

Dynamic asset allocation when bequests are luxury goods

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Abstract

Luxury bequests impart systematic effects of age to an investor's optimal allocation: the expected percentage allocation to equities rises throughout retirement. When bequests are luxuries the marginal utility of bequests declines more slowly than the marginal utility of consumption. This is essentially lower risk aversion. As a retiree approaches death, her expected remaining lifetime utility is increasingly composed of bequest utility, and thus generates progressively lower risk aversion. A retiree responds by increasingly favoring the higher-return risky asset. Compared to standard preferences, luxury bequests elevate a retiree's average exposure to risky assets, but the difference is small in early retirement.

Key words: bequests, luxury goods, dynamic asset allocation, Merton portfolio problem, European put option, retirement risk zone.

JEL classification: D14, G11, G23.

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1. Introduction

This paper offers the first analysis of the implications for dynamic asset allocation of bequests that are luxury goods.¹ Luxury bequests impart systematic effects of age to an investor’s optimal allocation. In particular, a retiree’s percentage allocation to equities rises with age.² By contrast, standard analysis highlights the case of a constant percentage exposure to equity. Sharper still is the contrast between the main result of this paper and a popular rule of thumb that says an investor’s percentage allocation to equity should be set at 100 minus her age, even in retirement. Luxury bequests elevate a retiree’s average exposure to risky assets, compared to the standard case where utility from bequests is treated as having the same power form as utility from non-bequest consumption. However, a numeric exercise suggests that the difference is small in early retirement, consistent with the notion that a retiree should have an upward-sloping equity-age profile.

Intuition for an expected high average exposure to investment risk late in life can be conveyed via a diagram for the stylized case when bequests are pure luxury goods.³ See Figure 1.

The horizontal axis shows wealth taken into retirement, before and after a negative wealth shock. The left-hand vertical axis shows the marginal utility of consumption in retirement. The

¹Atkinson (1971) and Davies (1982) invoke luxury bequests to explain persistent wealth inequality across generations in the face of regression to the mean in earnings. Menchik (1980) estimates that the elasticity of bequests with respect to lifetime resources is 2.5. Carroll (2002) invokes luxury bequests to explain the high average allocation to equity-type assets in the portfolios of the rich. Dynan et al. (2002) and Lockwood (2011, 2012) point out that luxury bequests and precautionary saving together help explain the facts of low voluntary annuitization and low take-up of long-term care insurance, especially by the upper half of the income distribution. They note that low drawdowns of financial assets by many retirees are consistent with this two-part explanation. Wachter and Yogo (2010) find that the share of risky assets in portfolios tends to rise with the investor’s wealth, noting that this is evidence against homothetic preferences such as constant relative risk aversion. De Nardi et al. (2010) and Lockwood (2012) find evidence for luxury bequests based on the Method of Moments.

²Merton (1969, fn5) says that if the bequest function is not of power form then “systematic effects of age will appear in the optimal decision-making.” Our contribution verifies this observation and builds on Merton’s analysis by characterizing the systematic effects of age when bequests are luxury goods.

³ An optimum problem that is consistent with Figure 1 is given by

$$\max_{b,c} -\frac{(\bar{c} - c)^2}{2} + \theta b$$

subject to

$$b + c = w$$

and

$$b, c, w > 0.$$

The notation is: b bequest, c consumption, w wealth taken into retirement, \bar{c} satiety level of consumption and θ marginal utility of bequests (assumed constant).

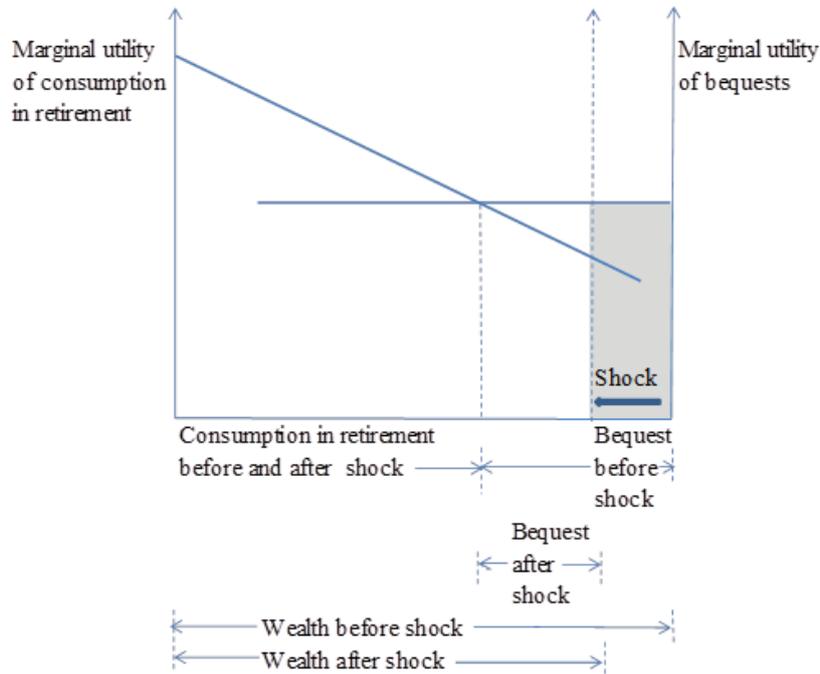


Figure 1: Bequests as shock absorbers.

right-hand vertical axis shows the marginal utility of the planned bequest, portrayed in the figure as a pure luxury good in the sense that the marginal utility of the planned bequest is constant, corresponding to a perfectly elastic demand for bequests with respect to wealth. Following a negative wealth shock, the planned bequest drops by an equal amount. This lowers the investor's welfare; the shaded area shows the welfare reduction. But consumption in retirement stays the same.

Figure 1 also helps with intuition in the case of dynamic asset allocation. When bequests are a luxury good the marginal utility of bequests declines more slowly than the marginal utility of consumption. But this is essentially lower risk aversion. As a retiree approaches death, her expected remaining lifetime utility is increasingly composed of bequest utility, and thus generates progressively lower risk aversion. A retiree responds by increasingly favoring the higher-return risky asset.⁴

⁴We thank a referee for suggesting this intuition for our main result.

2. Model

We embed a bequest function, studied by Carroll (2002), De Nardi *et al.* (2010) and Lockwood (2011, 2012) into the portfolio model introduced by Merton (1969, 1971) and amended by Cox and Huang (1989). The investor makes contingent plans for a bequest $b(T)$, consumption rates $c(t)$, and proportionate investments $x(t)$ in risky assets, that maximize expected utility

$$E\left[\int_0^T e^{-\rho t} \frac{(c(t) - h)^{1-\delta}}{1-\delta} dt + e^{-\rho T} \theta^\delta \frac{(\theta a + b(T))^{1-\delta}}{1-\delta}\right], \quad (1)$$

subject to a budget constraint

$$dw(t) = [(x(t)(\alpha - r) + r)w(t) - c(t)]dt + x(t)w(t)\sigma dz(t), \quad (2)$$

and initial condition

$$w(0) > \frac{h(1 - e^{-rT})}{r}, \quad (3)$$

where the notation is: E expectations operator, T age at death (assumed known), ρ rate of time preference, h nonnegative utility parameter with the interpretation of ‘subsistence’ or ‘protected’ or ‘habitual’ consumption⁵, δ positive utility curvature parameter, $\theta \equiv \phi/(1 - \phi)$ transformation of a utility parameter $\phi \in (0, 1)$ that has the interpretation of “the marginal propensity to bequeath in a one-period problem of allocating wealth between consumption and an immediate bequest” (Lockwood 2012, p. 6)⁶, a nonnegative bequest utility parameter, w wealth, α instantaneous expected return to risky assets, r return to safe assets, σ volatility of risky assets, and dz Wiener increment.

The bequest function in Eq. (1) can be related to Fig. 1. The first derivative of the bequest-

⁵Wachter and Yogo (2010) introduce nonhomothetic preferences by distinguishing between necessities and luxuries in non-bequest consumption, noting but not invoking luxury bequests. We follow Merton (1971) in distinguishing between ‘protected’ and ‘unprotected’ consumption. This also generates a high share of risky assets in the portfolios of the rich, but simplifies analysis as it avoids the complication of a relative price between pure necessities and other goods in non-bequest consumption. (‘Protected’ consumption is a pure necessity as its elasticity of demand with respect to wealth is zero).

⁶See also Footnote 3 above.

utility function with respect to the amount of bequest b is

$$\theta^\delta(\theta a + b)^{-\delta} \geq 0, \tag{4}$$

and the second derivative is

$$-\delta\theta^\delta(\theta a + b)^{-(1+\delta)} \leq 0. \tag{5}$$

It follows from Eq. (5) that the schedule portraying the marginal utility of bequests in Fig. 1 becomes flatter as the bequest-utility parameter a increases. The schedule is completely flat in the limiting case $a \rightarrow \infty$.

3. Solution

We solve the problem (1)–(3) in three steps. Step 1 changes variables and then uses dynamic programming to determine what Cox and Huang (1989) describe as an “unconstrained policy” for investing wealth. Unconstrained dynamic programming does not solve the problem (1)–(3), however, as a consequence of the nonnegative bequest shift parameter a in Eq. (1).⁷ Specifically, a negative bequest could arise if bequests are strong luxuries in the sense that the product term $a\theta$ is high relative to initial wealth net of protected consumption, $w(0) - h(1 - e^{-rT})/r$. Step 2 draws on Cox and Ross (1974) and Cox and Huang (1989) to replicate a European put option that insures against negative bequests. Following Cox and Huang, constrained wealth, i.e. wealth net of the insurance premium, is invested in the unconstrained policy. Step 3 invokes a theorem of Cox and Huang to deduce that we have determined an optimal consumption and investment policy.

Step 1 follows Merton (1971) by changes of variables that map the optimum problem described by Eqs (1) and (2) into the dynamic-programming problem involving constant relative risk aversion

⁷Cox and Huang (1989) emphasize this type of problem, as do Sethi et al. (1992). The fact that Eq. (1) subtracts (nonnegative) protected consumption h from unprotected consumption, together with the power form of the instantaneous utility function, rules out negative non-bequest consumption. This renders our analysis simpler than Cox and Huang’s, which allows negative values of h . Example 8.13 of Karatzas and Shreve (1998, pp. 132–133) solves a problem that looks at first sight to be close to the problem defined by Eqs (1)–(3) (Karatzas and Shreve use a pure martingale method to solve their problem). But its bequest-function shift parameter is of opposite sign to our shift parameter a , corresponding to the case of bequests that are necessities rather than luxuries. Cox and Huang explain why bequest utility functions with first derivatives that go to infinity as bequests go to zero do not need the complication of an option-type fund to rule out negative bequests.

that was posed and solved by Merton (1969), although the changes needed here are more extensive.⁸

Define protected wealth

$$\hat{w}(t) \equiv \frac{h}{r}[1 - e^{-r(T-t)}] - \theta a e^{-r(T-t)}, \quad (6)$$

surplus wealth

$$\tilde{w}(t) \equiv w(t) - \hat{w}(t), \quad (7)$$

transformed bequest

$$\tilde{b}(T) \equiv \theta a + b(T), \quad (8)$$

surplus consumption

$$\tilde{c}(t) \equiv c(t) - h, \quad (9)$$

and surplus investment

$$\tilde{x}(t) \equiv x(t)w(t)/\tilde{w}(t). \quad (10)$$

Step 2 ensures $w(T) \geq 0$, i.e.. $\tilde{w}(T) \geq \theta a$. It values and replicates a put option on an ‘optimally invested’ synthetic security $\tilde{w}_\lambda(t)$, where the terminology and the subscript follow Cox and Huang (1989). Initial surplus optimally-invested wealth, $\tilde{w}_\lambda(0)$, is just small enough to ensure that sufficient wealth remains to guarantee a nonnegative bequest. Remaining initial surplus wealth, $\tilde{w}(0) - \tilde{w}_\lambda(0)$, is invested in a European put option on optimally-invested wealth. The put’s value subsequently is given by

$$p(\tilde{w}_\lambda(t), t) \equiv E_t^Q \max[0, \theta a - \tilde{w}_\lambda(T)], \quad (11)$$

where the superscript Q on the right-hand side denotes the value of an expectation taken under the risk-neutral measure.

The option is self-funding through time, so that surplus wealth is conserved in the sense

$$\tilde{w}(t) = \tilde{w}_\lambda(t) + p(\tilde{w}_\lambda(t), t). \quad (12)$$

⁸Our main novelty in this transformation is the second term on the right-hand side of Eq. (6) below. Ingersoll (1987, p. 246) points out in a setting without bequests that Merton’s problem readily accommodates non-uniformity in the parameter corresponding to h in our setup.

Step 3 notes that Eq. (12) in conjunction with our solutions for optimally-invested surplus wealth and the put option fulfils the conditions for Theorem 2.4 of Cox and Huang (1989), thereby establishing that we have located an optimum.

The solution can be summarized as follows:

Proposition 1

Optimal surplus consumption in the problem described by Eqs (1)–(3) is a time-varying fraction of constrained optimally-invested surplus wealth,

$$\tilde{c}^*(t) = \beta(t)\tilde{w}_\lambda(t), \quad (13)$$

and optimal surplus investment is a constant proportion of constrained optimally-invested surplus wealth,

$$\tilde{x}^*(t) = \frac{\alpha - r}{\sigma^2 \delta}, \quad (14)$$

where

$$\beta(t) = \left[\frac{1 + (\nu\theta - 1)e^{\nu(t-T)}}{\nu} \right]^{-1} \quad (15)$$

$$\nu \equiv \mu/\delta, \quad (16)$$

and

$$\mu \equiv \rho - (1 - \delta) \left[\frac{(\alpha - r)^2}{2\sigma^2 \delta} + r \right]. \quad (17)$$

■

Step 1 of the proof finds values of $\tilde{b}(T)$, $\tilde{c}(t)$ and $\tilde{x}(t)$ that maximize

$$E\left[\int_0^T e^{-\rho t} \frac{\tilde{c}(t)^{1-\delta}}{1-\delta} dt + e^{-\rho T} \theta^\delta \frac{\tilde{b}(T)^{1-\delta}}{1-\delta}\right], \quad (18)$$

subject to

$$d\tilde{w}(t) = [(\tilde{x}(t)(\alpha - r) + r)\tilde{w}(t) - \tilde{c}(t)]dt + \tilde{x}(t)\tilde{w}(t)\sigma dz(t). \quad (19)$$

At this point we need not specify whether surplus wealth is constrained or unconstrained, i.e., gross or net of the insurance package. Eq. (18) is the same as Eq. (1) and Eq. (19) is the same as Eq. (2). Solving the optimum problem described by Eqs (1) and (2) is therefore the same as solving the optimum problem described by Eqs (18) and (19). Merton (1969, Sections 3 and 4) solves the latter problem by means of dynamic programming and begins by defining a value function. Our luxury-bequest counterpart is

$$J(\tilde{w}(t), t) \equiv \max_{\tilde{c}(t), \tilde{x}(t)} E_t \left[\int_t^T e^{-\rho s} \frac{\tilde{c}(s)^{1-\delta}}{1-\delta} ds + e^{-\rho T} \theta^\delta \frac{\tilde{w}(T)^{1-\delta}}{1-\delta} \right]. \quad (20)$$

Following Merton (1969, Section 4), the associated Hamilton-Jacobi-Bellman equation, upon plugging in the first-order conditions

$$\tilde{c}(t) = (e^{\rho t} J_{\tilde{w}})^{-1/\delta}, \quad (21)$$

and

$$\tilde{x}(t) = (-J_{\tilde{w}} / \tilde{w} J_{\tilde{w}\tilde{w}})(\alpha - r) / \sigma^2, \quad (22)$$

is given by

$$0 = \frac{\delta}{1-\delta} e^{-\rho t} (J_{\tilde{w}})^{\frac{\delta-1}{\delta}} + J_t + r\tilde{w}J_{\tilde{w}} - \frac{1}{2} \frac{(\alpha - r)^2}{\sigma^2} \frac{J_{\tilde{w}}^2}{J_{\tilde{w}\tilde{w}}}, \quad (23)$$

with boundary condition

$$J(\tilde{w}(T), T) = e^{-\rho T} \theta^\delta \frac{\tilde{w}(T)^{1-\delta}}{1-\delta}. \quad (24)$$

Merton (1969) shows that the solution of Eqs (23)–(24) is

$$J(\tilde{w}(t), t) = \frac{\beta(t)}{1-\delta} e^{-\rho t} \tilde{w}(t)^{1-\delta}. \quad (25)$$

Eqs (13) and (14) follow from Eqs (21), (22) and (25), with $\tilde{w}(t)$ replaced by $\tilde{w}_\lambda(t)$.

Steps 2 uses the Black-Scholes-Merton formula, in conjunction with the risk-neutral valuation argument of Cox and Ross (1976), to replicate a put option with strike θa on constrained optimally-invested surplus wealth $\tilde{w}_\lambda(t)$. Eqs (13), (14) and (19) show that the stochastic process for constrained optimally-invested wealth has deterministic drift $\tilde{x}^* \alpha + (1 - \tilde{x}^*) r - \beta(t)$ and constant

volatility $\tilde{x}^*\sigma$:

$$d\tilde{w}_\lambda(t) = [\tilde{x}^*(\alpha - r) + r - \beta(t)]\tilde{w}_\lambda(t)dt + \tilde{x}^*\sigma\tilde{w}_\lambda(t)dz, \quad (26)$$

where \tilde{x}^* is pinned down by Eq. (14). Here $\beta(t)$ is like a deterministic (as distinct from fixed) continuous payout from a commodity corresponding to the synthetic risky asset $\tilde{w}_\lambda(t)$, analogous to a variable-proportion storage cost. The risk-neutral specialization of the process defined by Eq. (26) has instantaneous return r and (constant) instantaneous volatility $\tilde{x}^*\sigma$. Standard theory says that replicating the option specified by Eq. (11) with this asset and the safe asset requires going long by an amount

$$N(-d_2)\theta a e^{-r(T-t)} \quad (27)$$

in the safe asset, and short an amount

$$N(-d_1)\tilde{w}_\lambda(t)e^{-\int_t^T \beta(s)ds} \quad (28)$$

in the synthetic risky asset, where

$$d_1 = \frac{\ln\left(\frac{\tilde{w}_\lambda(t)}{\theta a}\right) + \left[r + \frac{(\sigma\tilde{x}^*)^2}{2}\right](T-t) - \int_t^T \beta(s)ds}{\sigma\tilde{x}^*\sqrt{T-t}}, \quad (29)$$

$$d_2 = d_1 - \sigma\tilde{x}^*\sqrt{T-t}, \quad (30)$$

and $N(\cdot)$ denotes the Normal distribution.⁹ Replicating the put with the underlying risky asset at time t therefore requires going short an amount

$$\tilde{x}^*(t)N(-d_1)\tilde{w}_\lambda(t)\theta a e^{-\int_t^T \beta(s)ds} \quad (31)$$

in the underlying risky asset.

Step 3 invokes Theorem 2.4 of Cox and Huang (1989) to deduce that the solution characterized by Eqs (12), (13), (14), (27), (28), (29) and (30) is indeed an optimum.

⁹We have constructed a spreadsheet (available on request) to confirm numerically that Eqs (27) to (30) do in fact replicate in expectation the required put, and use these data to help draw Fig. 2 below.

4. Asset allocation

From Eqs (12), (14) and (31), followed by application of Eqs (6) and (7) to substitute out $\tilde{w}^*(t)$, the optimal dollar investment $A^*(t)$ in risky assets is

$$A^*(t) = \tilde{x}^*(t)\tilde{w}_\lambda(t) - \tilde{x}^*(t)\text{N}(-d_1)\tilde{w}_\lambda(t)e^{-\int_t^T \beta(s)ds} \quad (32)$$

$$= \tilde{x}^*(t)[\tilde{w}^*(t) - p(\tilde{w}_\lambda(t), t) - \text{N}(-d_1)\tilde{w}_\lambda(t)e^{-\int_t^T \beta(s)ds}] \quad (33)$$

$$= \left(\frac{\alpha - r}{\delta\sigma^2}\right) [\tilde{w}^*(t) - \text{N}(-d_2)\theta a e^{-r(T-t)}] \quad (34)$$

$$= \left(\frac{\alpha - r}{\delta\sigma^2}\right) \left[w(t) - \frac{h}{r}(1 - e^{-r(T-t)}) + \theta a e^{-r(T-t)}(1 - \text{N}(-d_2)) \right]. \quad (35)$$

Divide Eq. (35) through by $w(t)$ to arrive at our main result:

Proposition 2

The optimal proportionate investment $x^(t)$ in risky assets, in terms of the model's state variable and parameters, is given by*

$$x^*(t) = \left(\frac{\alpha - r}{\delta\sigma^2}\right) \left[1 - \frac{h}{rw(t)}(1 - e^{-r(T-t)}) + \frac{\theta a}{w(t)}e^{-r(T-t)}(1 - \text{N}(-d_2)) \right]. \quad (36)$$

In contrast to protected consumption without a luxury bequest ($h > 0$ and $a = 0$), luxury bequests ($a > 0$) elevate exposures to risky assets, at any point in time and for a given level of wealth. Like protected consumption, however, luxury bequests impart an upward tilt to the expected trajectory of proportionate investment in risky assets, again for a given level of wealth.

The right-hand side of Eq. (36) consists of three terms. The first, i.e., $(\alpha - r)/\delta\sigma^2$, is familiar from Merton (1969). The second was introduced by Merton (1971). Its implications for dynamic asset allocation are discussed by Ingersoll (1987) and Karatzas and Shreve (1998), among others. Ingersoll offers the useful analogy of an 'escrow' account, comprised of safe securities, set up at time zero, and then run down gradually, until time T . The third term is the main contribution of this paper. Cox and Huang (1988) give several worked-out examples containing option-related

components in their solutions. None of those examples addresses luxury bequests, however, although that case is implicitly covered by Cox and Huang’s Theorem 2.4.¹⁰ Carroll (2002) gives theory and evidence in support of the proposition that luxury bequests raise the average level of risky assets in portfolios, without considering dynamic asset allocation.

5. Numeric illustration

Figure 2 illustrates the effect of luxury bequests on expected asset allocation for a particular initial value of wealth and a particular set of model parameters.¹¹ The dashed line portrays the standard homothetic case (i.e., protected consumption is zero) where utility from bequests has the same power form as utility from consumption. The expected and actual share of risky assets is 38 per cent of the portfolio. The solid line portrays the case of zero protected consumption along with a positive shift parameter in the function describing utility from bequests (thereby rendering them a luxury good) and a synthetic put option on optimally-invested wealth. The dotted line strips out the effect of our synthetic put on optimally-invested wealth, thereby shedding light on the empirical importance of looking beyond the solution resulting from unconstrained dynamic programming.

At the initial age of 65, and in the case of the solution that rules out negative bequests (i.e., the solution that incorporates a synthetic put option), the expected and actual share of risky assets is 40 per cent, or just 2 percentage points higher than in the standard case of a bequest function of power form. So our example suggests that at the outset of retirement it is not important in practice to account for luxury bequests when allocating assets.

Had we disregarded the need to rule out nonnegative bequests, however, the allocation to risky assets would have been appreciably higher at the outset of retirement. This difference is consistent with the fact that the required synthetic put has considerable time value at the outset of retirement.

¹⁰A natural question regarding Eq. (36) is its extension to the case of uncertain lifetimes. As Cox and Huang’s Theorem 2.4 is invoked in the present paper, one would need to extend that theorem to the case of uncertain lifetimes. Even with certain lifetimes the derivation of Theorem 2.4 is lengthy—the natural way to extend it would be to introduce a complete market for mortality-contingent securities. Accordingly, this paper leaves the case of uncertain lifetimes to future research.

¹¹ In Fig. 2, initial wealth=\$600,000, initial age=65, final age=100, rate of time preference=2% p.a., real interest rate=2% p.a., utility curvature parameter=2, expected return to risky assets=5% p.a., volatility of risky assets =20% p.a., bequest utility parameter = \$20,400, and propensity to bequeath = 0.92. The latter two values are from Lockwood (2012, Table 3). Protected consumption is zero.

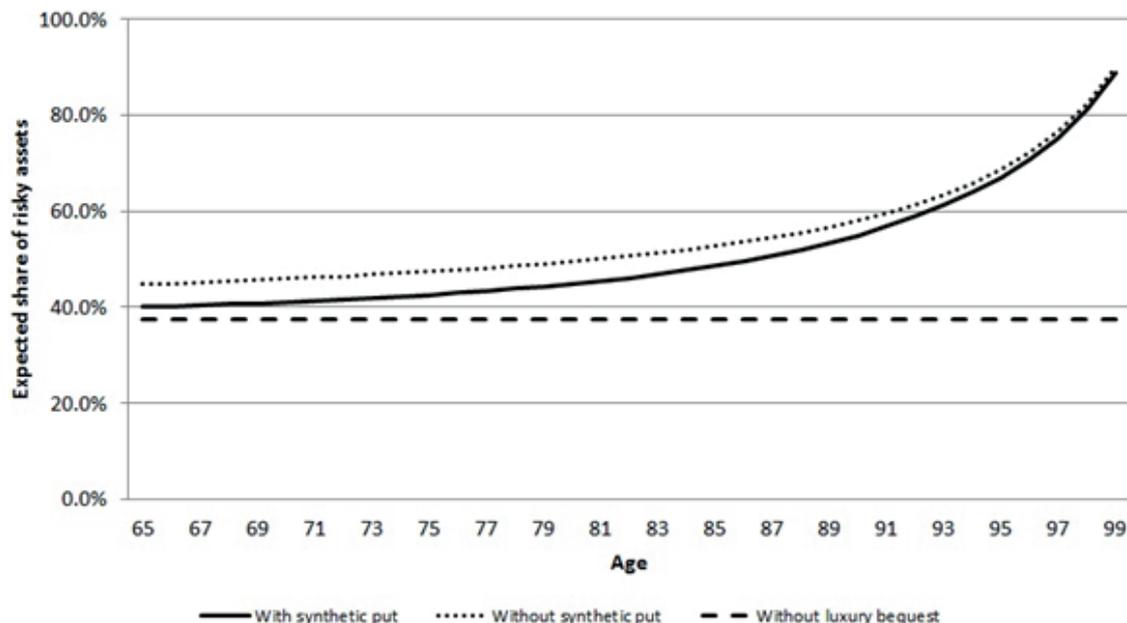


Figure 2: Risky assets share with and without luxury bequests. *Note:* assumed parameter values and initial wealth are detailed in Footnote 11.

At the final age of 100, and in the case of the solution that rules out negative bequests, the expected share of risky assets is 90 per cent, or 52 percentage points higher than for the standard case of a bequest function of power form. The expected bequest amount is 22 per cent of the wealth taken into retirement. In our example, then, assuming bequests are luxury goods makes a big difference to asset allocation late in retirement even though the planned bequest is not particularly big. On the other hand, the synthetic put makes scarcely any difference to asset allocation late in retirement, consistent with decay over time in its value.

6. Implications for investment advice

The standard case of constant relative risk aversion assumes preferences are homothetic, implying that advice on asset allocation is scalable. Luxury bequests are at odds with the assumption of homothetic preferences, in this way strengthening the case for customized advice. Notably, rich investors with bequest motives may need a more aggressive allocation, in line with Carroll's (2002) observations about actual allocations.

Concerning financial plans for people of middle means, Bengen (2001) and Milevsky and Sal-

isbury (2006) explore numerically the notion of a ‘retirement risk zone’ whereby allocations at the outset of retirement need to be conservative. Notably, if there is a bear market on the cusp of a retirement then it is difficult to recoup even if investment markets recover subsequently, as ongoing drawdowns deplete remaining wealth. Entering retirement with a high present value of protected consumption relative to wealth is one theoretical justification for a conservative allocation early on.¹² We showed that luxury bequests justify aggressive allocations later on, in this way strengthening the case for an upward tilt throughout retirement in an investor’s expected percentage exposure to equities.

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¹²Ingersoll (1987) and Karatzas and Shreve (1998) make this point.

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