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### Option Pricing When the Regime-Switching Risk is Priced

The option valuation problem under regime-switching has received considerable interest in the literature. The problem is motivated by both practical and theoretical interests. From the practical perspective, regime-switching models have better empirical performance than their constant-coefficient counterparts. From the theoretical point of view, there are two sources of risks underlying a regime-switching model, one being the diffusion risk and one being the regime-switching risk. The diffusion risk is attributed to the fluctuations of market prices or rates and can be regarded as market or financial risk. The regime-switching risk is due to the change in (macro)- economic conditions and can be thought of as economic risk. Due to the presence of two sources of risks, the market described by regime-switching models is, in general, incomplete. This means that there are more than one equivalent martingale measures and no-arbitrage prices.

The main challenge of the option valuation problem under regime-switching models is how to determine an equivalent martingale measure so that both the regime switching risk and the diffusion risk appropriately. This issue seems to be overlooked or not fully addressed in the existing literature. However, it is certainly an interesting and important one. First, with the regime-switching risk being priced directly and appropriately, one can incorporate the impact of switching regimes in the asset price dynamics on the behavior of option prices more completely and exactly. Second, we witness closer interaction between finance and macro-economics. By pricing the regime-switching risk appropriately, one can get some insights into how macroeconomic conditions affect the option prices. This is especially important if we consider pricing an option with a long maturity since macro-economic conditions can change over a long period of time.

In this talk, we shall discuss the pricing of an option when the price dynamic of the underlying risky asset is governed by a Markov-modulated geometric Brownian motion. We suppose that the drift and the volatility of the underlying risky asset are modulated by an observable continuous-time, finite-state Markov chain, whose states represent observable states of an economy. More specifically, one may interpret the states of the chain as proxies of observable macro-economic indicators, such as gross domestic product (GDP) and retail price index (RPI), or different stages of business cycles. For example, if the number of states of the chain is four, the states can be interpreted as "Peak", "Trough", "Recession", "Expansion" in a business cycle. Here, we introduce a novel two-stage pricing model to price both the diffusion risk and the regime-switching risk. The first stage of the method involves the use of a well-known tool in actuarial science, namely, the Esscher transform to determine a set of equivalent martingale measures satisfying a martingale

restriction. In general, there may be more than one equivalent martingale measures specified by the Esscher transform, which satisfy the martingale condition. So, in the second stage, we determine an equivalent martingale pricing measure by minimizing the maximum entropy between an equivalent martingale measure and the real-world probability measure over different economic states. This is a min-max entropy problem. By solving this problem, we pick an equivalent martingale measure which is "closest" to the real-world probability and contains the most informational content uniformly over different states. The minimization of relative entropy is an important approach to determine an equivalent martingale measure in an incomplete market. We conduct numerical experiments to illustrate the effect of pricing regime-switching risk.

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