ABSTRACT

We explore how the underlying informational frictions, or anonymity, that give rise to the existence of monetary exchange affect international exchange rate dynamics. Using a two-country, two-sector model, we show that information friction implies a particular restriction on domestic relative pricing dynamics and hence on international nominal and real exchange rate determination. Furthermore, if capital is utilized as a factor of production in both production sectors, there is a further restriction on asset pricing relations (money and capital), as result of monetary and real outcomes being interdependent in the model. Our perfectly flexible price model is capable of producing seemingly rigid aggregate prices, endogenously, in response to technology and monetary shocks. The model is thus capable of accounting for the empirical regularities that the real and nominal exchange rates are more volatile than U.S. output, and that the two are positively and perfectly correlated. These quantitative features are obtained with the model also being consistent with other standard real business cycle facts.

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1. Introduction

It is well known that the real and nominal exchange rate of the world’s largest economies are very volatile and persistent. Moreover, the two series are almost-perfectly and positively correlated. The seminal work of Chari, Kehoe, and McGrattan [2002] explored whether these features of the data could be understood in the context of a standard two country real business cycle model with sticky prices. They concluded that such models can explain the volatility of the real exchange rate but that they cannot match its persistence. In a nutshell, ad-hoc sticky price models are able to generate volatile real and nominal exchange rate processes, because, by assumption prices are made to not adjust too quickly to aggregate shocks. In an open economy, the nominal exchange rate and therefore, the real exchange rate, have to overreact. This is a manifestation of the textbook Dornbusch [1976] exchange rate overshooting hypothesis.

The key ingredient in modern monetary theory, and in our model, is a notion of “anonymity” of traders. Anonymity is a nutshell term for: (i) The lack of, or, imperfect record-keeping of individual trader’s histories; (ii) Nonexistence of public communication of individual trading histories; and (iii) Lack of enforcement of private contracts. Given this assumption of anonymity and coupled with a bilateral random matching environment (which gives rise to a lack of double coincidence of wants), one can thus rationalize an equilibrium theory of the demand for money. Money in this type of environment is thus a medium of exchange and a store of value (i.e. serves a precautionary asset function) since agents face individual random-matching uncertainty before trading in the DM. In contrast, existing monetary business-cycle models introduce money in more reduced-form ways using either money-in-the-utility (MIU) or cash-in-advance constraints (CIA) [e.g. Chari, Kehoe, and McGrattan, 2002; Schlagenhauf and Wrase, 1995]. These are not innocuous modelling choices. We show that a deeper model of domestic money demand matters for the dynamics of relative prices domestically and internationally.

In this paper, we examine whether a flexible price, two-country, search theoretic model of money is able to account for the empirical regularities observed in U.S. real and nominal exchange rates. We consider a two-country version of Lagos and Wright [2005] where international trade and asset flows occur in the model’s frictionless Walrasian centralized markets (CM). This assumption allows direct comparisons with existing international monetary business cycle models with flexible prices (e.g. Schlagenhauf and Wrase [1995]) and models with sticky prices (e.g. Chari, Kehoe, and McGrattan [2002]), while providing a foundation for money and an alternative equilibrium restriction on pricing processes. Following Aruoba, Waller, and Wright [2008], we allow for installed capital in each centralized market (CM) to be a productive input for sellers in each subsequent decentralized market (DM). This aspect of “capital complementarity” generates an equilibrium linkage between inflation and real economic activity across the DM and CM. There are thus two key mechanisms at work in this model which help amplify and propagate international business cycle shocks. The first mechanism is monetary friction (“anonymity”). This friction induces asset market incompleteness in the sense that individuals are unable to fully insure against their preference shocks in the DM.
The second mechanism is the notion of capital complementarity. The latter mechanism provides for an additional return on capital which places additional restriction on the equilibrium asset pricing relations with respect to money and capital.

To disentangle the contribution of monetary frictions and the role of the non-tradable sector on the exchange rate dynamics, we relax the anonymity assumption, as in Aruoba, Waller, and Wright [2008]. In particular, we introduce an exogenous probability that agents in each DM may be segmented into one of two kinds of trades: anonymous monetary trades or monitored credit trades. By considering the limit of pure credit trades in the DM, first-best allocation, we are able to shut down the role of monetary friction and to isolate the effect on exchange rate dynamics due to the traded versus non-traded sector. We show that this feature alone cannot account for the stylized facts on the exchange rates for the U.S.. However, when there is a small degree of anonymity, and hence monetary trades exist, relative pricing dynamics behave in such a way that aggregate relative prices are relatively non-volatile and persistent, in response to shocks. This contributes to the excess volatility and persistence in the real and nominal exchange rate. Thus without requiring exogenous price-stickiness nor additional shocks, the benchmark model is also able to rationalize near perfect positive correlation between the real and nominal exchange rate. To be sure that the second mechanism of capital complementarity is not a key driver of the results, we also consider the limit where this complementarity is not present. Again, we show that the real exchange rate exhibits the stylized fact of excess volatility only when there is a monetary or information friction. Thus the monetary friction, in the sense of Lagos and Wright [2005], is more than just a vehicle for a theoretical foundation of money. In a stochastic two-country environment, it restricts pricing relations such that the model is able to account for the stylized facts on real and nominal exchange rate fluctuations.

The paper is organized as follows. In section 2, we outline the details and assumption of the baseline quantitative-theoretical model. We then work through the equilibrium constructs and implications of the model in Section 3. Next, in Section 4, we provide some insight into the key mechanisms in the model. We then take the theory to the data in Section 5. We discuss the model’s business cycle features relative to the data and other existing models in Section 6. We then explain how the mechanisms interact to produce the business cycle features, by using the partial tool of impulse response analysis, in Section 7. We conclude in Section 8.

2. Environment

Consider a two-country model, each referred to as Home and Foreign. Variables and parameters without an asterisk (or with a subscript h) will refer to the Home country, and those without an asterisk (or with a subscript f), will refer to the Foreign country. Time is denumerable, and a time period is denoted by $t \in \mathbb{N} := \{0, 1, 2, \ldots\}$. Agents exist on a continuum $[0, 1]$ and have a common discount factor $\beta \in (0, 1)$. Each $t \in \mathbb{N}$ is composed of two arbitrary sub-periods, night and day. In the night, agents trade anonymously in bilateral random matches, in decentralized markets (DM). In the day, agents trade in centralized markets (CM). The nature of consumption, production and trade in each market will be explained in detail in sections 2.5 and 2.6.
2.1. Preferences and DM technology. Let $q^b \in \mathbb{R}_+$ be an agent’s consumption (if the agent is a buyer) and $q^f \in \mathbb{R}_+$ be an agent’s output (if the agent is a seller) of a “specialized”, or, agent-specific and non-storable good in the DM. Similar to Lagos and Wright [2005], an agent who is a producer of a $q^f$ is assumed to not value it, but may, with probability $\sigma \leq 1/2$ exchange it (for money) with another agent who, with symmetric probability $\sigma$, wishes to consume it (a buyer) – i.e. it is a $q^b$ to this buyer. Thus, with probability $1 - 2\sigma$, an agent will leave a DM with no exchange. For simplicity, assume that “doble-coincidence-of-wants” events, where buyers and sellers in the DM are able to barter and money is inessential, occur with probability zero.

Let $X \in \mathbb{R}_+, K \in \mathbb{R}_+$ and $H \in [0, \overline{H}]$, where $\overline{H} < +\infty$, denote consumption, individual capital stock and labor in the CM, respectively. Agents’ per-period preferences are given by

$$
(q, X, H) \mapsto \begin{cases} 
    u(q) + U(X) + h(H) & \text{if buyer} \\
    -c(q/z, k) + U(X) + h(H) & \text{if seller}
\end{cases}
$$

where $u(q)$ is the per-period payoff from $q$ if the agent is a buyer, $z$ is aggregate home total factor productivity, and $c(q/z, k)$ is the utility cost of producing $q$ with fixed within-period $k$ determined in the previous CM. $U(X)$ is the immediate payoff from consuming $X$ in the CM, and $-h(H)$ is the disutility of work effort in the CM. We make the following assumptions.

Assumption 1. The functions $u, U, h : \mathbb{R}_+ \to \mathbb{R}$ and $c : \mathbb{R}_+^2 \to \mathbb{R}$ have the following properties:

(i) First and second derivatives exist everywhere: $u, U \in C^2(\mathbb{R}_+)$ and $c \in C^2(\mathbb{R}_+)$;

(ii) $u_q > 0, c_q > 0, c_k < 0, U_X > 0$ and $h_H > 0$;

(iii) $u_{qq} < 0, c_{qq} > 0, c_{kk} < 0, U_{XX} < 0$ and $h_{HH} = 0$;

(iv) $u(0) = c(0, 0) = 0$; and

(v) $u(q) > c(q/z, k)$ for every $(q/z, k)$.

2.2. CM Technology. In the CM the final good in the Home country is produced according to a constant returns technology, $(y_h, y_f) \mapsto G(y_h, y_f)$, where $y_h$ denotes the input demand for an intermediate good produced in the home country, and, $y_f$ represents the demand of a substitutable input produced in the foreign country. Similarly, the foreign final good is given by, $(y_f^*, y_h^*) \mapsto G^*(y_f^*, y_h^*)$. Assume that $G \in C^2(\mathbb{R}_+^2)$ and that $G_i > 0, G_j > 0, G_{ii} < 0, G_{jj} < 0$, and, $F(i, 0) = F(0, j) = 0$, for some inputs $i, j$.

Let $K$ denote an aggregate capital stock in each home country. The production of the different intermediate goods are given by another constant returns technology, $(K, H) \mapsto zF(K, H)$ which is subject to a stochastic productivity shock, $z$. Assume $(z_i)_{i \in \mathbb{N}}$ is a strictly positive and bounded stochastic process. Assume that $F \in C^2(\mathbb{R}_+^2)$ and that $F_K > 0, F_H > 0, F_{KK} < 0, F_{HH} < 0$, and, $F(K, 0) = F(0, H) = 0$.

2.3. State variables. Let $m \in \mathbb{R}_+$ be the stock of an agent’s local nominal money holding in the Home country. Denote $b$ as the current stock of an internationally traded complete state-contingent money claim, held by an agent in the Home country. Each $b$ is denominated in the

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1 It will be immediately apparent to the serious monetary theorist that we are placing an ad hoc restriction that agents can only use the local currency to buy local goods, especially in the DM. We appeal to observed facts that this is indeed what we see— one does not pay for a haircut in the United States using the South Korean Won. Of course, what we expect to see in the model ought to be the result of a possible equilibrium in an environment where agents are not a priori restricted to hold a particular currency. One possible microfoundation lies in sellers’ unwillingness to
Home currency. Since these complete contingent claims require knowledge of traders’ histories, it is natural that they are not issued or traded in the DM with anonymous randomly matched trades. They are traded only during each CM subperiod.

Now we introduce a modelling device that will help us isolate the role of anonymity or monetary friction in the model. Following Aruoba, Waller, and Wright [2008], suppose that conditional on being a buyer or seller, the exogenous probability that a buyer or seller would engage in an exchange where record keeping is possible is \((1 - \kappa) \in [0, 1]\). That is, the event that a buyer or a seller can buy or sell a good in the DM using credit occurs with the discrete probability measure \(\sigma(1 - \kappa)\). Since credit is assumed to be enforceable in such an event, a buyer is willing to take (and a seller is willing to give) out the nominal loan \(l\) in exchange for a good, say \(\tilde{q}\). This loan is required to be repaid in full in the following CM. Thus there is no complication with discounting given the timing of the sub-markets. Then we let \(q\) denote a DM specialized good that is exchanged for money in events where exchange occurs with measure \(\sigma\kappa\) for a buyer or seller. Thus we have two distinct markets, one for anonymous traders where cash is needed and one where credit is available. In particular, a fraction \(\sigma\kappa\) of agents can trade in DM with credit. While a fraction \(\sigma\kappa\) of agents trade only using fiat money. This is useful because when \(\kappa = 0\), we are able to shut down the source of monetary friction – the anonymity assumption – and the resulting limit economy is a version of a two-sector real business cycle model with traded and nontraded goods.

Denote the vector of exogenous shocks as \(z \in Z\). For example, we consider Home and Foreign, technology (\(z\)) and money growth (\(\psi\)), shocks. Thus \(z := (z^*, \psi, \psi^*)\), and \(Z \subset \mathbb{R}^4\).

Let the time-\(t\) aggregate (global) CM state vector relevant to an agent in country \(i \in \{h, f\}\) be \(s := (M, M^*, B, B^*, K, K^*, \phi, \phi^*, e, \mu_h, \mu_f, z)\), consisting of, respectively, the global/aggregate Home and Foreign capital stocks, the global Home and Foreign DM-specific capital stocks, the total Home and Foreign holding of the state-contingent claims, Home and Foreign nominal money stock, the value of money in the Home CM (\(\phi := 1/p_X\)), the value of money in the Foreign CM (\(\phi^* := 1/p_X^*\)), the nominal exchange rate in Home CM currency terms (\(e\)), and, \(\mu_i(\cdot, z) : B_i(z) \to [0, 1]\) which is the time-\(t\) probability measure on the Borel \(\sigma\)-field \(B_i(z) := B([R_+ \times R \times R_+ \times R])\) generated by the product state space containing \((m, b, k, l)\), for each vector of exogenous state variables, \(z\).

Also, let the space of all such distributions be \(\mathcal{P}(B_i)\), for each fixed \(z\).

At the beginning of the time-\(t\) DM, the aggregate (global) state vector for an agent in country \(i \in \{h, f\}\) is \(\hat{s} := (M, M^*, B, B^*, K, K^*, \phi, \phi^*, e, v_{h0}, v_{f0}, z)\). The explicit switch in notation from \(v_i\) to \(\mu_i\) takes into account that, in general, the distribution of assets upon the economy \(i\) entering

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1. Accept a currency they do not recognize [see e.g., Lester, Postlewaite, and Wright, 2008]. However, these explorations are beyond the scope of this paper. We have nevertheless tried a version of the model where we do not restrict trades of local goods to local currencies. In this case, the composition of each agent’s currency portfolio, and therefore, the nominal exchange rate, will be indeterminate – i.e. we have a stochastic version of Kareken and Wallace [1981]. Unfortunately, this les stringent version of the model does not admit any stable rational expectations equilibrium, given a data consistent calibration of the model.

2. Note that if \(Z = \emptyset\), i.e. in the absence of aggregate exogenous shocks, then the solution of the Markov equilibrium is characterized by a deterministic difference equation system, as in Lagos and Wright [2005]. Also, note that the aggregate prices (\(\phi, \phi^*, e\)) are explicitly included as (auxiliary) state variables, following Duffie, Geanakoplos, Mas-Colell, and McLennan [1994], so that we can restrict our characterization of equilibria to stationary Markov equilibria.

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each period’s DM, \( \nu_i \), may be different to the distribution \( \mu_i \) upon its leaving the DM, and into the CM, in the same period.\(^3\)

2.4. **Timing.** Figure 1 depicts the sequence of events within each \( t \in \mathbb{N} \). The relevant aggregate state vector \( s \) is realized at the beginning of each \( t \). This is public information for all agents. An agent in the Home country, first entering the DM with assets \( (m, b, k) \) respectively, money, bonds, and capital, given \( \hat{s} \), is publicly known by the *individual* state \( (a, \hat{s}) := (m, b, k, 0, \hat{s}) \). For simplicity, we make the restriction that each country \( i \) agent does not hold another country’s currency as asset.\(^4\) Since bilateral matches in the DM are random, agents within each country \( i \) only know the state of their trade partners *ex post*. *Ex ante* they only know the probability distribution of traders in the DM, which is \( (\sigma, \sigma, 1 - 2\sigma) \) with support \{Buyer, Seller, Neither\}. Conditional on either events \{Buyer\} or \{Seller\}, there is an identical distribution \( \{\kappa, 1 - \kappa\} \) faced by the agent of a trade being either anonymous (monetary) or monitored (credit).

\[
\begin{array}{c|c}
\text{DM: } V(m, b, k, \hat{s}) & \text{CM: } W(a', s) \\
\hline
\end{array}
\]

\[
\begin{array}{c|c}
\text{DM*: } V(m^*, b^*, k^*, \hat{s}) & \text{CM*: } W(a'^*, s) \\
\hline
\end{array}
\]

\[
\text{Trade: } (b_+, b^*_+), (y_f, y_h^*)
\]

Upon leaving the DM, an agent’s *individual* state changes to

\[
(a', s) := \begin{cases} 
(m', b, k, 0, s) & \text{w.p. } 2\sigma \kappa \\
(m, b, k, l, s) & \text{w.p. } 2\sigma (1 - \kappa)
\end{cases}
\]

\(^3\)It is straightforward to prove that the probability measures \( \nu_i \) for each \( i \in \{h, f\} \), is degenerate in any equilibrium, as a stochastic extension to the original proof in Lagos and Wright [2004]. This affords us plenty of tractability and ease of computation later.

\(^4\)See Head and Shi [2003] for the environment where agents trade currency internationally.
reflecting the possibility that money had changed hands as a result of the agent being a buyer or seller. As a result of that, the distribution of assets (namely money) would also have changed from \( v_t \in \mathfrak{s} \) to \( \mu_t \in \mathfrak{s} \). The components \( (b,k) \) have not changed since they are predetermined at the beginning of \( t \). Thus, within \( t \), the agent enters the CM with possible state \( (a',s) \). Agents do not discount payoffs within each period \( t \).

In the next two sections we describe in detail the sub-period problems, DM and CM, in a backward fashion. To economize on notation, we use the following convention. A variable or vector with a “+” subscript will denote its time \( t+1 \) contingent outcome. A state with a “-” subscript will denote its time \( t-1 \) realization. However, in some cases, variables with a “+” subscript, such as capital and bonds, are predetermined at the beginning of time \( t+1 \). In such cases, these are decision or control variables which will be made obvious in the problems below. The same variable without the “+” subscript denotes its current or time-\( t \) realization.

2.5. Centralized markets. In the CM, an agent consumes a general good \( X \in \mathbb{R}_+ \) which is produced using CM-specific labor \( H \in \mathbb{R}_+ \) and capital \( k \). In contrast to Lagos and Wright [2005], we model a set of nominally complete state-contingent claims issued by both countries. Agents in each country’s CM who consume more (less) than their total wealth can also trade in these securities. Note that we do not model international trade in final consumption goods or intermediate inputs. This keeps the model manageable and more importantly, allows us to focus on the effect of money and capital on the only channel for international linkage.

Let \( \lambda > 0 \) be the constant marginal disutility of work effort. Let \( \delta \in [0,1] \) be the depreciation rate of capital and \( \tau_k \) a proportional tax rate on capital income. Denote \( \bar{r} := r(s) \equiv (1 - \tau_k) (\hat{r}(s) - \delta) \) and \( r^* := r^*(s) \equiv (1 - \tau_k) (\hat{r}^*(s) - \delta) \) be, respectively, the after-tax competitive rate of return to physical capital, \( \bar{r} \) net of depreciation, at Home and in the Foreign country. Similar, denote \( w(s) := (1 - \tau_H) \hat{w}(s) \) as the after-tax real wage rate. Finally, denote \( \tau_X \) as the proportional tax rate on CM consumption \( X \).

Let \( m_+ := m(a,s), k_+ := k(a,s), \) and \( b_+ := b(a,s), \) so that \( a_+ = (m_+, b_+, k_+, 0). \) \( Q(a_+, s_+ | a, s) \) is the domestic price of one unit of the state-contingent claim with nominal \( (a_+, s_+)-\)contingent payoffs, i.e. \( b(a_+, s_+ | a, s). \) Let \( \phi := \phi(s) = 1/p_X(s) \) and \( \phi := \phi^*(s) = 1/p^*_X(s) \) be the inverses of the prices of \( X \) (i.e. the value of a unit of each currency), in the respective Home and Foreign countries.

At each \( t \in \mathbb{N} \), a price-taking agent (at the beginning of the CM sub-period in the Home country) named \( (m,b,k,l,s) \) solves the recursive problem of:

\[
W(m,b,k,l,s) = \max_{X,H,m_+ ,k_+ ,b_+} \left\{ U(X) - AH \right. \\
+ \beta \int V(m_+, b_+, k_+, s_+) \lambda(s, ds_+) \right\}
\] (1)
subject to

\[ s_+ = G(s, v_+), \quad v \sim \varphi \]

\[ (1 + \tau_X)X(a, s) + k(a, s) - k - \phi(s)b + T(s) \]

\[ = \phi(s) [m - m(a, s) - l] + w(s)H(a, s) + r(s)k \]

\[ - \phi(s) \int_{s_+,a_+} b(a_+, s_+ | a, s)Q(a_+, s_+ | a, s)\mu_h(s_+, da_+) \lambda(s, ds_+) \]

where \( \lambda(s, \cdot) \), for each given \( s \), is induced by \( G \circ \varphi \), and defines an equilibrium product probability measure over Borel-subsets containing \( s_+ \). Constraint (2) describes a transition law, where the mapping \( G = \mathcal{G}_s(z) \circ \mathcal{G}_z(z) \), with component \( \mathcal{G}_s(z) \) inducing the \( z \)-dependent stochastic process for endogenous aggregate states, \( \{s\} \setminus \{z\} \), is to be pinned down in equilibrium, and \( (z, v_+) \mapsto \mathcal{G}_s(z, v_+) \) is an exogenous map for the aggregate shocks. Implicit in the constraint (2) is the equilibrium transition of the distribution of individual states from the period-\( t \) CM, to the period-(\( t + 1 \)) DM:

\[ v_h(s_+, \cdot) = G_v [\mu_h(s, \cdot), z_+] \]

such that the relevant conditional distribution of assets at the beginning of the time-(\( t + 1 \)) CM subperiod is given by:

\[ \mu_h(s_+, \cdot) = G_v [v_h(s_+, \cdot), z_+] \equiv G_v [G_s(s, z_+)] \]

where \( G_v \) and \( G_s \) are components of \( G_s(z) \setminus \{z\} \).

The sequence of state-contingent one-period budget constraints given by (3) say the following: For each given state \( (m, b, k, s) \), consumption of the general good \( X \) is to be financed by the variation in real money holdings, by real labor income \( wH \), net of investment flows to physical capital made in the CM, net of contingent claims in real terms, and net of lump-sum government taxes, \( T \).

2.5.1. Optimal individuals’ decisions in the CM. Eliminating \( H \) in (1) using the budget constraint (3), the optimal decision rules satisfy the following conditions for every state \((a, s)\) and every measurable continuation state \((s_+, \hat{s}_+)\).

The optimal trade-off between current CM consumption \( X \) and leisure \( -H \), given the competitive real wage \( w := w(s) \), is

\[ X : \quad U_X [X(a, s)] = \frac{A(1 + \tau_X)}{w(s)} \]

where the marginal utility of leisure is a constant \( A > 0 \). In what follows, note that the derivative functions of \( V \) is determined in general equilibrium. The optimal trade-off between a current increase in marginal utility of \( X \) in the CM and the present-value expected marginal value of entering the next-period DM with a marginal increment of money holdings is

\[ m_+ : \quad \frac{A\phi(s)}{w(s)} = \beta \int V_{m_+}(m_+, b_+, k_+, 0, \hat{s}_+) \lambda(s, ds_+) \]

Similar to condition (7), conditions (8)-(9) below provide the optimal trade-offs between the current utility of consumption of \( X \) and the expected discounted marginal value of entering
the DM with more assets. Specifically, the optimal choice of the complete state-contingent money claims, or bonds, is given by

\[
b_+ (\cdot ; s) : \frac{A\phi(s)}{w(s)} \left[ Q(a_+, s_+ | a, s) \mu_h(s_+, da_+) \right] \lambda(s, ds_+) = \beta V_{b_+} (m_+, b_+, k_+, 0, \hat{s}_+),
\]

which holds for every \( s \), every \( \hat{s}_+ \), and implicitly, every \( s_+ \).

The optimal choice of the Home-produced capital stock available for production in the next period satisfies

\[
k_+ : \frac{A}{w(s)} = \beta \int V_{k_+} (m_+, b_+, k_+, 0, \hat{s}_+) \lambda(s, d\hat{s}_+).
\]

2.5.2. Envelope conditions for \( W \) in the CM. At an optimum, the envelope conditions for the agent’s CM decision problem are as follows. The marginal value of money holdings upon entering the CM is

\[
W_m (m, b, k, l, s) = \frac{A\phi(s)}{w(s)},
\]

the marginal value of holding bonds upon entering the CM, respectively, are

\[
W_b (m, b, k, l, s) = \frac{A\phi(s)}{w(s)},
\]

and the marginal value of holding the each of the four types of capital stocks at the beginning of the CM are as follows. With respect to a Home agent’s holding of capital stock in the Home country, the marginal CM value is:

\[
W_k (m, b, k, l, s) = \frac{A}{w(s)} [1 + r(s)].
\]

With respect to a Home agent’s holding of credit in the Home country, the marginal CM value is:

\[
W_l (m, b, k, l, s) = -\frac{A\phi(s)}{w(s)}.
\]

The envelope conditions (10)-(13) imply that, \( W \) is linear in \((m, b, k, l)\), for each fixed aggregate state \( s \). So we can write \( W \) as

\[
W(m, b, k, l, s) = W(0, 0, 0, s) + \frac{A}{w(s)} \left[ \phi(m + b) + (1 + r)k \right].
\]

2.5.3. Firms. Let \( P_h \) be the Home currency price of the Home produced intermediate good, and \( P_y \) be that of the Foreign produced intermediate good use by the Home final-good firm. The Home final-good firm solves:

\[
\max_{y_h, y_f} \left\{ \frac{G(y_h(s), y_f(s))}{\phi(s)} - P_h(s)y_h(s) - P_f(s)y_f(s) \right\}.
\]

The profit-maximizing conditions are:

\[
\phi(s)P_h(s) = G_{y_h}[y_h(s), y_f(s)],
\]
and
\[ \phi(s) P_f(s) = G_f[y_h(s), y_f(s)]. \]  \hfill (16)

The Home intermediate goods producer solves
\[
\max_{H,K} \left\{ P_y h(s) \cdot z F_k[K(s_\cdot), H(s)] - \frac{[\omega(s) H(s) + r(s) K(s_\cdot)]}{\phi(s)} \right\},
\]
where the market for inputs to \( F \) is perfectly competitive. Profit maximization is characterized by the usual first order conditions where capital and labor are paid a respective rental rate which equals their marginal products in every aggregate state \( s \):
\[ \tilde{r}(s) = \phi(s) P_h(s) \cdot z F_k[K(s_\cdot), H(s)], \]  \hfill (17)
and
\[ \tilde{w}(s) = \phi(s) P_h(s) \cdot z F_H[K(s_\cdot), H(s)], \]  \hfill (18)
where
\[ H(s) = \int_a H(a,s) \mu_h(s,da) \]
is aggregate labor supply in the Home CM.

Without loss of generality for the rest of the model, we shall assume that \((z_t)_{t \in \mathbb{N}}\) is induced by some Markov process to be described in the application later on. A foreign country’s CM agent named \((m^*, b^*, k^*, l^*, s)\) and its firm have a symmetric problem to (1)-(3), (15)-(16), and (17)-(18).

### 2.6. Decentralized markets.

At the beginning of the CM at each \( t \in \mathbb{N} \), an agent named \((m, b, k, 0, \hat{s})\) enters the DM.\(^5\) With a fixed probability \( \sigma \) this agent is the buyer of the special good that some other agent produces, \( q^b \), where the other agent (seller) is indexed by the state \((\tilde{a}, \hat{s}) := (\tilde{m}, \tilde{b}, \tilde{k}, \hat{s})\), but not vice-versa. With probability \( \sigma \kappa \), the buyer parts with \( d^b \) “dollars” and realizes a payoff of \( u(q^b) \in \mathbb{R} \). The buyer then enters the day CM with a value of \( W(m - d^b, b, k, 0, s) \). With probability \( \sigma (1 - \kappa) \), the buyer does not use money, but takes out a nominal loan \( l \), from the seller he meets, and realizes a payoff of \( u(q^b) \in \mathbb{R} \). The buyer then enters the day CM with a value of \( W(m, b, k, l, s) \).

Symmetrically, with probability \( \sigma \kappa \), agent \((m, b, k, \hat{s})\) has a special good \( q^a \) which other buyers want to buy, but not vice-versa. This agent receives \( d^a \) dollars in exchange for exerting a production cost of \( c(q^a/z, k) \in \mathbb{R}_+ \). In the DM our agents have their capital physically fixed in place at production sites. Thus, a buyer must visit, random the location of a seller, and since capital is not portable, it cannot be used for payment, while currency can. This use of spatial separation is in the spirit of the “worker-shopper” idea. Notice that capital obtained

\(^5\)Note that \( m \) implicitly includes any aggregate monetary transfer or injection from the government, which we denote later as \( i(\hat{s}) \), so then, \( m(\hat{s}) = m(s_\cdot) + i(\hat{s}) \).
from the previous period’s CM, \( k \), accrues a return in the DM in the form of the marginal benefit to producing \( q (q^d \text{ or } \tilde{q}^d) \), i.e. \( c_k(q/z, k) \). This seller then enters the day CM with a value of \( W (m + d^s, b, k, s) \). With probability \( \sigma (1 - \kappa) \), a seller may sell \( \tilde{q}^d \) by extending a loan \( l \) to a matched buyer.

These four events described above are known as single-coincidence-of-wants meetings, where money is a portable medium of exchange in events that occur with probability \( 2\sigma \kappa \), and where credit \( l \) is the medium of exchange in events with probability \( 2\sigma (1 - \kappa) \). With probability \( 1 - 2\sigma \), agent \( (m, b, k, s) \) leaves the DM and enters the day with his assets intact, and begins his activity in the CM with value \( W(m, b, k, 0, s) \). For simplicity, we assume the probability of a “double-coincidence” meeting, and hence the occurrence of pure barter, is zero.

Formally, an agent named \( (m, b, k, 0, \tilde{s}) \) has a value \( V(m, b, k, 0, \tilde{s}) \) at the beginning of the DM that satisfies the following problem:

\[
V(m, b, k, 0, \tilde{s}) = \sigma V^b(m, b, k, 0, \tilde{s}) \\
+ \sigma V^s(m, b, k, 0) + (1 - 2\sigma)W(m, b, k, s). 
\]  
(19)

where, in general:

\[
V^b(m, b, k, 0) = \kappa \int [u(q^b) + W (m - d^b, b, k, 0, s)] v_h(d\tilde{a}, \tilde{s}) \\
+ (1 - \kappa) \int [u(\tilde{q}^b) + W (m, b, k, l^b, s)] v_h(d\tilde{a}, \tilde{s}),
\]

and,

\[
V^s(m, b, k, 0) = \kappa \int [-c(q^s, k) + W (m + d^s, b, k, 0, s)] v_h(d\tilde{a}, \tilde{s}) \\
+ (1 - \kappa) \int [-c(\tilde{q}^s, k) + W (m, b, k, -l^s, s)] v_h(d\tilde{a}, \tilde{s}),
\]

are the value functions of ex-post buyer and sellers respectively.

2.6.1. **Walrasian price taking.** Consider a version where \((q^b, q^s, d^b, d^s, q^b, q^s, l^b, l^s)\) are determined by Walrasian price taking. Then, we have:

\[
V^b(m, b, k, 0) = \kappa \max_{q^b \in [0, \rho/\beta]} \left[ u(q^b) + W (m - \tilde{p}q^b, b, k, 0, s) \right] \\
+ (1 - \kappa) \max_{q^b \in [0, \rho/\beta]} \left[ u(\tilde{q}^b) + W (m, b, k, l^b, s) \right],
\]

where \( d^b = \tilde{p}q^b \), and,

\[
V^s(m, b, k, 0) = \kappa \max_{q^s} \left[ -c(q^s/z, k) + W (m + d^s, b, k, 0, s) \right] \\
+ (1 - \kappa) \max_{q^s} \left[ -c(\tilde{q}^s/z, k) + W (m, b, k, -l^s, s) \right],
\]

where \( \tilde{p} \) and \( \tilde{p} \) are the respective prices of the special good, taken as given by all buyers and sellers.

---

\(^{6}\)This feature was first introduced by Aruoba, Waller, and Wright [2008, Appendix A.1]. The authors showed that whether there exist two kinds of capital goods, for use in the DM and in the CM production, respectively, is of negligible quantitative consequence in their model.
2.7. **Monetary policy.** New money supply is injected at the end of the period in the CM.\(^7\) Specifically, the monetary authority follows a monetary supply rule:

\[
M(s) = \exp(\psi)M(s_-),
\]

(20)

where \(\exp\{\psi\} - 1\) is the one-period money supply growth rate between time \(t\) and \(t+1\). Assume that \((\exp(\psi_t))_{t \in \mathbb{N}}\) follows a Markov process that lives in the compact set \([1, N]\), with \(N < +\infty\). We define this process later.

The monetary authority’s nominal budget constraint for each \(s\) is:

\[
M(s) = i(s) + M(s_-).
\]

(21)

Thus, aggregate money injection \(i(s)\) is given by:

\[
i(s) = [\exp(\psi) - 1] M(s_-).
\]

Also, note that if we set \(\exp(\psi) = 1\) for all periods, then money supply remains constant forever. In this case, \(i(s) = 0\) for all \(s\) so no new money is injected into the economy. Again, the Foreign country would have a symmetric description of its government policy.

---

3. **Stationary Markov Monetary Equilibrium**

In this section, we state a key result which is just an extension of Lagos and Wright [2005] to environments with aggregate uncertainty. We claim here that in an equilibrium, the endogenous distribution of agents’ asset holdings is degenerate at the start of each period (and hence DM), such that all agents in each country choose the same allocations that depend only on the global state. We further characterize the equilibrium conditions in the DM and list the conditions for market clearing in the CM. We then define the elements that constitute a *stationary Markov monetary equilibrium*, which includes non-Walrasian or decentralized bilateral random matches with alternative pricing mechanisms: Walrasian price-taking, generalized Nash bargaining, and proportional bargaining.

In general, because of the memory-less random matching process in the DM, we will need to track the history of aggregate distribution of assets held by agents in any equilibrium where money has value. However, because of the quasi-linear assumption on each agent’s per-period payoff function, it can be shown that in equilibrium asset holdings at the beginning of each \(t \in \mathbb{N}\) are identical across all agents within each country \(i\), so that,

\[
(m, b, k)(s) = \int (m, b, k)\nu_i(\tilde{s}, dm, db, dk)
\]

\[
:= (M, B, K)(\tilde{s})
\]

(22)

\[
= (M, B, K)(z).
\]

---

\(^7\)This is merely for mathematical convenience, so that within each DM, agents do not have to deal with a stochastic total payoff function, \(W\).
for each \( i \in \{h,f\} \), for all \( \hat{s} \). This implies that we can explicitly write \( v(\hat{s}, \cdot) \) as \( v(z, \cdot) \), and furthermore, for every \( z \), and every \( A \in B_i(z) \),

\[
v_i(z, A) = \begin{cases} 1 & \text{if } (m, b, k) = (M, B, K) \in A \\ 0 & \text{otherwise} \end{cases}
\]

However, we can see that even if \( v_i(z, \cdot) \) is degenerate at the end of the CM, \( \mu_i(z, \cdot) \) is not. Thus, explicitly, agents at the beginning of each CM will still face an aggregate state variable \( s \) that contains a non-degenerate distribution of individual states. Specifically, the non-degeneracy is along the dimension of money holdings out of the DM.

### 3.1. DM competitive pricing and equilibrium decisions.

In equilibrium, the constraints \( d \leq m \), and \( l \leq \hat{p} \hat{q} \) bind, and \( q^b = q^s = q \). Thus for the \( \sigma \kappa \) proportion of agents who are sellers that meet buyers and they trade with money, we have the equilibrium condition that the relative price of the special good in terms of the CM final good, is equal to the marginal cost of the DM seller:

\[
\frac{A \phi}{w} M = \frac{1}{z} c_q(q/z, K)q \equiv g(q, K).
\]  

(23)

Note that \( \hat{p} = M/q \) in equilibrium.

For the \( \sigma(1 - \kappa) \) proportion of buyers and sellers, we have:

\[
\frac{A \phi}{w} l = \frac{1}{z} c_q(\hat{q}/z, K)\hat{q} \equiv g(\hat{q}, K).
\]

(24)

Since by assumption contracts are enforceable for these agents, then credit attains the first best allocation in terms of \( \hat{q} \) satisfying

\[
u_q(\hat{q}) = \frac{1}{z} c_q(\hat{q}/z, K).
\]

(25)

Therefore we can substitute out credit in the equilibrium conditions later, using

\[l = \frac{w u_q(\hat{q})\hat{q}}{A \phi}.\]

(26)

### 3.2. Envelope conditions for \( V \) in the DM.

At an interior optimum consistent with equilibrium, we have the following envelope conditions. Utilizing the linearity of \( W \), the marginal value of money at the beginning of the DM is

\[
V_M(M, B, K, 0, \hat{s}) = \frac{A \phi}{w} \left[ (1 - \sigma \kappa) + \sigma \kappa z \cdot u_q(q) c_q(q/z, K) \right] > 0.
\]

(27)

The marginal value of the state-contingent money claims at the beginning of the DM is

\[
V_B(M, B, K, 0, \hat{s}) = W_b(M, B, K, 0, s) = \frac{A \phi}{w}.
\]

(28)

The DM marginal value of the capital stock is

\[
V_K(M, B, K, 0, \hat{s}) = \frac{A \phi}{w} (1 + r) - \sigma \kappa \gamma(q, K, z) - \sigma(1 - \kappa)\gamma(\hat{q}, K, z) > 0,
\]

(29)
where
\[
\gamma(q, K, z) = c_K(q/z, K) < 0. \tag{30}
\]

The function \(\gamma\) is strictly negative due to two effects that capture the reduction in marginal cost of production in the DM. The first term on the right of (30) is the indirect effect on marginal cost through the effect of an additional capital stock on the terms of trade \(q\).

This reflects the fact that parties to a monetary trade in the DM are not price takers. The terms of trade determined by the bargaining solution thus takes into account the seller’s stock of capital in the DM. The second term is the capital-effort complementarity effect in the production of \(q\) in the sense that marginally more capital stock reduces the cost of producing \(q\).

### 3.3. Market clearing in the CM

In an equilibrium, since agents within each country choose the same asset holdings, i.e. \((m, b, k) = (M, B, K)\), then they do not borrow from, or, lend to each other, only countries lend to each other. Therefore, in the global equilibrium, state-contingent money claims by Home and Foreign have zero excess demand:

\[
B(s) + B^*(s) = 0. \tag{31}
\]

in every state \(s\). The Home resource constraint is given by

\[
G[y_h(s), y_f(s)] = X(s) + I(s) + G^d(s), \tag{32}
\]

where \(I(s) = K(s) - (1 - \delta)K(s_-)\) is domestic capital investment, and,

\[
G^d(s) = [T(s) + (M(s) - M(s_-))\phi(s)]
+ \tau_X X(s) + \tau_H H(s) + \tau_K (\tilde{r}(s) - \delta)K(s_-).
\]

The Foreign resource constraint is given by

\[
G[y^*_h(s), y^*_f(s)] = X^*(s) + I^*(s) + G^{d*}(s), \tag{33}
\]

where \(I^*(s) = K^*(s) - (1 - \delta)K^*(s_-)\) is the Foreign country’s investment in its own capital stock, and,

\[
G^{d*}(s) = [T^*(s) + (M^*(s) - M^*(s_-))\phi^*(s)]
+ \tau_X X^*(s) + \tau_H H^*(s) + \tau_K (\tilde{r}^*(s) - \delta)K^*(s_-).
\]

Market clearing for the intermediate goods must hold:

\[
zF[K(s_-), H(s)] = y_h(s) + y^*_h(s) \tag{34}
\]
\[
z^*F[K^*(s_-), H^*(s)] = y^*_f(s) + y_f(s) \tag{35}
\]

**Definition 1.** A monetary stationary Markov equilibrium (SME), given any feasible monetary policy rule \((\psi, \psi^*)\), is a set of time-invariant maps consisting of

- **E1.** strictly positive pricing functions \((\phi, \phi^*, e)\) and \((w, r, w^*, r^*, Q)\),
- **E2.** transition laws \((G, \psi)\) and \((G^*, \psi^*)\),
- **E3.** value functions \(V, W\) and \(V^*, W^*\),
- **E4.** CM decision rules \((X, X^*, m, m^*, b, k, b^*, k^*)\), and
E5. DM terms of trade (decision rules), \((d, q, \ddot{q})\) and \((d^*, q^*, \ddot{q}^*)\),
such that:

1. given prices (E1), the value functions \(V\) and \(W\) satisfy the functional equations (1), (2), (3), and (19) and symmetrically \(V^*, W^*\) solve the Foreign country counterpart problems;
2. given the value functions \(V\) and \(W\), and prices (E1), the decision rules E4 solve (1), (2), (3) in the CM, for the Home country and symmetrically for the Foreign country, given \(V^*\) and \(W^*\);
3. Firms optimize: (17) and (18);
4. given the value functions \(W\) and \(V\), the decision rules E5 solve (23), (25), and (26) in the DM, and symmetrically for the Foreign country, given \(W^*\);
5. The government budget constraint (21) is satisfied for Home and symmetrically for Foreign.
6. Markets clear in the CM and CM*: (31), (32) and (33), where \(m = M, b = B\) and \(k = K, m^* = M^*, b^* = B^*\) and \(k^* = K^*\).

3.4. Other variable definitions. Since the model features a DM sector that is akin to a non-traded goods sector, we will need to define a relevant price index in order to define a real exchange rate. Empirically, the relevant exchange rate will have to come to some measure of a broad index, such as The U.S. Federal Reserve Board of Governor’s Broad Index used by Heathcote and Perri [2002]. First we define a DM price index as the convex combination of the pricing outcome in monetary and credit trades:

\[ p_{DM} := \kappa \ddot{p} + (1 - \kappa) \ddot{\dot{p}}. \]

The foreign counterpart will be \(p^*_{DM}\). Denote the aggregate DM consumption as

\[ q_{DM} := \kappa q + (1 - \kappa) \ddot{q}. \]

Now we can define our measure of aggregate price index (or output deflator) as

\[ P_Y = \zeta \phi^{-1} + (1 - \zeta) p_{DM}, \]

where

\[ \zeta = \frac{X}{X + \sigma q_{DM}}, \]

is the CM consumption share in total domestic consumption. Note that this share is time-varying in the sense that it is dependent on the aggregate state \(s\). The foreign price index is defined analogously as \(P^*_Y\). Now we define the real exchange rate as

\[ RER(s) := \frac{e(s)P^*_Y(s)}{P_Y(s)}. \] (36)

4. Equilibrium Exchange Rate Dynamic Implications

We are now in a position to gain further insights into the international asset pricing properties of the model and the implication of the assumption of anonymity, \(0 < \kappa \leq 1\), for the exchange rates. For ease of notation and exposition, and without loss of generality, we set \(\kappa = 1\) for now and \(\tau_X = \tau_H = \tau_K = 0\). Using the first-order conditions in the CM and DM, the
corresponding envelope conditions, and imposing equilibrium, we can derive a set of stochastic Euler functional equations necessary for characterizing a stationary Markovian equilibrium (SME). We can write the SME conditions as ones that characterize the solutions as \( s \)-dependent processes.\(^8\) The full details are given in Appendix A.

First, from (6), we can easily deduce that in equilibrium, \( X(a, s) = X(s) \) and \( X^*(a_f, s) = X^*(s) \), for all \( s \). Also, we have, in equilibrium, \( q(m, \hat{k}, s) = q(M, K, s) \equiv q(s) \) and \( q^*(m^*, \hat{k}^*, s) = q^*(M^*, K^*, s) \equiv q^*(s) \). Together with (7) and (86), we have the SME version of the Euler functional equation for optimal money holdings in the Home country:

\[
U_X[X(s)] = \beta E_\lambda \left\{ U_X[X(s_+)] \frac{\phi(s_+)}{\phi(s)} \left[ (1 - \sigma) + \sigma \frac{z_+ u_q[q(s_+) / \lambda]}{c_q[q(s_+) / z_+, K(s) / \lambda]} \right] \right\},
\]

where, \( E_\lambda \) denotes the expectation operator with respect to the conditional distribution \( \lambda(s, \cdot) \), and, the term in the square brackets is the state-contingent one-period nominal gross return on money holding. This return is made up of two terms: (i) With measure 1 \(-\sigma\), the positive one-for-one effect on the value of entering the CM with more money holding in the case that the agent does not trade in the DM net of the negative one-for-one effect on the value of entering the CM with less money holding in the case that the agent spent his money holding as buyer; and (ii) the positive marginal effect of money holding on utility of consumption via the specific good \( q \), conditional of the agent being a buyer with probability \( \sigma \). Alternatively, we can write (37) as an integral-functional equation solely in terms of the functions \( q \) and \( K \):

\[
g[q(s), K(s_+), z] = \frac{M(s_+)}{M(s)} E_\lambda \left\{ g[q(s_+), K(s), z_+] \left[ (1 - \sigma) + \sigma \frac{z_+ u_q[q(s_+) / \lambda]}{c_q[q(s_+) / z_+, K(s) / \lambda]} \right] \right\}.
\]

A parallel of this equation for the Foreign country characterizing equilibrium \( q^* \) is:

\[
g[q^*(s), K^*(s_+), z] = \frac{M^*(s_+)}{M^*(s)} E_\lambda \left\{ g[q^*(s_+), K^*(s), z_+] \left[ (1 - \sigma) + \sigma \frac{u_q[q^*(s_+) / \lambda]}{c_q[q^*(s_+) / z_+, K^*(s) / \lambda]} \right] \right\}.
\]

Second, since in equilibrium, \( X(a, s) = X(s) \) for all \( s \), along with (8) and (28), we then have an Euler equation for optimal Home bond holdings:

\[
Q(s_+ | s) := \left[ \int_{a_+} Q(a_+, s_+ | a, s) \mu_h(s_+, d a_+) \right] \lambda(s, ds_+)
\]

\[
= \beta \frac{U_X[X(s_+)]}{U_X[X(s)]} \frac{\phi(s_+)}{\phi(s)} \lambda(s, ds_+), \quad \forall s, s_+.
\]
Third, Foreign agents would also have a first order condition for bonds similar to (40), which, in Home currency terms is:

\[ Q(s_+ | s) := \left[ \int_{a^+} Q(a^+ , s_+ | a ; s) \mu_f (s_+ , d a^+) \right] \lambda(s, ds_+) \]

\[ = \beta \frac{U_X[X^*(s_+)] \phi^*(s_+) e(s)}{U_X[X^*(s)]} \left[ \frac{e(s_+)}{\phi^*(s_+)} \lambda(s, ds_+) \right], \quad \forall s, s_. \]  

(41)

From (6), (9) and knowing \( V_K \), we have an Euler equation for optimal Home capital holdings:

\[ U_X[X(s)] = \beta \mathbb{E}_h \left\{ U_X[X(s_+)] \left[ (1 + r(s_+)) - \sigma \frac{\gamma[q(s_+), K(s), z_+]}{U_X[X(s_+)]} \right] \right\}. \]  

(42)

The equation characterizing optimal holding of capital by Foreign agents is:

\[ U_X[X^*(s)] = \beta \mathbb{E}_\lambda \left\{ U_X[X^*(s_+)] \left[ (1 + r^*(s_+) - \delta) - \sigma \frac{\gamma^*[q^*(s_+), K^*(s), z_+]}{U_X[X^*(s_+)]} \right] \right\}. \]  

(43)

4.1. Inspecting the mechanism. Equating the pricing kernel from (40) and (41) and iterating, we have

\[ \frac{U_X[X(s)] \phi(s)}{U_X[X(s)] \phi(s_0)} = \frac{U_X[X^*(s)] \phi^*(s)}{U_X[X^*(s_0)] \phi^*(s_0)} \]  

(44)

where \( s_0 \) is the initial aggregate state. Assume that the initial condition, given by

\[ \kappa_0 := \frac{e(s_0) U_X[X(s_0)] \phi(s_0)}{U_X[X^*(s_0)] \phi^*(s_0)} \]

is fixed. We can re-write the above expression in (44) as the equilibrium determination of the nominal exchange rate:

\[ e(s) = \kappa_0 \frac{U_X[X^*(s)] \phi^*(s)}{U_X[X(s)] \phi(s)}. \]  

(45)

This warrants some remark. Up to this point, in terms of equilibrium complete state-contingent money claims, we have derived a standard complete markets result for the nominal exchange rate [see e.g. Chari, Kehoe, and McGrattan, 2002]. Specifically, what equation (45) says is that the nominal exchange rate, at each state of the world, is proportional to the within-period the relative value of the marginal rate of substitution of the general good between Home and Foreign consumers. Intuitively, and quite obviously, in the absence of frictions, we will obtain similar dynamics for the nominal exchange rate, \( x \), as in the standard international real business cycle model with money [see Schlenkenhauf and Wrase, 1995].

Note however, in equilibrium, the DM price-taking protocol implies that the utility value of an additional dollar must equal the utility value of the expenditure on purchasing a special good \( q \), which by anonymity, must be purchased with money:

\[ U_X[X(s)] \phi(s) M(s) = \frac{1}{z} c_q \left( \frac{q(s)}{z}, K(s_-) \right) q(s) \equiv g[q(s), K(s_-), z]. \]  

(46)
In terms of stationary variables – i.e. normalizing by $M(s_-)$ – and imposing log utility we have:

$$\frac{\hat{\phi}(s)}{X(s)} = \frac{1}{\exp\{\psi\}} c q\left(\frac{q(s)}{z}, K(s_-)\right) \frac{q(s)}{z} \equiv g[q(s), K(s_-), z],$$

where $\hat{\phi}(s) := \phi(s) M(s_-)$ and $M(s)/M(s_-) = \exp\{\psi\}$.

In contrast now, consider a version of our model where money would have been introduced via a cash-in-advanced (CIA) constraint, à la Cooley and Hansen [1989]. In a monetary equilibrium where the CIA constraint binds almost surely, we would have in terms of the equivalent stationary variables:

$$\frac{\hat{\phi}(s)}{X(s)} = \frac{1}{\exp\{\psi\}}.$$  

(48)

The interpretation in the CIA version is obviously quite different. In such an economy, agents are constrained to hold money to buy goods by assumption. What equation (48) then implies is that a positive increase in money supply (on the right) must be followed by a virtually one-for-one increase in the price level (or decrease in the value of a dollar, $\hat{\phi}$), if equilibrium consumption $X$ is smooth (or equivalently if agents are risk-averse). In short, the relative price of a unit of $X$ is extremely flexible in response to a monetary shock. If so, from an inspection of the nominal exchange rate determination condition in (45), we can immediately deduce that there would be very little volatility in the nominal exchange rate. Hence there would be very little connection between the nominal and the real exchange rates as well. These have been verified by the earlier work of Schlagenhauf and Wrase [1995] in the context of a two-country CIA monetary model.

However, consider our example with the equilibrium condition (47). It would be a mistake to think of this as a CIA constraint, as it is obviously not. It is part of the equilibrium characterization. However, with log utility, we have a direct comparison with a model possessing the CIA constraint (48). So now, in contrast, even in the presence of consumption smoothing, the DM equilibrium pricing condition (47) implies that an increase in money supply need not be followed by a one-for-one increase in the price level, or a decrease in the value of money. Since positive monetary injection will mean that current $q$ will increase, on the left side of the equilibrium money Euler equation (38), holding the conditional expectations on the right of (38) constant. As current $q$ increases immediately, this has an opposing effect to an increase in money supply. That is, on the one hand, an increase in money supply has a tendency to reduce the marginal utility value of holding a dollar (the left side of (47)), an increase in $q$ tends to increase the utility value of that dollar purchasing the special good $q$ (the right side of (47)). Depending on the nature of the pricing protocol and parameterization, and hence the shape of $g$, it may be that the value of a dollar $\hat{\phi}$ need not fall as much as the increase in money supply. In other words, it may be possible that the equilibrium pricing process to appear rather rigid or “sticky” as an equilibrium phenomenon, rather than by assumption.

Consider also a supply-side or technology shock, $z$. An increase $z$, has a tendency to raise the current marginal product of labor and hence labor demand in the CM. Equating (6) and (18), implying equilibrium labor market clearing in the CM, we can see that if consumption
increases but by not as much, then labor allocation would also increase. This would imply an increase in current CM investment into productive capital stock next period. Since \( c(q/z, K) \) is the dual cost function to a homogeneous of degree one production technology in the DM, we can deduce that an increase in \( z \) will lower the marginal cost of producing \( q \). This will lower the term on the right of the equilibrium monetary pricing condition (47). However, the technology shock also affect the left side of (47) via lowering the marginal product of labor, and hence raising the marginal utility of \( X \), \( U_X(X) \). Again, depending on the shape of \( g \), the value of a dollar, \( \phi \), need not respond so sensitively to a technology shock. Therefore, consistent with the nominal exchange rate determination condition (45), the nominal exchange rate ought to be quite volatile too.

Since the real exchange rate in our two-sector model is defined by (36), we would expect the real exchange rate to co-move with the nominal exchange rate. Finally, if these shocks are persistent, then we may expect that this is also propagated into persistent equilibrium relative pricing processes too.

5. Computational Exercise

As a first exercise, we consider a simpler pricing mechanism in the DM – Walrasian price taking. We can show analytically that in this case, the SME nests a unique non-stochastic steady state equilibrium.\(^9\) For our numerical experiments, we consider the following specific functions to represent the model primitives. In the CM, preferences and technology are described by

\[
U(X) = B \frac{X^{1-\gamma} - 1}{1-\gamma}, \quad zF(K, H) = zK^\alpha H^{1-\alpha},
\]

respectively, where \( B > 0, \gamma > 0, \) and \( \alpha \in (0,1) \). The symmetric description holds for the Foreign country. Note however, the notation for the final goods production function \( G \) is such that

\[
G(y, y_f) = \left[ \theta(y) + (1 - \theta)(y_f)^{1\epsilon} \right]^{1/\epsilon},
\]

for the Home country, and,

\[
G(y_f, y_h) = \left[ \theta(y_f) + (1 - \theta)(y_h)^{1\epsilon} \right]^{1/\epsilon},
\]

for the Foreign country, where \( \theta \in (0,1) \) and \(-\infty \leq 1/\epsilon \leq 1 \). The elasticity of substitution between the inputs to \( G \) is given by \( \sigma_e = \epsilon/(\epsilon - 1) \). These functional forms are quite standard in models with international trade in intermediate goods [see e.g. Heathcote and Perri, 2002; Chari, Kehoe, and McGrattan, 2002].

In the DM, preferences and technology are given respectively by

\[
u(q) = C \frac{(q + q)^{1-\eta} - b^{1-\eta}}{1-\eta}, \quad c(q, K) = q^\omega (K)^{1-\omega}
\]

where \( C = 1 \) without loss of generality, \( q \downarrow 0 \) (\( b = 0 \) if DM trade is determined by Walrasian price taking), \( \eta > 0 \) and \( \omega \geq 1 \).

---

\(^9\) Later, in the bargaining case, we have to rely on numerical methods to find the non-stochastic steady state solutions.
5.1. Baseline model calibration. Table 1 summarizes the baseline parameter values for the model. To discipline our numerical exercise, we calibrate the model to match long run stylized facts. First, we discuss parameters that can be easily estimated or fixed independently. Similar to Aruoba, Waller, and Wright [2008], we calibrate $\alpha$ to match the target of labor share in output, which is about 0.7 in the data [see also Aruoba, 2010]. We fix $\delta = 0.1$ as estimated in Heathcote and Perri [2002] for a two country model. Following Aruoba, Waller, and Wright [2008] and Aruoba [2010], we calibrate $\sigma$ to match the long-run money demand semi-elasticity with respect to the nominal interest rate, where money is defined by M1 for the U.S.. This elasticity is about $-0.23$. The risk aversion parameters $\eta$ and $\gamma$ imply that both $U$ and $u$ are natural log functions of $X$ and $q$, respectively. This restriction is required for the baseline model to have a balanced growth path, since the per-period utility function is linearly separable in consumption and leisure [see Waller, 2010]. The constant marginal taxes on capital, labor and CM-consumption, $(\tau_K, \tau_H, \tau_X) = (0.548, 0.242, 0.069)$, are chosen as in Aruoba, Waller, and Wright [2008]. The estimate of $\vartheta$ is from Backus, Kehoe, and Kydland [1994].

Second, we calibrate simultaneously the remaining parameters $(A, B, \varpi)$ to match the targets of proportion of hours worked, $H$, a measure of non-traded consumption goods share in total consumption, $NTS$, and the long run capital output ratio, $K/Y$. The value of $H$ is roughly 0.33, which is standard. This value can be thought of as pinning down the marginal utility of labour parameter $A$. $B$ is calibrated, in this model, to match a DM consumption (interpreted as a nontradable good in this model) share of total consumption to be close to 0.50 for the U.S., a share estimated by Stockman and Tesar [1995]. This is in contrast to the closed-economy models in Aruoba, Waller, and Wright [2008] and Aruoba [2010], where intuitively, $B$ is calibrated to match the velocity of money. The target capital-output ratio, $K/Y$, is 2.23 in annual terms. Given other parameters, this ratio can be thought of as pinning down the calibration for $\varpi$ from the Euler equation characterizing equilibrium capital accumulation along the steady state path. The calibrated value of $\omega > 1$, implies that the more capital is installed for use in the DM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Fixed</td>
</tr>
<tr>
<td>$\eta = \gamma$</td>
<td>1</td>
<td>Fixed</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>$I/K$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1/3</td>
<td>Total capital income share, 1/3</td>
</tr>
<tr>
<td>$A$</td>
<td>0.4858</td>
<td>Total labor hours fraction, 1/3</td>
</tr>
<tr>
<td>$\varpi$</td>
<td>1.2766</td>
<td>$K/Y = 8.92$ per quarter (2.23 per annum)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.13</td>
<td>Real money demand interest elasticity, $-0.23$ (AWW)</td>
</tr>
<tr>
<td>$B$</td>
<td>0.1686</td>
<td>Non-traded good consumption share, 0.50</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>0.9397</td>
<td>Share of imports in net exports (BKK)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>3</td>
<td>Estimated: Symmetry; imports share of GDP 1.6% (CKM)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.15</td>
<td>Estimated, AWW</td>
</tr>
<tr>
<td>$\tau_K$</td>
<td>0.548</td>
<td>Estimated, AWW</td>
</tr>
<tr>
<td>$\tau_H$</td>
<td>0.242</td>
<td>Estimated, AWW</td>
</tr>
<tr>
<td>$\tau_X$</td>
<td>0.069</td>
<td>Estimated, AWW</td>
</tr>
</tbody>
</table>

Notes:
(a) Aruoba, Waller, and Wright [2008]: (AWW)
(b) Backus, Kehoe, and Kydland [1994]: (BKK)
(c) Chari, Kehoe, and McGrattan [2002]: (CKM)
production, the lower the cost of producing a unit of DM output $q$. By duality, this implies that capital is a complementary input to labor effort in DM production.

In the baseline model, we assume that all the TFP levels (and their shocks), in both CM and DM, are uncorrelated with each other [see also Chari, Kehoe, and McGrattan, 2002]. In parameterizing the exogenous TFP autocorrelation parameters $(\rho_Z, \rho_{Z^*})$ we borrow values from Chari, Kehoe, and McGrattan [2002]. The money supply growth stochastic processes are the estimates from Schlagenhauf and Wrase [1995].

6. INTERNATIONAL BUSINESS CYCLE FEATURES

In this section, we discuss the business cycle dynamics of the calibrated baseline model. We report the quantitative predictions of our benchmark model relative to a business cycle model with sticky prices [Chari, Kehoe, and McGrattan, 2002, labelled CKM in the tables], and a real business cycle model of Heathcote and Perri [2002] (HP in the tables).

As we can see from Table 2, the benchmark model can account for the volatilities of the key business cycle data for the U.S. quite well. In particular, the model can account for up to 81% of the consumption volatility, 80% of the volatility in domestic investment, and about 90% of labor volatility. The model over-predicts the nominal exchange rate volatility.

Overall, in terms of the nominal and real exchange rate volatilities, the model is able to reproduce qualitatively the observation that both exchange rates are much more volatile than U.S. GDP. As opposed to Chari, Kehoe, and McGrattan [2002] and Heathcote and Perri [2002], our benchmark model does not rely on large relative risk aversion parameters, sticky prices nor imperfections in international risk sharing to generate volatility.\(^{10}\) Furthermore, in contrast, standard flexible price two-country CIA models [see Schlagenhauf and Wrase, 1995] are unable to reproduce any realistic volatilities in the real and nominal exchange rates.

Next, consider the first order autocorrelation coefficients of the equilibrium processes in Table 3. In terms of consumption in the traditional Walrasian (or RBC) sector of the model, investment, labor allocation, and GDP, the model matches the empirical persistence in the data very well. However, in terms of the real and nominal exchange rates, the model under predicts the persistence observed in the data. However, the baseline model is able to do just as well as the

---

\(^{10}\)On the other hand, the competitive equilibrium in our model features incomplete markets as a result of idiosyncratic shocks to agent types each period as they enter the DM. Since there is a link between the DM and CM outcomes via capital, not all consumption risk can be fully insured.
TABLE 3. First-order autocorrelations

<table>
<thead>
<tr>
<th>Data</th>
<th>PT (%)</th>
<th>PT (% data)</th>
<th>CKM (% data)*</th>
<th>HP (% data)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal E.R., e</td>
<td>0.83</td>
<td>0.66</td>
<td>83 [53, 80]</td>
<td>n.a.</td>
</tr>
<tr>
<td>Real E.R., RER</td>
<td>0.84</td>
<td>0.66</td>
<td>79 [70, 80]</td>
<td>n.a.</td>
</tr>
<tr>
<td>Consumption, C</td>
<td>0.87</td>
<td>0.97</td>
<td>78 [54, 68]</td>
<td>n.a.</td>
</tr>
<tr>
<td>Investment, I</td>
<td>0.90</td>
<td>0.84</td>
<td>111 [52, 66]</td>
<td>n.a.</td>
</tr>
<tr>
<td>Hours, H</td>
<td>0.94</td>
<td>0.92</td>
<td>98 [53, 76]</td>
<td>n.a.</td>
</tr>
<tr>
<td>Output, Y</td>
<td>0.89</td>
<td>0.90</td>
<td>101 [56, 80]</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

Notes:
(a) [*] Percentage of respective data set’s statistics accounted for by respective models.
(c) Heathcote and Perri [2002] (HP) did not report these statistics.

TABLE 4. Contemporaneous correlations

<table>
<thead>
<tr>
<th>Data</th>
<th>PT (%)</th>
<th>PT (% data)</th>
<th>CKM (% data)*</th>
<th>HP (% data)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>(RER, e)</td>
<td>0.99</td>
<td>0.99</td>
<td>100 [75,88]</td>
<td>n.a.</td>
</tr>
<tr>
<td>(RER, NX)</td>
<td>0.14</td>
<td>0.19</td>
<td>136 [534,628]</td>
<td>n.a.</td>
</tr>
<tr>
<td>(RER, C/C*)</td>
<td>0.23</td>
<td>0.12</td>
<td>52 -286</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

* Percentage of respective data set’s statistics accounted for by respective models.

models of Chari, Kehoe, and McGrattan [2002], without requiring any exogenous sticky-price assumption.

In terms of the other correlations in the data, Table 4 shows that the model is able to generate realistic cross-country output, and consumption correlations, that are weakly positive. Moreover, the model is able to generate a real-nominal exchange rate correlation that is very close to the data. The model is able to explain just under 50% of the mild positive correlation between the real exchange rate and net exports in the data. To see why, we consider the partial explanations given in Figures 2 and 3. Figures 2 depicts the impulse response of the components of the real-exchange-rate definition in the model, \( RER := eP_Y/P^*_Y \) to a 1% total factor productivity shock in the home country. Figure 3 considers that of a 1% home money supply growth shock. The resulting dynamics of the relative cross-country aggregate price deflators are such that they are not so sensitive to technology shocks. By definition then, the dynamics of the real exchange rate must be tracking that of the nominal exchange very well. Hence the near perfect correlation between the two. In standard sticky-price models [see e.g. Chari, Kehoe, and McGrattan, 2002], the assumption of price stickiness plays a similar, but more obvious, role. However, in our model, this appears to be an equilibrium outcome arising from monetary friction and its resulting restriction of asset and relative pricing dynamics. These figures thus confirm our conjecture in Section 4.

What stands out is that the model breaks away from the Backus and Smith [1993] puzzle—that relative consumption across countries is counterfactually perfectly correlated with the real exchange rate. This is not surprising since now, since our DM sector where the special good is assumed to be produced and traded locally, behaves exactly a nontraded goods sector. This essentially breaks the perfect link between the real exchange rate and the real terms of trade induced by the marginal rate of subsitution of consumption across countries in the traded goods sector (even with complete international asset markets).
NOW WE WILL EXPLAIN THE MECHANISM IN THIS MODEL THAT CONTRIBUTES TO THE BUSINESS CYCLE PROPERTIES OF THE BENCHMARK MODEL REPORTED IN TABLES 2-4 AND DISCUSSED IN SECTION 6. RECALL THAT IN SECTION 4, WE PROVIDED THE EXPLANATION OF THE POTENTIAL EFFECTS OF THE ASSUMPTION OF ANONYMITY (AND ITS RESULTING MONETARY EQUILIBRIUM DETERMINATION) ON RELATIVE PRICING PROCESSES AND THEREFORE EQUILIBRIUM EXCHANGE RATES. IN THIS SECTION, WE REVISIT OUR EXPLANATIONS, BY CONDUCTING A FEW VERIFICATION EXPERIMENTS.

SPECIFICALLY, SINCE WE SHOWED THAT THE KEY IS IN THE MONETARY PRICING PROTOCOL, I.E. THE SHAPE OF THE $g$ FUNCTION, IN AFFECTING RELATIVE PRICING AND EXCHANGE RATE DYNAMICS, ONE WAY TO TEST THIS CLAIM IS TO TAKE THE MODEL TO ITS LOGICAL LIMITS. TABLE 5 SUMMARIZES THESE LIMIT CASES OF OUR ECONOMY, WHICH ARE: (I) FB: NO ANONYMITY (OR EQUIVALENTLY A FIRST BEST EQUILIBRIUM); (II) FB ($\omega = 1$): FB WITHOUT CAPITAL SERVICE IN DM PRODUCTION; (III) ME-PT: THE BASELINE MONETARY EQUILIBRIUM WITH DM PRICE-TAKING ASSUMPTION; AND (IV) ME-PT ($\omega = 1$): ME-PT WITHOUT DM CAPITAL SERVICE.

<table>
<thead>
<tr>
<th>Capital utilization in DM</th>
<th>$\omega &gt; 1$</th>
<th>$\omega = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-monetary equilibria $\kappa = 0$</td>
<td>FB</td>
<td>FB ($\omega = 1$)</td>
</tr>
<tr>
<td>Monetary equilibria, $\kappa &gt; 0$</td>
<td>ME-PT</td>
<td>ME-PT ($\omega = 1$)</td>
</tr>
</tbody>
</table>

Consider the limit economy FB with pure credit trades ($\kappa = 0$) in the DM, equivalent to a first-best allocation relative to our baseline calibrated ME-PT model. This acts to shut down completely the role of anonymity and hence monetary friction. This limit economy, FB, also highlights a remainder structure: a (equivalently separable-utility) version of standard two-sector real-business-cycle model with traded and nontraded goods. However, as the column

<table>
<thead>
<tr>
<th>Economy</th>
<th>ME-PT$^*$</th>
<th>ME-PT ($\omega = 1$)</th>
<th>FB</th>
<th>FB ($\omega = 1$)</th>
<th>ME-PB</th>
<th>ME-NB</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal E.R., $e$</strong></td>
<td>6.01</td>
<td>0.86</td>
<td>n.a.</td>
<td>n.a.</td>
<td>1.47</td>
<td>0.52</td>
</tr>
<tr>
<td><strong>Real E.R., $RER$</strong></td>
<td>2.17</td>
<td>3.67</td>
<td>0.73</td>
<td>0.95</td>
<td>1.96</td>
<td>0.24</td>
</tr>
<tr>
<td><strong>Consumption, $C$</strong></td>
<td>0.58</td>
<td>1.21</td>
<td>0.61</td>
<td>0.76</td>
<td>1.03</td>
<td>0.19</td>
</tr>
<tr>
<td><strong>Investment, $I$</strong></td>
<td>2.04</td>
<td>0.01</td>
<td>2.68</td>
<td>4.28</td>
<td>0.48</td>
<td>0.14</td>
</tr>
<tr>
<td><strong>Hours, $H$</strong></td>
<td>0.59</td>
<td>0.25</td>
<td>0.74</td>
<td>0.56</td>
<td>0.07</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Standard deviation:**

| Nominal E.R., $e$ | 0.66 | 0.64 | n.a. | n.a. | 0.67 | 0.67 |
| Real E.R., $RER$ | 0.66 | 0.65 | 0.99 | 0.99 | 0.69 | 0.78 |
| Consumption, $C$ | 0.97 | 0.90 | 0.99 | 0.99 | 0.62 | 0.72 |
| Investment, $I$ | 0.84 | 0.29 | 0.91 | 0.90 | 0.72 | 0.79 |
| Hours, $H$ | 0.92 | 0.87 | 0.90 | 0.89 | 0.87 | 0.66 |
| Output, $Y$ | 0.90 | 0.60 | 0.95 | 0.96 | 0.63 | 0.75 |

**Autocorrelation:**

| Nominal E.R., $e$ | 0.66 | 0.64 | n.a. | n.a. | 0.67 | 0.67 |
| Real E.R., $RER$ | 0.66 | 0.65 | 0.99 | 0.99 | 0.69 | 0.78 |
| Consumption, $C$ | 0.97 | 0.90 | 0.99 | 0.99 | 0.62 | 0.72 |
| Investment, $I$ | 0.84 | 0.29 | 0.91 | 0.90 | 0.72 | 0.79 |
| Hours, $H$ | 0.92 | 0.87 | 0.90 | 0.89 | 0.87 | 0.66 |
| Output, $Y$ | 0.90 | 0.60 | 0.95 | 0.96 | 0.63 | 0.75 |
FB versus column ME-PT in Table 6 show, these alone cannot account for the RER stylized fact: That the RER is more volatile than U.S. output.

Note that columns FB and ME-PT of Table 6 represent economies with capital linking both the DM (nontraded good sector) and the CM (traded good sector). We would also like to see what additional contribution the assumption of capital utilization in the DM (nontraded good sector) plays in generating the excess-volatility stylized fact of the RER in the models. This exercise is shown in Columns ME-PT (\(\varpi = 1\)) and FB (\(\varpi = 1\)) of Table 6. These are the respective economies represented in columns FB and ME-PT with capital no being a factor of production for sellers in the DM (nontraded good sector).

Again, the same pattern arises, without monetary frictions, and hence a monetary equilibrium, the limit economy FB (\(\varpi = 1\)) cannot account for the stylized fact of excess-volatility in the real exchange rate. In contrast, however, the monetary frictions result in lower persistence of the RER relative to the limit FB economies.

Thus we have verified that, relative to our baseline calibration of the ME-PT model, the key informational friction of anonymity is not only a means of introducing money into models after Lagos and Wright [2005], but they also matter for stochastic equilibrium relative pricing dynamics. In our case of a most parsimonious price-taking protocol, our \(g\) function indeed is able to produce what we conjectured from analyzing the model’s SME conditions in Section 4. We also consider, in the last two columns labeled ME-PB (monetary model with DM proportional bargaining) and ME-NB (with DM Nash bargaining) in Table 6, as alternative pricing protocols in the monetary DM. Thus it matters what determines the \(g\) function, i.e. the assumed DM protocol for determining the terms of trade in stochastic bilateral matches.

8. CONCLUSION

In this paper, we examine whether a flexible price, two-country, search theoretic model of money is able to account for the empirical regularities observed in U.S. real and nominal exchange rate dynamics. We propose a two-country version of Aruoba, Waller, and Wright [2008] where international trade and asset flows occur in the model’s Walrasian centralized markets. Moreover, we allow for “capital complementarity” across tradable and non-tradable sectors. This feature generates an equilibrium linkage between inflation and real economic activity.

There are two key mechanisms at work in this model that help amplify and propagate international business cycle shocks. The first mechanism is monetary friction (i.e. “anonymity”). This friction induces asset market incompleteness in the sense that individuals are unable to fully insure against their preference shocks in the DM. The second mechanism is the notion of capital complementarity. The latter mechanism provides for an additional return on capital which places additional restriction on the equilibrium asset pricing relations with respect to money and capital.

We show that the relative pricing dynamics of the baseline model behave in such a way that aggregate relative prices are relatively non-volatile and persistent. This contributes to the excess volatility and persistence in the real and nominal exchange rate. Thus without requiring exogenous price-stickiness, we are also able to rationalize near perfect positive correlation between the real and nominal exchange rate. Thus monetary friction, in the sense of Lagos and
Wright [2005], is more than just a vehicle for a theoretical foundation of money. In a stochastic two-country environment, it restricts asset pricing relations such that the model is able to account for the stylized facts on real and nominal exchange rate fluctuations.

**APPENDIX A. SME CHARACTERIZATION**

Consider a simplification of the model with $\kappa = 1$ and $\tau_K = \tau_X = \tau_H = 0$. Since the processes $(\psi)$ and $(\psi^*)$ are bounded below by zero, this implies that nominal variables, namely $M, M^*, \phi$ and $\phi^*$ will grow unboundedly. We can perform a change of variables in the equilibrium conditions for nominal variables as follows. We normalize Home and Foreign nominal variables by $M(s_-)$ and $M^*(s_-)$, respectively, such that

$$\hat{i}(s) := \frac{i(s)}{M(s_-)}$$
$$\hat{i}^*(s) := \frac{i^*(s)}{M^*(s_-)}$$
$$\hat{\Phi}(s) := \frac{\phi(s)M(s_-)}{M(s_-)}$$
$$\hat{\Phi}^*(s) := \frac{\phi^*(s)M^*(s_-)}{M(s_-)}$$
$$\hat{P}_h(s) = P_h(s)/M(s_-)$$
$$\hat{P}_f(s) = P_f(s)/M(s_-).$$

Then our SME conditions can be equivalently written as follows. The Home and Foreign money supply injection are respectively given as:

$$i(s) = \exp(\psi) - 1$$
$$i^*(s) = \exp(\psi^*) - 1$$

Labor market clearing in the CM in Home and Foreign, respectively, are

$$U_X[X(s)] = A\hat{\Phi}(s)\hat{P}_h(s)z_F[H(s)]$$
$$U_X[X^*(s)] = A\hat{\Phi}^*(s)\hat{P}_f(s)z_F[H^*(s)]$$

The Home resource constraint in equilibrium is given by

$$G(y_h(s), y_f(s)) = X(s) + K(s) - (1 - \delta)K(s_-).$$

The Foreign resource constraint is given by

$$G(y^*_f(s), y^*_h(s)) = X^*(s) + K^*(s) - (1 - \delta)K^*(s_-).$$

Complete international risk sharing entails

$$\frac{\hat{\epsilon}(s)\hat{\Phi}(s)}{\hat{\Phi}^*(s)} = \kappa_0 \frac{U_X[X^*(s)]}{U_X[X(s)]}.$$

where $\kappa_0 = 1$, implying a symmetric initial steady state, without loss of generality.

Aggregate general-good price levels in Home and Foreign, respectively, are pinned down by

$$\frac{A\hat{\Phi}(s)}{w^*(s)} \exp\{\psi\} = g[q(s), K(s_-), z],$$
The equilibrium Euler equations for Home are:

\[ g[q^*(s), K^*(s), z^*] = \beta \mathbb{E}_h \left\{ g[q(s_+), K(s), z_+] \exp\{ -\psi \} \right\} \times \left( 1 - \sigma + \sigma \frac{u_q[q(s_+) - q^*(s_+), K(s), z_+]}{g[q(s_+), K(s), z_+]} \right) \]  

\[ U_X[X(s)] = \beta \mathbb{E}_h \left\{ U_X[X^*(s)] \left[ (1 + r(s_+) - \delta) - \sigma \gamma \frac{[q(s_+), K(s), z_+]}{U_X[X^*(s_+) - q^*(s_+) \cdot K^*(s), z^*] \right] \right\} \].

The equilibrium Euler equations for Foreign are:

\[ g[q^*(s), K^*(s), z^*] = \beta \mathbb{E}_h \left\{ g[q(s_+), K(s), z_+] \exp\{ -\psi \} \right\} \times \left( 1 - \sigma + \sigma \frac{u_q[q(s_+) - q^*(s_+), K(s), z_+]}{g[q(s_+), K(s), z_+]} \right) \]  

\[ U_X[X^*(s)] = \beta \mathbb{E}_h \left\{ U_X[X^*(s)] \left[ (1 + r^*(s_+) - \delta) - \sigma \gamma \frac{[q(s_+), K(s), z_+]}{U_X[X^*(s_+) - q^*(s_+) \cdot K^*(s), z^*] \right] \right\} \].

Note that capital and labor rental pricing functions are given by:

\[ r(s) = \hat{\phi}(s) \hat{P}_h(s) \cdot z F_k[K(s_+), H(s)], \]  

for Home, and

\[ w(s) = \hat{\phi}(s) \hat{P}_h(s) \cdot z F_H[K(s_+), H(s)], \]  

\[ r^*(s) = \frac{\hat{\phi}^*(s) \hat{P}_f(s)}{e(s)} \cdot z^* F_k[K^*(s_+), H^*(s)], \]  

and

\[ w^*(s) = \frac{\hat{\phi}^*(s) \hat{P}_f(s)}{e(s)} \cdot z^* F_H[K^*(s_+), H^*(s)], \]  

for Foreign, where we have made use of the law of one price for intermediate goods.

Intermediate goods trade and market clearing are given by:

\[ \hat{\phi}(s) \hat{P}_h(s) = G_{y_h}[y_h(s), y_f(s)], \]  

and

\[ \hat{\phi}(s) \hat{P}_f(s) = G_{y_f}[y_h(s), y_f(s)]. \]
for Home, and
\[
\frac{\hat{\phi}^*(s)\hat{P}_f(s)}{e(s)} = G_{y_f}[y_f(s), y_h^*(s)],
\]
and
\[
\frac{\hat{\phi}^*(s)\hat{P}_h(s)}{e(s)} = G_{y_h^*}[y_f(s), y_h^*(s)].
\]

for Foreign, where we have again made use of the law of one price for intermediate goods.

Market clearing for intermediate goods are:
\[
z_F[K(s-), H(s)] = y_h(s) + y_h^*(s) \quad(68)
\]
\[
z^*F[K^*(s-), H^*(s)] = y_f(s) + y_f^*(s). \quad(69)
\]

**Definition 2.** A stationary Markov monetary equilibrium (with decentralized bargaining) is given by time-invariant functions of \(s\), i.e.

1. **Consumption functions** \((X, X^*, H, H^*, q, q^*, y_h, y_f, y_f^*, y_h^*)\),
2. **Savings functions** \((K, K^*)\), and,
3. **Pricing functions** \((w, w^*, r, r^*, \hat{e}, \hat{\phi}, \hat{\phi}^*, \hat{P}_h, \hat{P}_y)\),

that induce bounded stochastic processes satisfying the recursions (49)-(69), for given policies \((\hat{\iota}(s), \hat{\iota}^*(s))\).

**APPENDIX B. FIRST-BEST PROBLEM**

In this appendix, we outline the first-best equilibrium solution. This is equivalent to the limit of our monetary economy where there is no monetary friction (i.e anonymity), or \(\kappa = 0\). Hence the allocation will be equivalent to a version of a real business cycle model with a traded (CM) and a non-traded (DM) goods sector, labeled as FB in the paper. Variables are defined in Table 7.

**Table 7. Variable definition**

<table>
<thead>
<tr>
<th>Mnemonic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K)</td>
<td>Home capital stock</td>
</tr>
<tr>
<td>(K^*)</td>
<td>Foreign capital stock</td>
</tr>
<tr>
<td>(z)</td>
<td>Home TFP state</td>
</tr>
<tr>
<td>(z^*)</td>
<td>Foreign TFP state</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Home money supply growth</td>
</tr>
<tr>
<td>(\mu^*)</td>
<td>Foreign money supply growth</td>
</tr>
<tr>
<td>(X)</td>
<td>Home CM good</td>
</tr>
<tr>
<td>(X^*)</td>
<td>Foreign CM good</td>
</tr>
<tr>
<td>(q)</td>
<td>Home DM good</td>
</tr>
<tr>
<td>(q^*)</td>
<td>Foreign DM good</td>
</tr>
<tr>
<td>(a)</td>
<td>Home produced intermediate good, Home use</td>
</tr>
<tr>
<td>(a^*)</td>
<td>Home produced intermediate good, Foreign use</td>
</tr>
<tr>
<td>(b)</td>
<td>Foreign produced intermediate good, Home use</td>
</tr>
<tr>
<td>(b^*)</td>
<td>Foreign produced intermediate good, Foreign use</td>
</tr>
</tbody>
</table>

Variety \(a + a^*\) (used by Home and Foreign) is produced by the Home country’s technology \(F\), and vice-versa for variety \(b + b^*\). \(c : \mathbb{R}^2_+ \rightarrow \mathbb{R}_+\) is the cost function describing the technology.
of producing $q$. Let $G$ be the technology that aggregates the inputs $(a, b)$ for the Home country and $(b^*, a^*)$ for the Foreign country into a final general good of the same characteristic as $X$ and $X^*$ respectively.

B.1. The planner’s problem. Define an allocation function by

$$a := (X, X^*, H, H^*, q, q^*, K, K^*, a, a^*, b, b^*).$$

Denote $s_t$ as the vector of relevant state variables. Here, we have $s_t := (K, K^*, z, z^*)$. Let $s_t \mapsto J(s_t)$ be the planner’s value function. A Pareto allocation $\{a(s_t)\}_{t \in \mathbb{N}}$ in this economy is generated by an $a$ satisfying the following Bellman equation:

$$J(s) = \max_a \left\{ U(X) - AH + U(X^*) - AH^* + \sigma[u(q) - c(q/z, K)] + \sigma[u(q^*) - c(q^*/z^*, K^*)] + \beta E[J(s')|s] \right\}$$

subject to

$$K' = G(a, b) + (1 - \delta)K - X,$$  \hspace{1cm} (71)

$$K'^* = G(b^*, a^*) + (1 - \delta)K^* - X^*,$$  \hspace{1cm} (72)

$$a + a^* = zF(K, H),$$  \hspace{1cm} (73)

$$b + b^* = z^*F(K^*, H^*).$$  \hspace{1cm} (74)

Let $(\lambda, \lambda^*, v, v^*)$ be the state-by-state Lagrange multipliers on the respective constraints above. The first-order conditions for the RHS problem in the Bellman equation are:

$$X : \quad U_X(X) = \lambda,$$  \hspace{1cm} \text{and}  \hspace{1cm} \quad X^* : \quad U_X^*(X^*) = \lambda^*,$$

$$K' : \quad -\lambda + \beta E[J_{K'}(s')|s] = 0,$$  \hspace{1cm} \text{and}  \hspace{1cm} \quad K'^* : \quad -\lambda^* + \beta E[J_{K'^*}(s')|s] = 0,$$

$$H : \quad -A + vF_H(K, H) = 0,$$  \hspace{1cm} \text{and}  \hspace{1cm} \quad H^* : \quad -A + v^*z^*F_H(K^*, H^*) = 0,$$

$$q : \quad \sigma[u_q(q) - c_q(q/z, K)/z] = 0,$$  \hspace{1cm} \text{and}  \hspace{1cm} \quad q^* : \quad \sigma[u_q^*(q^*) - c_q^*(q^*/z^*, K^*)/z^*] = 0,$$

$$a : \quad \lambda G_a(a, b) - v = 0,$$  \hspace{1cm} \text{and}  \hspace{1cm} \quad a^* : \quad \lambda^* G_a^*(b^*, a^*) - v = 0,$$

$$b : \quad \lambda G_b(a, b) - v^* = 0,$$  \hspace{1cm} \text{and}  \hspace{1cm} \quad b^* : \quad \lambda^* G_b^*(b^*, a^*) - v^* = 0,$$

and feasibility conditions are given in (71)-(74).

Under regularity assumptions $J$ is continuously differentiable.\textsuperscript{11} Then the envelope conditions, with respect to $K$ and $K^*$, at an interior maximum are

$$J_K(s) = -\sigma c_K(q/z, K) + \lambda (1 - \delta) + v[zF_K(K, H)],$$

and

$$J_{K^*}(s) = -\sigma c_{K^*}(q^*/z^*, K^*) + \lambda^*(1 - \delta) + v^*[z^*F_{K^*}(K^*, H^*)].$$

\textsuperscript{11}Given (i) the state space is a convex and compact Borel subset of $\mathbb{R}^4$; (ii) and appropriate assumptions of the stochastic processes on $(z, z^*)$ – i.e. the transition probability functions have the Feller property; (iii) continuous differentiability of the per-period payoff on the state space; and (iv) given assumptions that $F$ and $G$ are continuous, and define convex production sets, then $J(\cdot, z, z^*)$ is continuously differentiable in $(K, K^*)$ at some $(K_0, K_0^*)$ in the interior of the state space.
From these optimality conditions, we have the characterization of a Pareto allocation \( \{\alpha(s_t)\}_{t \in \mathbb{N}} \). More precisely, after some straightforward substitution, we have the following definition.

**Definition 3.** A **Pareto allocation** \( \{\alpha(s_t)\}_{t \in \mathbb{N}} \) is given by a list of allocation functions

\[
\alpha := (X, X^*, H, H^*, q, q^*, K, K^*, a, a^*, b, b^*)
\]

satisfying the following conditions:

\[
\begin{align*}
\lambda &= U_X(X) \\
\lambda^* &= U_X(X^*) \\
\lambda &= \beta \mathbb{E} \left\{ \lambda' \left[ G_a(a', b') z' F_K(K', H') + 1 - \delta \right] - \sigma c_K(q'/z', K') \middle| s \right\} \\
\lambda^* &= \beta \mathbb{E} \left\{ \lambda'^* \left[ G_{b^*}(b^*, a^*) z'^* F_K(K'^*, H'^*) + 1 - \delta \right] - \sigma c_{K^*}(q'^*/z'^*, K'^*) \middle| s \right\} \\
A &= z F_H(H, K) G_a(a, b) \\
A &= z^* F_{H^*}(K^*, H^*) G_{b^*}(b^*, a^*) \\
u_q(q) &= c_q(q/z, K)/z \\
u_{q^*}(q^*) &= c_q^*(q^*/z^*, K^*)/z^* \\
G_a(a, b) \lambda &= G_{a^*}(b^*, a^*) \lambda^* \\
G_{b^*}(b^*, a^*) \lambda^* &= G_b(b^*, a^*) \lambda^* \\
z F(K, H) &= a + a^* \\
z^* F(K^*, H^*) &= b + b^* \\
G(a, b) &= X + K' - (1 - \delta) K \\
G(b^*, a^*) &= X^* + K'^* - (1 - \delta) K^*.
\end{align*}
\]

**Remark 1.** The planner allocates \( q \) and \( q^* \) efficiently. That is for all states \( s_t \) and dates \( t \in \mathbb{N} \), the marginal utility of a buyer consuming \( q \) in the Home country is equal to a seller's marginal cost of producing it, \( u_q(q) = c_q(q/z, K)/z \). Likewise for \( q^* \). This coincides with the outcome of a barter economy if there were no double coincidence of wants problem [see also Lagos and Wright, 2005; Aruoba, Waller, and Wright, 2008].

**Remark 2.** The terms of trade and international relative price for tradable intermediate goods is given by:

\[
\frac{\nu^*}{\nu} = \frac{G_{b^*}(a, b)}{G_a(a, b)} = \frac{G_{b^*}(b^*, a^*)}{G_{a^*}(b^*, a^*)}.
\]

**Remark 3.** Let \( X^* \) be the numeraire good. Denote the non-traded special good \( q \) share of total consumption as:

\[
\chi := \frac{\sigma q}{\sigma q + X^*}
\]
for the Home country, and

\[ \chi^* := \frac{\sigma q^*}{\sigma q^* + X^*}, \]

for the foreign. Denote \( p_X := U_X(X^*)/U_X(X) \) as the general good real terms of trade. Note that since \( X^* \) is numeraire, then \( p_X^* := 1 \). The relative prices between special and general goods are then

\[ p_q := U_X(X^*)/u_q(q), \]

and

\[ p_q^* := U_X(X^*)/u_q(q^*), \]

respectively, for the Home and Foreign, special goods. Then the real exchange rate is defined as

\[ \text{RER} := \chi^* p_q^* + (1 - \chi^*) \cdot 1_{\chi^* p_q^* + (1 - \chi) p_X}. \] (75)

**Appendix C. Bargaining**

Our modeling strategy proceeds from the baseline model with decentralized market (DM) price taking, to two alternative bargaining protocols (which have increasing sources of frictions) for determining the terms of trade in the DM. The former baseline environment has the minimal number of frictions introduced into the DM trading environment (i.e. degree of anonymity, \( \kappa \)). The latter two alternatives – proportional bargaining (PB) and generalized Nash bargaining (GNB) – are respectively, increasing in their sources of frictions. In the following two alternatives, there will be one additional key parameter, \( \theta \in [0, 1] \), which indexes the bargaining power of the buyer. The PB environment generally introduces a holdup problem in terms of capital whenever \( \theta > 0 \), and, the GNB setup introduces both money (when inflation in some states of nature is away from the Friedman rule) and capital holdup frictions, whenever \( 0 < \theta < 1 \). In this appendix, we outline these two alternatives to the baseline model.

In section C.1 we consider closing the DM pricing and allocation mechanism using the proportional bargaining solution due to Kalai [1977]. In section C.2 we consider the generalized Nash bargaining solution used originally by Lagos and Wright [2005]. Finally in section D, we detail the nonstochastic steady state conditions in the baseline model, and also how we calibrate a subset of the baseline model’s parameters that are not estimated elsewhere. In this section we also show where departures and additions occurs in the case of the PB and GNB alternative models.

---

12As discussed in Aruoba, Waller, and Wright [2008], if we set \( \theta = 1 \), the buyer takes all the surplus in a GNB outcome, and this resolves the money holdup inefficiency on the buyer’s part, but creates the extreme holdup problem in terms of capital for the seller who ends up having the marginal benefit of more capital for production in the DM exactly offset by the marginal cost of increased production. If we set \( \theta = 0 \), the capital holdup problem disappears as ex-post sellers can expropriate all the GNB surplus. However, in this case the buyer’s money holdup problem is extreme. Thus there is no \( \theta \) in the GNB case which can eliminate all holdup frictions.
C.1. **Proportional bargaining.** We first discuss the example of proportional bargaining.\(^{13}\) We begin by fixing some notation to ease the exposition. Recall that \((m, b, k, l, s) \mapsto W(m, b, k, l, s)\) is the value function for an agent \((m, b, k, l, s)\) who begins the subperiod centralized market (CM). We shall now denote \(W(m, \cdot) := W(m, b, k, 0, s)\) for revelant bilateral matches where money, \(m\), will the medium of exchange; and \(W(l, \cdot) := W(m, b, k, l, s)\) where credit, \(l\), is used instead.\(^{14}\) Let \((m_i, d_i, l_i)\) denote the nominal money holdings, monetary payment, and nominal loan, respectively, held by an agent \(i \in \{b, s\}\), where \(b\) indicates a buyer and \(s\) indicates a seller. Naturally, \(l_i \leq 0\).

C.1.1. **DM monetary exchange.** Let each buyer in a bilateral anonymous monetary exchange have bargaining strength \(\theta \in [0, 1]\). Denote the solution \((q, d)\) by the decision rules \(q := q(m_b, k_s, s)\) and \(d := d(m_b, k_s, s)\). The proportional bargaining solution in a bilateral match is given by

\[
(q, d) \in \arg \max_{q \in R_s, d \in [0, m_b]} \left\{ u(q) + W(m_b - d, \cdot) - T_b \right\}
\]

\[
= \frac{\theta}{1 - \theta} \left[ -c(q/z, k_s) + W(m_s + d) - T_s \right],
\]

where \(T_b := W(m_b - 0, \cdot)\) and \(T_s := W(m_b + 0, \cdot)\). Since we had shown that \(W\) is linear in the individual states, and more specifically, here we have

\[
W(m_b - d, \cdot) = \frac{A\phi}{w} [m_b - d + b_b] + (1 + r)k_b,
\]

and,

\[
W(m_s + d, \cdot) = \frac{A\phi}{w} [m_s + d + b_s] + (1 + r)k_s,
\]

then the bargaining problem simplifies to

\[
(q, d) \in \arg \max_{q \in R_s, d \in [0, m_b]} \left\{ u(q) - \frac{A\phi}{w}d \right\}
\]

\[
u(q) - \frac{A\phi}{w}d = \frac{\theta}{1 - \theta} \left[ -c(q/z, k_s) + \frac{A\phi}{w}d \right].
\]

An optimum is achieved when \(d = m_b = m\) and the constraint is satisfied with equality. Simplifying, we obtain

\[
\frac{A\phi}{w} m = (1 - \theta)u(q) + \theta c(q/z, k_s) \equiv g(q, k_s, z).
\]

\(^{13}\)It has been shown by Aruoba, Rocheteau, and Waller [2007] that generalized Nash bargaining (GNB) as the solution concept for the DM terms of trade determination has a very different welfare implication. Specifically, Aruoba, Rocheteau, and Waller [2007] show that GNB lacks a property of strong monotonicity—agents’ payoffs do not increase with the bargaining set. Therefore, under GNB, there is still inefficiency at the Friedman rule where there is no money holdup problem. This inefficiency is thus attributed to the lack of strong monotonicity problem. In contrast, Aruoba, Rocheteau, and Waller [2007] show that the egalitarian, or more generally, the proportional solution concept of Kalai [1977], belongs to a class of axiomatic bargaining solutions that satisfy the strong monotonicity property.

\(^{14}\)Again, one could conjecture an equilibrium where credit trades are supplemented with money. However, it is easily shown that money in this case is redundant.
Since we assumed that \( u \) and \( c \) are twice continuously differentiable functions, respectively, on \( \mathbb{R}_+ \) and \( \mathbb{R}^2_+ \), applying the implicit function theorem, we have

\[
\frac{\partial q}{\partial m_b} = \frac{A\phi}{w} \left[ \frac{1}{s_q(q,k_s,z)} \right] > 0,
\]

and,

\[
\frac{\partial q}{\partial k_s} = -\frac{\theta c_k(q/z,k_s)}{s_q(q,k_s,z)} > 0,
\]

where \( s_q(q,k_s,z) := (1 - \theta)u_q(q) + \theta c_q(q/z,k_s)/z \).

**C.1.2. DM credit trades.** We assume that the bargaining problem is identical if buyers and sellers happen to be classified into trades that are completely monitored. Denote the solution \((\tilde{q},l)\) by the decision rules \(\tilde{q} := \tilde{q}(k_s,s)\) and \(l := l(k_s,s)\). Then, the equivalent bargaining solution on the terms of trade \((\tilde{q},l)\) in a bilateral match that is not anonymous is characterized by a first-best allocation:

\[
u_q(\tilde{q}) = c_q(\tilde{q}/z,k_s)/z, \tag{77}\]

and by,

\[
\frac{A\phi}{w} l = (1 - \theta)u(\tilde{q}) + \theta c(\tilde{q}/z,k_s) = g(\tilde{q},k_s,z). \tag{78}\]

Again, applying the implicit function theorem, and noting that \( \partial k_s/\partial k = \partial k_b/\partial k = 1 \), we obtain

\[
\frac{\partial l}{\partial k} = \frac{w}{A\phi} \left[ (1 - \theta)u_q(\tilde{q}) \frac{\partial \tilde{q}}{\partial k_b} + \theta c_q(\tilde{q}/z,k_s) \frac{1}{z} \frac{\partial \tilde{q}}{\partial k_s} + \theta c_k(\tilde{q}/z,k_s) \right]
\]

\[
= \frac{w}{A\phi} \left[ \frac{g_q(\tilde{q},k_s,z) - \theta c_q(\tilde{q}/z,k_s)/z}{s_q(\tilde{q},k_s,z)} \right] \theta c_k(\tilde{q}/z,k_s)
\]

\[
= \frac{w}{A\phi} \left[ (1 - \theta)u_q(\tilde{q}) \right] \theta c_k(\tilde{q}/z,k_s),
\]

where the second equality obtains from the fact that \( \partial \tilde{q}/\partial k_b = 0 \).

**C.1.3. Envelope conditions.** Exploiting the linearity of \( W \), we can write the ex-ante marginal value of money at the start of the DM as

\[
V_m(m,b,k,0,s) = \sigma \kappa \left[ u_q(q_b) \frac{\partial q_b}{\partial m} - \frac{A\phi}{w} \frac{\partial d_b}{\partial m} \right] + \sigma \kappa \left[ -c_q(q_s,k) \frac{\partial q_s}{\partial m} + \frac{A\phi}{w} \frac{\partial d_s}{\partial m} + \frac{A\phi}{w} \sigma \kappa \frac{u_q(q)}{s_q(q,k,z)} \right] \tag{79}
\]

where the last equality has made use of \( \partial q_s/\partial m = \partial d_s/\partial m = 0 \), and, then symmetry in equilibrium, \( q_b = q_s = q \).
The marginal value of capital at the start of the DM is

\[ V_k(m, b, k, 0, \hat{s}) = \frac{A}{w}(1 + r) + \sigma \kappa \left[ u_q(q_b) \frac{\partial q_b}{\partial k} - \frac{A \phi}{w} \frac{\partial d_b}{\partial k} \right] + \sigma \kappa \left[ -c_q(q_s/z, k) \frac{1}{z} \frac{\partial q_s}{\partial k} - c_k(q_s/z, k) + \frac{A \phi}{w} \frac{\partial d_s}{\partial k} \right] + \sigma'(1 - \kappa) \left[ u_q(q_b) \frac{\partial \tilde{q}_b}{\partial k} - \frac{A \phi}{w} \frac{\partial d_b}{\partial k} \right] + \sigma'(1 - \kappa) \left[ -c_q(q_s/z, k) \frac{1}{z} \frac{\partial q_s}{\partial k} - c_k(q_s/z, k) + \frac{A \phi}{w} \frac{\partial d_s}{\partial k} \right]. \]

Note that \( \frac{\partial q_b}{\partial k} = \frac{\partial d_s}{\partial k} = \frac{\partial \tilde{q}_b}{\partial k} = \frac{\partial l_b}{\partial k} = 0 \). Therefore, the envelope condition above simplify to

\[ V_k(m, b, k, 0, \hat{s}) = \frac{A}{w}(1 + r) - \sigma \kappa \left[ c_q(q_s/z, k) \frac{1}{z} \frac{\partial q_s}{\partial k} + c_k(q_s/z, k) \right] - \sigma'(1 - \kappa) \left[ c_q(q_s/z, k) \frac{1}{z} \frac{\partial q_s}{\partial k} + c_k(q_s/z, k) - \frac{A \phi}{w} \frac{\partial d_s}{\partial k} \right] \]

\[ = \frac{A}{w}(1 + r) - \sigma \kappa \gamma(q, k, z) - \sigma'(1 - \kappa)(1 - \theta) \gamma(\tilde{q}, k, z). \]

where

\[ \gamma(q, k) := \left[ \frac{(1 - \theta) u_q(q)}{s_q(q, k, z)} \right] c_k(q/z, k), \]

and,

\[ \gamma(\tilde{q}, k) := \left[ \frac{(1 - \theta) u_q(\tilde{q})}{s_q(\tilde{q}, k, z)} \right] c_k(\tilde{q}/z, k). \]

C.2. Generalized Nash bargaining. In each single-coincidence meeting that occurs with probability \( \sigma \kappa \), the money exchanged \( d \) and quantity traded \( q \), solve a generalized Nash bargaining problem:\(^{15}\)

\[
\max_{q \in \mathbb{R}_+, d \in [0, m]} \left\{ [u(q) + W(m_b - d, \cdot) - T_b]^\theta \right. \\
\left. \times [-c(q/z, k_s) + W(m_s + d, \cdot) - T_s]^{1-\theta} \right\},
\]

(79)

where \( T_b = W(m_b, \cdot) \) and \( T_s = W(m_s, \cdot) \) are the respective threat points of the buyer and the seller –i.e. their individual values of entering the next CM with empty trades from the DM. The parameter \( \theta \in [0, 1] \) is the bargaining strength of the buyer, and, is also the probability that the buyer gets to make an offer in the subsequent round of an equivalent sequential bargaining game.

By the linearity of the value function \( W \), at each given \( s \), the problem can be further simplified to

\[
\max_{q \in \mathbb{R}_+, d \in [0, m]} \left\{ [u(q) - \frac{A \phi}{w} d]^\theta \left[ -c(q/z, k_s) + \frac{A \phi}{w} d \right]^{1-\theta} \right\}.
\]

(80)

\(^{15}\)The Nash bargaining solution has been shown to be approximately the unique subgame perfect equilibrium in an equivalent complete-information sequential bargaining game [see e.g. Binmore, Rubinstein, and Wolinsky, 1986; Howard, 1992].
C.2.1. *DM monetary exchange.* Consider bilateral single-coincidence trades where money is essential as a medium of exchange. In equilibrium, the constraint \( d \leq m_h = m \) binds. So then, a solution to the programming problem in (80) is necessarily and sufficiently given by the decision rules \( q(m, k_s, \$) \) and \( d(m, k_s, \$) \) satisfying:

\[
d(m, k_s, \$) = m, \tag{81}
\]

\[
\frac{A\varphi}{w}m = \frac{\theta u_q(q) + (1 - \theta)u(q) c_q(q/z, k_s)/z}{\theta u_q(q) + (1 - \theta)c_q(q/z, k_s)/z} = g(q, k_s, \$). \tag{82}
\]

Note that the first order condition (82) defines an implicit function of the solution \( q = q(m, k_s, \$) \). That is \( q \) depends only on the money holding of the buyer and the DM-specific capital stock of the seller. This result is identical to Aruoba, Waller, and Wright [2008]. Therefore, we have the following everywhere \((q, k_s)\)-smooth partial derivatives:

\[
g_q := \frac{u_q(c_q/z)}{[\theta u_q + (1 - \theta)c_q/z]^2} \left[ \theta u_q + (1 - \theta)(u - c)[u_q(c_q/z)^2 - c_q u_q/z] \right] > 0, \tag{83}
\]

and

\[
g_k := \frac{u_q c_k}{[\theta u_q + (1 - \theta)c_q/z]^2}[\theta u_q + (1 - \theta)(u - c)u_q c_q/z] < 0. \tag{84}
\]

Moreover, since \( u \in C^2(\mathbb{R}_+) \) and \( c \in C^2(\mathbb{R}_+^2) \), by the Implicit Function Theorem, this implies that \( q \in C^1(\mathbb{R}_+^2) \). Specifically, we can sign the following partial derivatives:

\[
\frac{\partial d}{\partial m} = 1, \quad \frac{\partial d}{\partial m_s} = 0, \quad \frac{\partial q}{\partial m} = \frac{A\varphi}{w} \frac{1}{g_q} > 0, \tag{85}
\]

\[
\frac{\partial q}{\partial m_s} = 0, \quad \frac{\partial d}{\partial k} = \frac{\partial m_s}{\partial k} = 0, \quad \frac{\partial q}{\partial k} = -\frac{g_k}{g_q} > 0.
\]

C.2.2. *DM credit trades.* Assuming the buyer in these events have the same bargaining power \( \theta \), the outcome under monitored trades will be characterized by a first best allocation and a loan schedule, respectively, as

\[
u_q(\bar{q}) = c_q(\bar{q}/z, k_s)/z,
\]

and

\[
\frac{A\varphi}{w}l = (1 - \theta)u(\bar{q}) + \theta c(\bar{q}, k_s, z) \equiv g(\bar{q}, k_s).
\]

Note that the allocation and loan determination \((\bar{q}, l)\) is exactly the same as in the proportional bargaining case, given the same events where trades are monitored.

C.2.3. *Envelope conditions.* At an optimum, the envelope conditions are as follows. The marginal value of money simplifies to

\[
V_m(m, b, k, 0, \$) = \frac{A\varphi}{w} \left[ \sigma_k \frac{u_q(q)}{g_q(q, k)} + (1 - \sigma_k) \right] > 0, \tag{86}
\]

where now \( g_q \) is defined in (83).
The DM marginal value of the capital stock above simplify to
\[
V_k(m, b, k, 0, s) = \frac{A}{w}(1 + r) - \sigma\kappa \left[ c_q(q_z/z, k)z^{-1} \frac{\partial q_s}{\partial k} + c_k(q_z/z, k) \right] \\
- \sigma(1 - \kappa) \left[ c_q(q_z/z, k)z^{-1} \frac{\partial q_s}{\partial k} + c_k(q_z/z, k) - \frac{A\phi \partial l_s}{w} \right] \\
= \frac{A}{w}(1 + r) - \sigma\kappa \gamma(q, k, z) - \sigma(1 - \kappa)(1 - \theta) \left[ \frac{(1 - \theta)u_q(\hat{q})}{g_q(\hat{q}, k, z)} \right] c_k(\hat{q} / z, k).
\]
where
\[
\gamma(q, k, z) = -c_q(q / z, k) \frac{1}{z} g_k(q, k, z) + c_k(q / z, k) < 0.
\]

APPENDIX D. NONSTOCHASTIC STEADY STATES AND CALIBRATIONS

In this section we outline how we calibrate the models. We consider first the baseline model with DM price taking. In section D.1, we discuss the model’s definition of output from each sector and the resulting aggregate output for a country. Then we outline the steady state calculations for the baseline model in section D.2. In subsections D.3 and D.4, we discuss the differences in the steady state conditions and an additional calibration target in terms of an aggregate markup of price over marginal cost.

D.1. Measuring output. For each country, the CM total (production) output in units of the final CM good, is
\[
Y_{CM} = \hat{\phi} \hat{P}_h z F(K, H).
\]
The DM total nominal output is \(\sigma\kappa M + \sigma(1 - \kappa)l\). Total real output in the DM, using \(\phi^{-1}\) as the unit of account is
\[
Y_{DM} = \sigma\kappa M\phi + \sigma(1 - \kappa)\hat{\phi} \\
= \sigma \left[ \frac{(1 - \tau_H)}{\lambda} \right] \left[ \hat{\phi} \hat{P}_h z F_H(K, H) \right] \left[ \kappa g(q, K, z) + (1 - \kappa)\hat{g}(\hat{q}, K, z) \right],
\]
where \(g(q, K, z)\) is defined accordingly for each case, and
\[
\hat{g}(\hat{q}, K, z) = \begin{cases} 
\hat{q} \cdot c_q(\hat{q}, K, z) & \text{if Price Taking} \\
(1 - \theta)u(\hat{q}) + \theta c(\hat{q}, K, z) & \text{if PB or GNB}.
\end{cases}
\]
Total GDP, measured in terms of the CM final goods is:
\[
Y = Y_{CM} + Y_{DM}.
\]
D.2. **Baseline nonstochastic steady state characterizations.** From the stationary equilibrium demand for intermediate goods we have at steady state:

\[ \hat{\phi}h = G(y_h, y_f) := \left( \theta y_h^{1-\epsilon} \right) \left[ G(y_h, y_f) \right]^{\epsilon-1} \]  

(87)

\[ \hat{\phi}f = G(y_f, y_f) := \left( (1 - \theta) y_f^{1-\epsilon} \right) \left[ G(y_h, y_f) \right]^{\epsilon-1} \]  

(88)

\[ \hat{\phi}h = G(y_h^*, y_f^*) := \left( (1 - \theta) y_h^* \right) \left[ G(y_h, y_f) \right]^{\epsilon-1} \]  

(89)

\[ \hat{\phi}f = G(y_f^*, y_f^*) := \left( (1 - \theta) y_f^* \right) \left[ G(y_h, y_f) \right]^{\epsilon-1} \]  

(90)

The law of one price holds for intermediate goods, so that equating (87) and (89), we have

\[ y_f = \left( \frac{\theta}{1 - \theta} \right)^{\frac{1-\epsilon}{\epsilon}} y_h. \]  

(91)

Using (92) in the aggregator \( G \), we have

\[ G(y_f, y_f) = \left[ \theta y_h^1 + (1 - \theta) y_f^1 \right] = \omega_1 y_h, \]  

(92)

where

\[ \omega_1 := \left[ \theta + (1 - \theta) \left( \frac{\theta}{1 - \theta} \right)^{1/(1-\epsilon)} \right]^{\epsilon}. \]

From market clearing for Home-produced intermediate goods, we have

\[ zK^\alpha H^{1-\alpha} = y_h + y_f^* \equiv \omega_F y_h, \]  

(93)

where

\[ \omega_F := \left[ 1 + \left( \frac{\theta}{1 - \theta} \right)^{\epsilon/(1-\epsilon)} \right]. \]

The resource constraint is

\[ G(y_h, y_f) = (1 + \tau_X) X + \delta K + \tau_H w H + \tau_K r K \equiv \omega_1 y_h. \]  

(94)

Equating (94) and (93) in terms of \( y_h \), we have a relationship between CM production and final demand:

\[ \left( \frac{\omega_1}{\omega_F} \right) zK^\alpha H^{1-\alpha} = (1 + \tau_X) X + [(1 - \alpha) \tau_H + \alpha \tau_K] \hat{\theta} \omega_1^{\epsilon-1} zK^\alpha H^{1-\alpha} \]

\[ + (1 - \tau_X) \delta K. \]

Now dividing the above expression by \( H \) and defining \( \kappa := K / H \), we obtain

\[ X = \frac{1}{1 + \tau_X} \left\{ \left[ \frac{\omega_1}{\omega_F} - ((1 - \alpha) \tau_H + \alpha \tau_K) \hat{\theta} \omega_1^{\epsilon-1} \right] z\kappa^\alpha - (1 - \tau_X) \delta \kappa \right\} H. \]  

(95)
Also, from the labor market clearing condition in the CM, we have, after evaluating $U_X$ using the CRRA functional form indexed by parameters $(B, \gamma)$:

$$X = \left[ \frac{(1 - \tau_H)(1 - \alpha)B \theta \omega_i^{\epsilon / \gamma} \tau}{A (1 + \tau_X)} \right]^{1/\gamma} z k^a. \tag{96}$$

**D.2.1. Other side equations.** The following relations will be used in various other equations pinning down calibrations below. First, from (95), we can divide through by $K$ to re-write as

$$\frac{X}{K} = \frac{1}{1 + \tau_X} \left\{ \frac{\omega_i}{\omega_F} - ((1 - \alpha)\tau_H + \alpha \tau_K) \theta \omega_i^{\epsilon / \gamma} \right\} z k^a - (1 - \tau_K)\delta \tag{95.a}$$

Further substitution of $X$ out using (96) yields a relation between $K$ and $k$:

$$K = \frac{1}{1 + \tau_X} \left\{ \frac{\omega_i}{\omega_F} - ((1 - \alpha)\tau_H + \alpha \tau_K) \theta \omega_i^{\epsilon / \gamma} \right\} z k^a - (1 - \tau_K)\delta \tag{97}$$

From the DM credit trade outcomes we have $u(q) = c(q, \bar{q}, K)$. Given the the parameterization of $u$ and $c$, indexed by parameters $(C, \eta)$ and $\omega$, respectively, we then have

$$\bar{q} = \left( \frac{C}{\omega} \right)^{1/\theta - 1} K^{\alpha / (1 - \theta - 1)}. \tag{98}$$

From the Euler equation for money holdings at steady state we also get

$$\frac{1}{\sigma \kappa} \left[ \beta^{-1} - (1 - \sigma \kappa) \right] c(q, K) = u(q),$$

where there is a wedge $(\sigma \kappa)^{-1} \left[ \beta^{-1} - (1 - \sigma \kappa) \right]$ arising from matching frictions, relative to a first-best characterization for the allocation of $q$. Using the parameterization of $u$ and $c$, we have explicitly a relation between $q$ and $K$:

$$q = \left( \frac{\sigma \kappa \cdot C}{\omega \left[ \beta^{-1} - (1 - \sigma \kappa) \right]} \right)^{1/\theta - 1} K^{\alpha / (1 - \theta - 1)}. \tag{99}$$

From the Euler equation for capital, we have the steady state relation between $k$ and $K$:

$$\delta = \frac{1 - \beta^{-1}}{1 - \tau_K} + (\theta \omega_i^{\epsilon / \gamma} \tau) z k^a - \frac{\sigma (1 + \tau_X)}{(1 - \tau_K) U_X(X)} [k \gamma(q, K) + (1 - \kappa) \gamma(q, K)]. \tag{100}$$

**D.2.2. Calibrating $A$.** Equating (95) and (96), we get an expression that allows us to calibrate (given target $H$ along with other parameters), the marginal disutility of labor in the CM:

$$A = \left[ (1 + \tau_X)^{-1}(1 - \tau_H)(1 - \alpha)B \theta \omega_i^{\epsilon / \gamma} z k^a \right] \left( \frac{1 + \tau_X}{H} \right)^{1/\gamma} \times \left\{ \frac{1}{\omega_i^{\epsilon / \gamma} - ((1 - \alpha)\tau_H + \alpha \tau_K) \theta \omega_i^{\epsilon / \gamma}} z k^a - (1 - \tau_K)\delta k \right\}^{1/\gamma}. \tag{101}$$
D.2.3. **Calibrating $\omega$.** From our definition of real GDP, we have

$$Y = \hat{\phi} \hat{P}_h z F(K, H) + \sigma \frac{(1 - \tau_H)}{A} \left[ \hat{\phi} \hat{P}_h z F_H(K, H) \right] \left[ \kappa g(q, K) + (1 - \kappa) \breve{g}(q, K) \right].$$

Divide both sides by $K$, knowing that $\hat{\phi} \hat{P}_h = \theta \omega^{\frac{\epsilon}{1 - \epsilon}}$. Then we have a relation between another calibration target, the output-to-capital ratio $s^{-1}_K := Y/K$, and the capital complementarity parameter $\omega$, given other calibrations:

$$s^{-1}_K = \theta \omega^{\frac{\epsilon}{1 - \epsilon}} z \kappa^{-1} \left\{ 1 + \sigma \frac{(1 - \tau_H)\omega}{A} [\kappa q^\omega + (1 - \kappa) \breve{q}^\omega] K^{-\omega} k \right\}. \quad (102)$$

D.2.4. **Calibrating $\alpha$.** Following Aruoba, Waller, and Wright [2008], we calibrate $\alpha$ to match the labor share of CM output, denoted as $LS$. Assuming the Cobb-Douglas parameterization of $F$, we have the relations

$$\alpha = -\left( \frac{\ln(z \cdot LS)}{\ln(K) - \ln(H)} \right). \quad (103)$$

D.2.5. **Calibrating $\sigma$.** In this section, we describe how we derive the calibration target variable – the nominal-interest-rate semi-elasticity of money demand, $\xi$ – in the search-theoretic models. This target is used for calibrating the value of $\sigma$.

The steps below apply to all three types of decentralized market (DM) pricing mechanism assumptions, with the appropriate definitions for partial derivatives. Computationally, these are modular objects that are easily applied. These steps are similar to Aruoba, Waller, and Wright [2008] with the exception that we now have to account for traded goods relative prices as well.

Consider a generic equilibrium pricing condition for trades involving money in the DM:

$$\frac{A \phi M}{\bar{w}} = g(q, K, z),$$

where we had defined $\bar{w} := \bar{w}(1 - \tau_H) = (1 - \tau_H) \phi P_h z F_H(K, H)$ in the paper, and, $(q, K) \mapsto g(q, K)$ depends on the pricing mechanism assumed.

[To be Typed ...]

D.2.6. **Calibrating $B$.** In Aruoba, Waller, and Wright [2008], the authors calibrate $B$ to match a measure of money demand elasticity. In our setting, since the DM sector also behaves like a nontraded goods sector where both money and credit are used, we choose to calibrate $B$ to a calibration target of the nontraded-goods consumption share. In our model this is just the DM consumption to total consumption ratio:

$$NTS = \frac{Y_{DM}}{X + Y_{DM}}. \quad (104)$$

□ *Calibration summary.* Along with (96), (97), (98) and (99), we have a system in (100), (101), (102), (103), (??) and (104), characterizing the solutions $(A, \alpha, B, \omega, \sigma, k)$. We minimize a quadratic loss criterion in terms of deviations from the targets $(H, LS, NTS, K/Y, v)$ subject to the system of nonlinear equations (100), (101), (102), (103), (??) and (104).

D.3. **PB and nonstochastic steady state characterizations.** The only difference in the characterization of steady state allocations appears in terms of the determination of steady state $(q, k)$.
where $\kappa := K/H$ is the capital-labor ratio. Specifically, from the Euler equation for money at steady state, we can derive a relation between $q$ and $K$ at steady state, assuming the functional forms for preferences and technology as in the baseline model’s example:\footnote{In deriving these expressions, we acknowledge that they are only limiting approximations, since we have set the lower bound on $q, q = 0$, when technically, for existence of equilibria, we require that $q > 0$ such that the threat points of the agents are well-defined.}

\[
q = \left( \frac{(1 - \theta)\beta^{-1} - (1 - \sigma\kappa)}{\theta(\beta^{-1} - (1 - \sigma\kappa))} C \right)^{-\frac{1}{\gamma}} K^{\frac{\kappa - 1}{\sigma\gamma}}.
\]

Recall that $\eta$ and $C$ are parameters governing the utility function $u$, and, $\omega$ is a parameter governing the production technology implied by the cost function $c$.

**Remark 4.** When $\theta = 1$, the characterization for the allocation of $q$ in relation to $K$ above is identical to the baseline price taking case.

Likewise, from the Euler equation characterizing equilibrium capital accumulation, we can derive a steady state relation solving implicitly for $\kappa$ as:

\[
\delta = \frac{1 - \beta^{-1}}{1 - \tau_K} + (\theta\omega_I^{\epsilon_1})az\kappa^{\sigma - 1} - \frac{\sigma(1 + \tau_X)}{(1 - \tau_K)U_X(X)}[\kappa\gamma(q, K) + (1 - \kappa)\gamma(\bar{q}, K)],
\]

where $\omega_I := [\theta + (1 - \theta)(\theta/(1 - \theta))^{1/(1 - \epsilon)}]^{1/\epsilon}$, and,

\[
\gamma(\bar{q}, K) := \left[\frac{(1 - \theta)u(q)}{g_q(\bar{q}, K)}\right]c_K(\bar{q}, K), \quad \gamma(q, K) := \left[\frac{(1 - \theta)u(q)}{g_q(q, K)}\right]c_K(q, K).
\]

Note that we know $X$ and $K$ can be written analytically as functions of $\kappa$, exactly, as in the baseline model.

**Remark 5.** When $\theta = 1$, the characterization for the allocation of $\kappa$ above is identical to the baseline price taking case, where the function $\gamma = c_K.$
where \( g(\bar{q}, K) := (1 - \theta)u(\bar{q}) + \theta c(\bar{q}, K) \).

So average markup coming from the DM is \( \mu_{DM} = \kappa \mu_M + (1 - \kappa) \mu_I \). Following Aruoba, Waller, and Wright [2008], we will define the aggregate markup \( \mu \) as \( \mu_{DM} \) weighted by the DM output share in total output, i.e.

\[
\mu := (Y_{DM} / Y) \mu_{DM} + (Y_{CM} / Y) \cdot 0, \quad \text{where} \quad Y = Y_{CM} + Y_{DM}.
\]

D.4. **GNB and nonstochastic steady state characterizations.** The only difference in the characterization of steady state allocations now appears in terms of the determination of steady state \((q, \bar{k})\) where \( \bar{k} := K / H \) is the capital-labor ratio. Specifically, from the Euler equation for money at steady state, we can derive a relation between \( q \) and \( K \) at steady state, assuming the functional forms for preferences and technology as in the baseline model’s example:

\[
\frac{1}{\bar{c} \bar{k}} [\beta^{-1} - (1 - \sigma \bar{k})] g_q(q, K) = \bar{u}_q(q).
\]

Now, with GNB, the \( g_q \) function involves second-order derivative functions of \( u \) and \( c \), so that the relation above cannot be explicitly written in terms of a exact relation between \( q \) and \( K \). Nevertheless, we can find the steady state points numerically.

Likewise, from the Euler equation characterizing equilibrium capital accumulation, we can derive a steady state relation solving implicitly for \( \bar{k} \) as:

\[
\delta = \frac{1 - \beta^{-1}}{1 - \tau_k} + (\theta \omega_1)^{\frac{\epsilon - 1}{\epsilon}} a z (1 - \theta)^{\alpha} \cdot \frac{\sigma(1 + \tau_X)}{(1 - \tau_k) U_X(X)} [k \bar{\gamma}(q, K) + (1 - \kappa) c_k(\bar{q}, K)],
\]

where \( \omega_1 := (\theta + (1 - \theta)(\theta / (1 - \theta)))^{1/(1-\epsilon)} \), and, where

\[
\bar{\gamma}(q, K) = -c_q(q, K) \frac{\bar{g}_k(q, K)}{\bar{g}_q(q, K)} + c_k(q, K) < 0.
\]

Note that we know \( X \) and \( K \) can be written analytically as functions of \( \bar{k} \), exactly, as in the baseline model.

D.4.1. **Calibrating \( \theta \).** Just as in the proportional bargaining case, the markup \( \mu_M \) in monetary trades in the DM satisfies the definition

\[
1 + \mu_M = \frac{M / q}{\bar{c} q(\bar{q}, K)} = \frac{g(q, K)}{\bar{c} q(\bar{q}, K)},
\]

where \( g(q, K) \) is now defined by (82).

Similar to the proportional bargaining case, the markup in credit trades \( \mu_I \) satisfies

\[
1 + \mu_I = \frac{1 / \bar{q}}{\bar{c} \bar{q}(\bar{q}, K)} = \frac{\bar{g}(\bar{q}, K)}{\bar{c} \bar{q}(\bar{q}, K)},
\]

where \( \bar{g}(\bar{q}, K) := (1 - \theta)u(\bar{q}) + \theta c(\bar{q}, K) \).

So average markup coming from the DM is still \( \mu_{DM} = \kappa \mu_M + (1 - \kappa) \mu_I \). The aggregate markup is \( \mu := (Y_{DM} / Y) \mu_{DM} + (Y_{CM} / Y) \cdot 0, \quad \text{where} \quad Y = Y_{CM} + Y_{DM} \).

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FIGURE 2. Real and nominal exchange rates versus relative aggregate prices: 1% Home TFP increase $z$. 
FIGURE 3. Real and nominal exchange rates versus relative aggregate prices: 1% Home money supply growth increase, $\psi$. 