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A Modification of the Traditional Analysis of Surplus Formula
by
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Abstract

The traditional formula used for analysis of surplus in the presence of a conservative valuation basis is compared to a similar formula used for analysis of profit under the margin on services valuation method using a realistic basis. Deficiencies of the former formula relative to the latter are noted. A revised formula is presented which overcomes these deficiencies while still allowing the use of a conservative valuation bases.

The Traditional Analysis of Surplus

In this section we will derive formulae to carry out an analysis of surplus as it would have been performed prior to the invention of the Margin on Services technique. The general approach used to derive these formulae is likely to be familiar to anyone who has studied analysis of surplus in life insurance. The approach used here is similar to that use in Carr (1996) and is no doubt also similar to many other actuarial books which discuss analysis of surplus.

We will assume that we are dealing with a statutory fund containing a single cohort of identical traditional policies. A premium is payable at the start of the year under consideration and the policies do not mature during this year. Expenses and surrenders will be ignored. Thus the only assumptions required are those for interest and mortality and the policy value is effectively a net premium policy value. On this simple basis both the premium and the policy value for a policy are proportional to its sum insured.

This scenario is quite a simple one and has been chosen to allow us to concentrate on some important basic principles without being distracted by complexities.

Time will be measured in years from the start of the year we are analysing.

Define:

s = Total sum insured for policies in force at time 0.
S_t = Surplus in the statutory fund at time t.
V_t = Policy value per unit sum insured at time t.
P_0 = Premium payable at time 0 per unit sum insured.

i = Expected interest rate over the year.
q = Expected mortality rate over the year.
(The symbols i and q are not best estimate assumptions but rather are the conservative assumptions used to determine V_t.)

i' = Actual interest rate over the year.
q' = Actual mortality rate over the year.

As is usual in the context of analysis of surplus, when we refer to 'interest' we include capital gains and losses as well as investment income. We will also employ some standard simplifying assumptions often encountered in the derivation of analysis of surplus formulae. We will assume all death claims occur in the middle of the year. We will also assume that all the resulting $(1+i)^{\frac{1}{2}}$ terms can be replaced by $\left(1+\frac{1}{2}i\right)$.

We begin by noting:

$$\left(s.V_0 + s.P_0\right)(1+i) - q.s\left(1+\tfrac{1}{2}i\right) = \left(1-q\right)s.V_1 \qquad \ldots(1)$$

That is, the policy values at the start of the year and the premiums received then, when accumulated with expected interest, are sufficient to pay the expected death claims occurring during the year and to set up the required policy values for the policies expected to survive the year. The mortality rate and interest rate used in this equation are conservative rates, but the equation holds because the policy values are calculated using those same conservative rates.

The Actual surplus at time 1 which we seek to analyse is:

$$S_1 = \left\{S_0 + s.V_0 + s.P_0\right\}(1+i') - q'.s\left(1+\tfrac{1}{2}i'\right) - \left(1-q'\right)s.V_1$$

Also define:

$$a = \left\{S_0 + s.V_0 + s.P_0\right\}(1+i) - q'.s\left(1+\tfrac{1}{2}i\right) - \left(1-q'\right)s.V_1$$

$$\begin{aligned} b &= \left\{S_0 + s.V_0 + s.P_0\right\}(1+i) - q.s\left(1+\tfrac{1}{2}i\right) - \left(1-q\right)s.V_1 \\ &= S_0(1+i) \end{aligned}$$

The second expression for *b* follows from combining equation (1) with the first expression for *b*.

The surplus can then be subdivided using:

$$\begin{aligned} S_1 &= \left(S_1 - a\right) + (a-b) + b \\ &= (i'-i)\left\{S_0 + s.V_0 + s.P_0 - \tfrac{1}{2}q'.s\right\} \\ &\quad + \left(q-q'\right)\left\{s\left(1+\tfrac{1}{2}i\right) - s.V_1\right\} \\ &\quad + S_0(1+i) \end{aligned}$$

This is usually rearranged to:

$$\begin{aligned} S_1 &= S_0(1+i') \\ &\quad + (i'-i)\left\{s.V_0 + s.P_0 - \tfrac{1}{2}q'.s\right\} \\ &\quad + \left(q-q'\right)\left\{s\left(1+\tfrac{1}{2}i\right) - s.V_1\right\} \end{aligned}$$

That is, the three components of surplus are:

1. Surplus brought forward, incorporating interest at the actual rate.
2. Interest Surplus
3. Mortality Surplus

The Modern MoS Analysis of Profit

Under the traditional approach we analysed the 'surplus', which was the difference between the assets at the end of the year and the conservative policy values required. By contrast the Margin on Services (MoS) method reveals the profit earned over the year and it is this profit that we now seek to analyse.

Define:

AP_1 = Actual Profit earned over the year (from time 0 to time 1.)
R_t = Reserves in the statutory fund at time *t*.
PL_t = Policy liability per unit sum insured at time *t*.
i'' = Expected interest rate over the year (Best Estimate Basis).
q'' = Expected mortality rate over the year (Best Estimate Basis).
PP_1 = Planned Profit for the year, assumed to occur at time 1.

Note that we are using the word 'reserves' in the correct accounting sense of the word, meaning the assets in excess of the liabilities. Thus, where the traditional approach subdivided the assets at time *t* into the conservative policy values of $s.V_t$ and the surplus of S_t, the modern approach subdivides those same assets into realistic policy liabilities of $s.PL_t$ and reserves of R_t. (We are assuming that the statutory fund under consideration has no liabilities other than the policy liabilities.)

The policy liability includes allowance for future planned profits. The incidence of the planned profit over the year depends on the profit carrier chosen. For our simple example, we will assume that these planned profits emerge at the end of the year and so:

$$\{s.PL_0 + s.P_0\}(1+i'') - q''.s(1+\tfrac{1}{2}i'') = (1-q'')s.PL_1 + PP_1 \qquad \ldots(2)$$

That is, the policy liabilities at the start of the year and the premiums received then, when accumulated with expected interest, are sufficient to pay the expected death claims occurring during the year, set up the required policy values for the policies expected to survive the year and allow for the release of the planned profit for the year. The mortality rate and interest rate used in this equation are the best estimate assumptions employed in the calculation of the policy liabilities.

The actual profit to be analysed at the end of the year can be determined as:

Actual Profit
= Revenue – Expenses
= Premiums + Interest – Death Claims – Increase in Policy Liabilities

That is:

$$\begin{aligned} AP_1 &= s.P_0 + i'.\left\{R_0 + s.PL_0 + s.P_0 - \tfrac{1}{2}q'.s\right\} - q'.s - \left\{(1-q')s.PL_1 - s.PL_0\right\} \\ &= i'.R_0 + \left\{s.PL_0 + s.P_0\right\}(1+i') - q'.s\left(1+\tfrac{1}{2}i'\right) - (1-q')s.PL_1 \end{aligned}$$

Also define:

$$\begin{aligned} d &= i''.R_0 + \left\{s.PL_0 + s.P_0\right\}(1+i'') - q'.s\left(1+\tfrac{1}{2}i''\right) - \left(1-q'\right)s.PL_1 \\ e &= i''.R_0 + \left\{s.PL_0 + s.P_0\right\}(1+i'') - q''.s\left(1+\tfrac{1}{2}i''\right) - \left(1-q''\right)s.PL_1 \\ &= i''.R_0 + PP_1 \end{aligned}$$

The second expression for *e* follows from combining equation (2) with the first expression for *e*.

The subdivision of the profit can then be carried out by noting

Actual Profit

$$\begin{aligned} AP_1 &= \left(AP_1 - d\right) + (d-e) + e \\ &= (i'-i'')\left\{R_0 + s.PL_0 + s.P_0 - \tfrac{1}{2}q'.s\right\} \\ &\quad + \left(q''-q'\right)\left\{s\left(1+\tfrac{1}{2}i''\right) - s.PL_1\right\} \\ &\quad + i''.R_0 + PP_1 \end{aligned}$$

This can be rearranged to:

$$\begin{aligned} AP_1 &= i'.R_0 \\ &\quad + PP_1 \\ &\quad + (i'-i'')\left\{s.PL_0 + s.P_0 - \tfrac{1}{2}q'.s\right\} \\ &\quad + \left(q''-q'\right)\left\{s\left(1+\tfrac{1}{2}i''\right) - s.PL_1\right\} \end{aligned}$$

This gives the 4 components of profit as:

1. Interest on Reserves
2. Planned Profit
3. Interest Profit
4. Mortality Profit

Comparison of the Traditional and Margin on Services Formulae

When we compare the traditional analysis of surplus formula and the MoS analysis of profit formulae, there are obviously many different symbols involved. Much of this is merely due to different terminology under the two systems. For example, under the traditional method we have referred to a Policy Value, while the MoS method refers to a Policy Liability.

However, let's look beyond these semantic differences, and consider major differences in structure between these two formulae.

The first apparent difference is that where the former has a surplus brought forward term of $S_0(1+i')$, the later has an interest on reserves term of $i'.R_0$. Though this may appear to be a significant difference in structure, it is really only a difference in presentation. The MoS method is primarily concerned with the profit generated during the year, an income statement item. Due to the way life insurance company accounts were constructed under the Life Insurance Act 1945, the traditional method was more concerned with analysing the accumulated surplus at the end of the year, a balance sheet item. That is, while the former was concerned with the changes within the year, the latter was concerned with a running total since the inception of the statutory fund. Hence when analysing the differences between the two formulae it is more useful to compare the MoS profit AP_1, with the change in surplus over the year $S_1 - S_0$ under the traditional method. This would give us the following two formulae to compare.

Traditional Analysis of Surplus:

$$\begin{aligned} S_1 - S_0 &= i'.S_0 \\ &\quad + (i'-i)\left\{s.V_0 + s.P_0 - \tfrac{1}{2}q'.s\right\} \\ &\quad + \left(q-q'\right)\left\{s\left(1+\tfrac{1}{2}i\right) - s.V_1\right\} \end{aligned}$$

Margin on Services Analysis of Profit

$$\begin{aligned} AP_1 &= i'.R_0 \\ &\quad + PP_1 \\ &\quad + (i'-i'')\left\{s.PL_0 + s.P_0 - \tfrac{1}{2}q'.s\right\} \\ &\quad + \left(q''-q'\right)\left\{s\left(1+\tfrac{1}{2}i''\right) - s.PL_1\right\} \end{aligned}$$

There are now two remaining structural differences between the two formulae.

Firstly, the MoS approach involves a planned profit term which is absent from the traditional approach.

Secondly, in the MoS approach, the interest profit term involves a comparison of i', the actual interest rate, to i'', the expected interest rate on the best estimate basis. The number resulting from this calculation is inherently meaningful and useful. For example, a small absolute value for the interest surplus indicates that the actuary's best estimate interest assumption was close to reality. By contrast, the traditional method's interest surplus involves a comparison of i', the actual interest rate, to i, the expected interest rate on a conservative basis. Under the minimum valuation basis specified by the Life Insurance Act 1945, the resulting interest profit was not a very useful figure. The maximum interest rate which could be used to value traditional policies was quite low, and hence an interest surplus would invariably result.

Similar comments can be made in respect of the mortality profit terms in the above formulae. Had we also included expenses and withdrawals in the formulae, similar comments would also apply to them. When policy values were determined in accordance with the Life Insurance Act 1945, the analysis of surplus would usually result in:

(a) a large interest surplus, since the valuation interest rate was conservatively low;
(b) a mortality surplus, since the valuation mortality rates were conservatively high;
(c) a large expense loss, since the valuation basis assumed no expenses; and

(d) a potentially large withdrawal profit or loss, (depending on how the losses on short duration forfeitures and surrenders compared to profits on longer duration surrenders) due to the basis making no allowance for withdrawals.

Thus, it can be suggested that the only real use of an analysis of surplus under the Life Act 1945 was as a check on the valuation calculations, (and not a totally independent check at that.)

On the above grounds, it seems clear that the MoS Analysis of Profit formula can be regarded as significantly more useful than the traditional formula, since it compares actual experience to the best estimate assumptions rather than to conservative assumptions.

The Revised Conservative-Basis Analysis of Surplus

There appears to be a belief that the useful properties inherent in the MoS formula and absent from the traditional formula are a direct result of the MoS philosophies of using best estimate assumptions and determining a profit carrier and calculating planned profits. That is, it may be thought that any analysis of surplus based on a conservative valuation basis (such as that in the Life Act 1945) is forever doomed to produce meaningless comparisons of experience to conservative assumptions.

This is not the case. It is in fact possible to adjust the traditional analysis of surplus formula so that, for example, the resulting interest profit compares the actual interest rate to a best estimate interest rate, even though the policy values are determined using a different conservative interest rate. We will now derive this formula.

The symbols we will use are all as defined above.

The change in surplus which we wish to analyse is

$$S_1 - S_0 = i'.S_0 + \{s.V_0 + s.P_0\}(1+i') - q'.s\left(1+\tfrac{1}{2}i'\right) - (1-q')s.V_1$$

The traditional derivation gradually changes ‘actual’ to ‘expected’ where the ‘expected’ items are those in the conservative policy value basis. This time we will change to ‘expected’ items using the best estimate basis.

Define:

$$f = i''.S_0 + \{s.V_0 + s.P_0\}(1+i'') - q'.s\left(1+\tfrac{1}{2}i''\right) - (1-q')s.V_1$$

$$g = i''.S_0 + \{s.V_0 + s.P_0\}(1+i'') - q''.s\left(1+\tfrac{1}{2}i''\right) - (1-q'')s.V_1$$

Then

$$\begin{aligned} S_1 - S_0 &= (S_1 - S_0 - f) + (f - g) + g \\ &= (i' - i'')\{S_0 + s.V_0 + s.P_0 - \tfrac{1}{2}q'.s\} \\ &\quad + (q'' - q')\{s\left(1+\tfrac{1}{2}i''\right) - s.V_1\} \\ &\quad + i''.S_0 + \{s.V_0 + s.P_0\}(1+i'') - q''.s\left(1+\tfrac{1}{2}i''\right) - (1-q'')s.V_1 \end{aligned}$$

This can be rearranged to

$$\begin{aligned} S_1 - S_0 &= i'.S_0 \\ &+ \left\{s.V_0 + s.P_0\right\}(1+i'') - q''.s\left(1+\tfrac{1}{2}i''\right) - \left(1-q''\right)s.V_1 \\ &+ (i'-i'')\left\{s.V_0 + s.P_0 - \tfrac{1}{2}q'.s\right\} \\ &+ \left(q''-q'\right)\left\{s\left(1+\tfrac{1}{2}i''\right) - s.V_1\right\} \end{aligned}$$

The second of the four terms on the right hand side may at first look rather threatening. However, it is merely the surplus which we would expect to emerge over the year if the experience follows the best estimate assumptions.

That is, the change in surplus consists of:

1. Actual interest on the surplus at the start of the year.
2. The surplus expected to emerge during the year on the best estimate basis.
3. Interest Profit, comparing actual interest to best estimate assumption
4. Mortality Profit, comparing actual mortality to best estimate assumption.

(It may also be noted that though we have referred to *i* and *q* as the conservative assumptions used to calculate the conservative policy values V_0 and V_1, in fact, there was nothing in the above derivation of the Revised 'Conservative-Basis' Analysis of Surplus formula which required them to be conservative. This formula remains valid, (though misnamed) even if those rates are best estimate or optimistic rather than conservative.)

Comparison of the Revised Approach With the Previous Two Methods

Traditional Analysis of Surplus:

$$\begin{aligned} S_1 - S_0 &= i'.S_0 \\ &+ (i'-i)\left\{s.V_0 + s.P_0 - \tfrac{1}{2}q'.s\right\} \\ &+ \left(q-q'\right)\left\{s\left(1+\tfrac{1}{2}i\right) - s.V_1\right\} \end{aligned}$$

Margin on Services Analysis of Profit

$$\begin{aligned} AP_1 &= i'.R_0 \\ &+ PP_1 \\ &+ (i'-i'')\left\{s.PL_0 + s.P_0 - \tfrac{1}{2}q'.s\right\} \\ &+ \left(q''-q'\right)\left\{s\left(1+\tfrac{1}{2}i''\right) - s.PL_1\right\} \end{aligned}$$

Revised Conservative-Basis Analysis of Surplus

$$\begin{aligned} S_1 - S_0 &= i'.S_0 \\ &+ \left\{s.V_0 + s.P_0\right\}(1+i'') - q''.s\left(1+\tfrac{1}{2}i''\right) - \left(1-q''\right)s.V_1 \\ &+ (i'-i'')\left\{s.V_0 + s.P_0 - \tfrac{1}{2}q'.s\right\} \\ &+ \left(q''-q'\right)\left\{s\left(1+\tfrac{1}{2}i''\right) - s.V_1\right\} \end{aligned}$$

Comparing the Revised Conservative-Basis Analysis of Surplus to the Traditional Analysis of Surplus, the interest and mortality profit figures are now more useful since they compare the actual to the best estimate, rather than comparing it to the conservative basis used to determine policy values. As a consequence of this change a new item appears: the amount of

surplus expected to emerge over the year if experience follows the best estimate assumptions. Since the policy value basis is conservative, this will be a positive amount. That is, if we set up policy values on a conservative basis but experience follows best estimate assumptions, we expect surplus to emerge over the year.

Comparing the Revised Conservative-Basis Analysis of Surplus to the MoS Analysis of Profit, we see that the 3rd and 4th terms in the two formulae, interest and mortality profit, match closely. (The only difference is that where one refers to a policy value, the other refers to a policy liability.) The 1st terms, interest on surplus or reserves, also achieve much the same purpose in each case. It can also be argued that there are some similarities between the 2nd terms, though the reasons for their appearance stem from very different philosophies. The MoS approach spreads the expected profit from a cohort of policies over the expected life of those policies and the 2nd term in the MoS Analysis of Profit states how much profit we expect to emerge this year. If we set up policy values on a conservative basis we expect that surplus will emerge and the 2nd term from the Revised Conservative-Basis Analysis of Surplus states how much is expected to emerge this year from that source.

Practical Use of the Revised Formula

By now the reader may be saying: "Well, yes; the algebra is all very pretty. However, given that we no longer carry out valuations using the conservative basis of the Life Insurance Act 1945, does the Revised Conservative-Basis Analysis of Surplus formula have any practical uses?"

In the new realistic reporting regime it is true that the primary focus of assessing the financial effect of experience has been on the analysis of profit. However, life insurance companies are also required to satisfy the Capital Adequacy Standard which incorporates as its starting point a conservative policy value. Since it is possible that the profit arising from the policy liabilities may follow different patterns to the surplus arising from the capital adequacy requirement, it is conceivable that an actuary may at some stage wish to analysis the conservative surplus as well as the realistic profit. The revised formula presented in this paper may prove a useful starting point for the analysis of the surplus.

References

CARR, P.S et al. (1996) The Practice of Life Insurance in Australia. The Institute of Actuaries of Australia.