A Flexible Approach to Modeling Ultimate Recoveries on Defaulted Loans and Bonds

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Research Objectives

1. To characterise the distribution of recoveries on defaulted debt as a finite mixture of distributions that:
   1.1 accommodates features of empirical recovery distributions;
   1.2 enables adaptation to conditioning information
2. Apply the mixture framework to evaluate:
   2.1 the impact of conditioning information on the shape of recovery distributions;
   2.2 the out-of-sample performance of mixture estimates (time and cross-sectional variation)

Why are these Questions Interesting/Important?

1. The estimation of recoveries is equally as significant to the accuracy of default risk model as default likelihood
2. Important features of the empirical data challenge standard methods

Stylised Facts

1. Bi-modality: renders the average misleading. Hlawatsch and Ostrowski [2011] summarise findings from previous studies:

Time Span	Left Mode	Right Mode	Description
1970-2003	0.1-0.3	0.7-0.9	Bonds and Loans
1982-99	0.05-0.15	1.0-1.1	Commercial Loans
1993-2004	0-0.1	0.9-1.1	Vehicle Leases
1993-2004	0-0.2	0.8-1.0	IT Leases
1995-2000	0-0.05	0.95-1.0	SME Loans
1980-2004	0-0.1	0.9-1.0	Loans
1970-96	0-0.15	0.95-1.0	Comm. & Ind. Loans
1985-2006	0-0.05	0.95-1.0	Bonds and Loans

(Empirical Modes of LGD Densities)

Stylised Facts

2. Collateralisation, degree of subordination or the value of claimants to subordinate to a given debt claim *Debt Cushion* (Keisman and Van de Castle [1999]) matter
3. Recoveries are lower in recessions, when the rate of defaults increase (Frye 2000, Altman, Brady, Resti and Sironi [2005]):
   - Altman et al show that the default rate on high yield debt explains up to 65% of the variation in *average* contemporaneous recovery rates

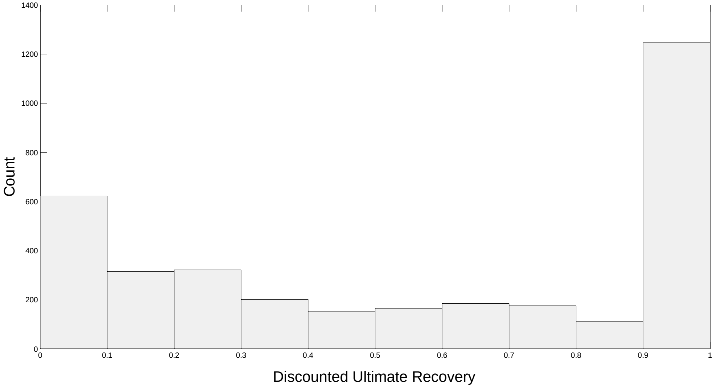
Stylised Facts

4. Macroeconomic effects may not be significant after accounting for industry effects (Acharya, Bharath, Srinivasan [2007])
5. Intra-class variability of recoveries is high, even within sub-portfolios of like subordination and collateralisation (Schuermann [2005])

Ultimate Recoveries on Defaulted Loans & Bonds

- Data source: Moody's Ultimate Recoveries Database (2007)
- Discounted value of instruments at the time of settlement or emergence from Chapter 11 proceedings discounted at the pre-petition coupon rate
- Reported numbers are those considered by Moody's to be most representative of economic value

Ultimate Recoveries on Defaulted Loans & Bonds



Modeling Responses

Examples include:

- **Parametric:** Beta distributions (commercial models such as KMV Portfolio Manager, CreditMetrics), Regressions (Acharya et al [2007])
- **Semi-Parametric:** Mixtures of Beta Distributions (Hlawatsch and Ostrowski [2011])
- **Non-Parametric:** Neural Networks, Regression Trees, (Qi and Zhao [2011]) Kernel Density Estimation (De Servigny and Renault [2004]

NOTE: the site www.defaultrisk.com lists 117 papers on recoveries!

Current Approach

1. Recoveries are usually bounded by the unit interval, so:
   1.1 We constrain recoveries to the interval (0,1)
   1.2 Map the constrained series to the real number line using a inverse CDF of a Student T with $v = 20$
2. We model the distribution of transformed recoveries $g(y)$ using a discrete mixture of m normal (Gaussian) components:

$$g(y) \approx \hat{g}(y) = \sum_{j=1}^{m} p_j f(y|\theta_j)$$

where $f(y|\theta_j)$ are the component distributions and p_j the (posterior) probability weightings.

Likelihood Form

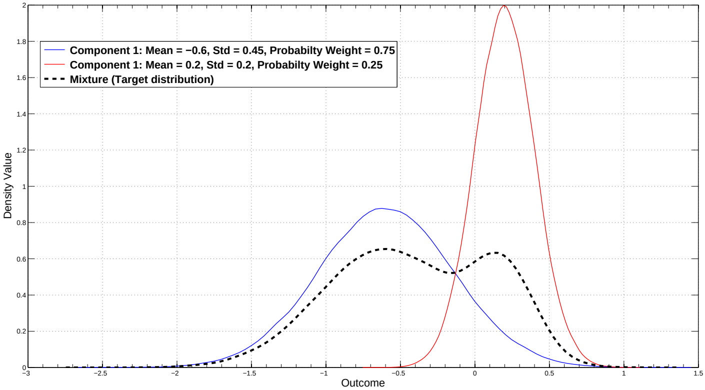
$$P(y|x,\beta,h,\alpha,p) = \frac{1}{(2\pi)^{\frac{N}{2}}} \prod_{i=1}^{N} \left\{ \sum_{j=1}^{m} p_j \sqrt{h_j} \exp\left[-\frac{h_j}{2} (y_i - \alpha_j)^2 \right] \right\}$$

Or equivalently,

$$P(y|x,\beta,h,\alpha,e) = \frac{1}{(2\pi)^{\frac{N}{2}}} \prod_{i=1}^{N} \left\{ \sum_{j=1}^{m} e_{ij} \sqrt{h_j} \exp\left[-\frac{h_j}{2} (y_i - \alpha_j)^2 \right] \right\}$$

where e_{ij} is a latent indicator variable that assigns observations i to mixture components j.

A Two-Component Example



Current Approach: Immediate Issues

1. How to simultaneously estimate the parameters of the component distributions and the probability weightings?
   - We take a Bayesian perspective and use Gibbs Sampling
2. How to determine the optimal number of mixture components?
   - Our choices are guided by model selection criteria
3. How to handle conditioning information?
   - Estimate the unconditional distribution and infer the effects of conditioning information
     OR
   - Explicitly model the dependence of component distributions and/or probability weights on conditioning information

Estimation Approach: Gibbs Sampling

- We use Gibbs sampling to characterise the posterior distribution of the parameters
- The approach requires us to draw successively from the marginal posterior distribution of each parameter, conditional on the current (sampled) values of all other parameters
- This is easy to do – especially when the requisite distributions are standard (as in the current case)

Estimation Approach: Gibbs Sampling

For example, in the case of the two-component normal mixture we would cycle through the following scheme many times:

1. Draw: $\alpha | h, p, e, y \sim TruncatedNormal$
2. Draw: $h | \alpha, p, e, y \sim Gamma$
3. Draw: $p | \alpha, h, e, y \sim Dirichlet$
4. Draw: $e | \alpha, h, p, y \sim Multinomial$
5. Back to step 1

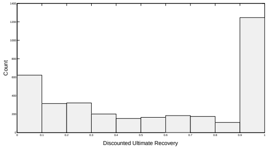
- In each step the new parameter draw is conditioned on the most recent draw of the other parameters.
- Provided some (generally innocuous) assumptions are not violated, the sampling scheme yields draws from $\alpha, h, e, p, | y$.

Unconditional Estimates

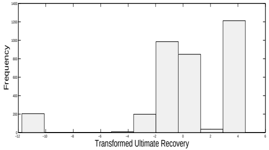
Table: Mixture Components: Summary Statistics

Mixture Component			
$i =$	1	2	3
Component Distributions: Normal Parameters			
$E(\alpha_i \mid y)$	-11.74	-0.55	4.54
$E(h_i \mid y)$	0.007	1.06	0.003
Component Distributions: Mapped to Recoveries			
Mean Recovery	0%	35.18%	100%
Median Recovery	0%	30.20%	100%
IQR	0%	46.40%	0%
Std Dev	0%	27.50%	0%
Unconditional Posterior Probability Weights			
$E(p_i \mid y)$	6%	59%	35%
$\sigma(p_i \mid y)$	0.4%	0.80%	1%

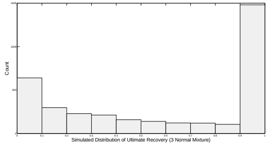
Unconditional Estimates



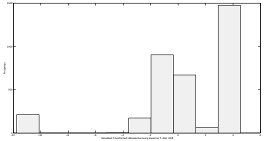
(a) Discounted Ultimate Recovery



(b) Transformed Data



(c) Discounted Ultimate Recovery (Simulated)



(d) Transformed Data (Simulated)

Inferring the Effect of Conditioning Information

- Recall that observation i is associated with mixture component j through e_{ij}
- To infer the effect of conditioning information we compute:

$$\hat{p}_{Qj} = \sum_{g=1}^{G} \frac{e_{Qj}^{[g]}}{n(Q)G}$$

  - where e_{Qj} denotes all e_{ij} such that $i \in Q$
  - G is the total number of post burn-in iterates from the Gibbs sampler
  - $n(Q)$ is the number of observations in Q.

In English: We are computing the proportion of draws associated with observations falling into a category (e.g. Subordinated bonds with no collateral) assigned to each mixture component j

Representative Finding

Table: Mixture Probabilities based on Debt Cushion and Industry Default Conditions

		Debt Cushion (DC)		
		$DC < 0.25$	$0.25 \leq DC < 0.75$	$0.75 \leq DC$
Industry Distress $\geq +1.7\sigma$	p1	0.10	0.01	0.02
	p2	0.79	0.68	0.33
	p3	0.11	0.31	0.65
Industry Distress $< +1.7\sigma$	p1	0.09	0.01	0.00
	p2	0.71	0.62	0.19
	p3	0.20	0.38	0.81

Out-of-Sample Performance?

- To evaluate the predictive value of the approach, we conduct the following resampling experiment:
  1. We **split the sample**. 1477 observations up to and including 2001 are set aside as an estimation sample and the remaining 2015 (5 years of data) comprise the test sample.
  2. We **draw a random set of 150 recoveries** on defaulted loans and bonds from the test sample and compute the ultimate recovery on an equally-weighted portfolio of the selected exposures. This value is stored as an outcome of the empirical loss distribution.
  3. We then **draw and store a predicted portfolio loss outcome** from each of the following models, estimated using the estimation sample:

Out-of-Sample Performance?

1. *3-Mix Comp:* Mixture components are re-weighted based on the Debt Cushion associated with the sampled exposures as well as the expectations of default in the borrowers' industries at the time of default.
2. *3-Mix Base:* is a draw from the posterior recovery distribution based on the 3-mixture specification.
3. *Regression:* is a draw from the posterior distribution of recoveries implied by a single mixture regression model incorporating all the the conditioning variables.
4. *Beta Comp:* is a draw from the Beta distribution calibrated to the subset of outcomes observed during the estimation period that match the sampled observations in terms of their Debt cushion category and industry default expectations.

Performance Evaluation: Portfolio Recoveries

	Actual	*3-Mix Comp*	*3-Mix Base*	*Regress*	*Beta Comp*
Median	83.8	87.4	90.6	80.6	71.7
Std	4.7	4.9	4.9	5.3	5.0
IQR	6.4	6.5	6.7	7.1	6.7
Lower Tail Percentile Cut-Offs					
5%	76.0	79.2	82.6	71.6	63.6
2%	74.0	77.2	80.5	69.3	61.5
1%	72.9	76.0	79.1	67.8	60.2
0.50%	71.5	74.7	77.8	66.6	58.9
0.10%	69.4	71.7	75.1	64.5	56.7
% Absolute Error in Estimation: Lower Tail Percentiles					
5%	-	4.2%	8.7%	5.8%	16.3%
2%	-	4.3%	8.7%	6.3%	16.9%
1%	-	4.4%	8.5%	7.0%	17.3%
0.50%	-	4.4%	8.8%	6.9%	17.7%
0.10%	-	3.4%	8.2%	7.0%	18.3%

Parametric Conditioning

- Two approaches:
  1. Model as a regression and the errors as a mixture
  2. Model explicit links between conditioning information and mixture assignments
- Our empirical findings strongly suggest that conditioning information is best used to capture shifts in the probability of assignment to mixture components rather than variation in the mean

Parametric Conditioning (3-Mixture Case)

We re-specify the likelihood as follows:

$$
\begin{aligned}
p(y_t|x_{t-1},\theta,z_t^*) &= \phi(y_t;\alpha_1,\sigma_1^2)^{I(c_0<z_t^*\leq c_1)}\phi(y_t;\alpha_2,\sigma_2^2)^{I(c_1<z_t^*\leq c_2)} \\
&\ldots \quad \phi(y_t;\alpha_3,\sigma_3^2)^{I(c_2<z_t^*\leq c_3)},
\end{aligned}
$$

$$
z_t^* = \beta_0 + \beta_1 x_{t-1} + \epsilon_t, \qquad \epsilon_t \overset{iid}{\sim} N[0,1].
$$

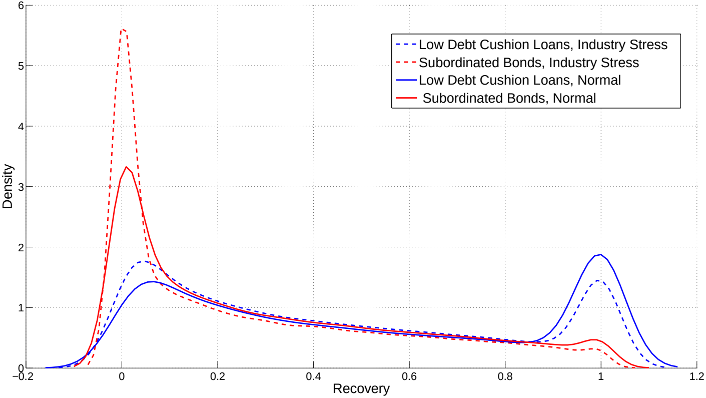
where $\alpha_1 < \alpha_2 < \alpha_3$, and the cut-points $c_0 = -\infty$, $c_1 = 0$, and $c_3 = +\infty$ are set to enable unique identification.[1]

[1]The Gibbs sampler now incorporates steps to draw: β, the latent variable z^* and the cut point c_2 – with normal, truncated normal and uniform posteriors respectively.

Parametric Conditioning: Indicative Result

	$\bar{\beta}$	$\sigma(\beta)$
Constant	2.39	0.09
Debt Cushion	1.82	0.10
1yr $<$ Time in Default $\leq$ 2yr	-0.20	0.06
2yr $<$ Time in Default $\leq$ 3yr	-0.28	0.08
3yr $<$ Time in Default	0.50	0.08
Rank 2	-0.27	0.06
Rank 3	-0.61	0.09
Rank $\geq$4	-0.63	0.13
No Collateral	-0.40	0.11
Default and Cure	3.01	0.42
Other Restructure	1.10	0.09
Distressed Exchange	0.68	0.30
Junior	-0.75	0.21
Revolver	0.31	0.08
Senior Secured	-0.29	0.09
Senior Subordinated	-0.42	0.14
Subordinated	-0.61	0.14
Industry Default	-0.23	0.02

Parametric Conditioning: Indicative Result



NOTE: Estimation Risk Ignored in this Example!

Summary

Our mixture based approach to modelling recoveries provides:

1. Flexibility in accommodating the shape of the recovery distribution and empirical features
2. A rich set of economically interesting data
3. Good out of sample performance relative to popular alternatives

We are currently:

- Updating our empirical findings using the latest version of the Moody's database using our new approach to conditioning information