a guaranteed payment to the private sector partner, an absolutely fixed payment. In this case, the covariance between the cash flow and the return on the market is zero – there is no relationship between these variables because the cash flow is fixed. Substituting this into our equation for beta produces a beta estimate of zero, which implies that these cash flows should be discounted at the risk free rate. This accords with basic intuition. A series of cash flows that are guaranteed by government should be discounted at the relevant risk free rate. In other words, a government bond should be valued as a government bond.

Negative cash flow, better than expected when the market is up

In this final case, we extend our example from above but consider a negative cash flow that is better than expected when the market is up (i.e., the reverse of the previous example of a risky payment to be made by government). For example, government may have agreed to pay a subsidy to the private sector partner so that the cash flow (from the perspective of government) will be –36 if the market is up and the economy is doing well, or –44 if the market is down and the economy is doing poorly. That is, government is effectively subsidising the losses of the private partner and those losses will be lower if the market is up and the economy is doing well. We continue to assume that there is a 50/50 chance of the market return being +30% or -10% over the period.

In this case we have:

$$E[CF_1] = 0.5 \times (-36) + 0.5 \times (-44) = -40$$

and the outcome (from the perspective of government) is better than expected if the market is up and worse than expected if the market is down.

This is similar to the standard case for a profitable project with positive cash flows – cash flow outcomes tend to be better than expected if the market is up and worse than expected if the market is down. Intuition might lead one to conclude that it would be appropriate in these circumstances to use the same beta that would be applied to a comparable, but profitable, project. However, this is not correct. The sign of the key covariance term, and consequently beta, is altered by the fact that the expected cash flow is negative. In particular, we have:

$$\operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right) = 0.5(0.9-1)(0.3-0.1) + 0.5(1.1-1)(-0.1-0.1) = -0.02.$$

We can compute the beta for this case as:

$$\beta = \frac{\operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right)(1+r_f)}{\operatorname{var}(r_m) - MRP \times \operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right)} = \frac{-0.02(1+0.04)}{0.04 - 0.06 \times (-0.02)} = -0.50 .$$

Using the CAPM to estimate the required return yields:

$$E[r_p] = 4\% + (-0.50) \times 6\% = 1.0\% .$$

Note that the beta estimate in this case is not the exact opposite of the previous case involving risky cash flows. In the previous case, the cash flow was 10% worse than expected when the market was up and 10% better than expected when the market was down. In this case, the reverse is true. In the previous case, the key covariance term

$$\operatorname{cov}\left(\frac{CF_1}{E[CF_1]}, r_m\right)$$

was +0.02 and in this case it is –0.02. That is, this covariance term has the same magnitude but a different sign. However, beta is not a perfect linear function of the key covariance term. In particular, the denominator of the expression for beta contains a second order term that involves the product if the covariance term and the MRP. But this results in a minor variation so that the beta is 0.54 for the former case and –0.50 in the latter.

In this regard, we note that beta cannot be estimated with any great precision (certainly not with respect to the second decimal place) and that the *Guidelines* propose that beta estimates be obtained from the relevant broad risk band. The minor variation in the magnitude of beta in the example above would certainly not result in a movement from one risk band to another. The obviously more important effect is in obtaining the correct sign on the beta estimate. Consequently, the focus of our proposed approach is on obtaining the correct sign for the beta estimate and takes the magnitude of beta from the relevant risk band (consistent with the approach set out in the *Guidelines* in this respect).

Summary

This appendix shows that beta depends upon the covariance between (a) a cash flow relative to its expected value, and (b) market returns. It also shows that this covariance (and consequently beta) depends crucially on:

- Whether the expected cash flow is positive or negative; and
- Whether the cash flow is more likely to be better than expected when the market is up and worse than expected when the market is down, or vice versa.

Consequently, when determining the sign for beta (positive or negative) these two considerations must be addressed. Our proposed approach sets out a simple 2×2 grid for this purpose, as illustrated in Table 7 above.

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