Risk, Restructuring, and Investing in Distressed Mortgage Debt

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Collaborators:

Seoyoung Kim (Purdue University)
Ray Meadows (The Recovery Company)

@Financial Risk Day, Sydney, 30\textsuperscript{th} March, 2012.

Papers at http://algo.scu.edu/~sanjivdas/research.htm
Outline

1. A model for restructuring distressed mortgages (the Principle Principal)

2. A reduced-form model with closed-form pricing for fast implementation to stem strategic default (Strategic Loan Modification)

3. An investment model for yield pick up and risk parameterization (Optimizing Restructured Debt Portfolios)
Landscape of Failure: Dec 2011

Foreclosure Rate Heat Map
What are new foreclosures as a percentage of the housing market?
More info

1 in every 634 housing units received a foreclosure filing in December 2011

Foreclosure Actions to Housing Units

1 in 177 Housing Units

1 in 316,435 Housing Units

High
Med
Low
Statistics

• 1 in 380 households received a foreclosure filing in August 2010 (RealtyTrac).
• Sep 2010: pre-sale foreclosure inventory 2.038 million homes (LPS).
• Sep 2010: 4.9 million mortgages 30-days overdue, 2.374 million 90-days past due (LPS).
• Sep 2010: 11 million borrowers (23% of households with a mortgage) have negative equity (CoreLogic).
• Jan 2011: 1.24 million non-HAMP mods in 2010 (HOPE NOW).
• Feb 2011: 87,083 mods in one month (183,241 HAMP mods in first half of 2011).
• May 2011: 28.4% single family homes with negative equity.
• May 2011: 20-city Case-Shiller down 30% from Jan 2006 (33% Jan 2012).
• July 2011: 1 in 611 homes received a foreclosure filing (improvement).
• Nov 2011: Case-Shiller down 3.7% year over year.
• Q4 2011: Residential investment 2.5% of GDP (6.3% in 2005).
• Jan 2012: US Q3 house prices forecast -2.7%, +3.8% by Q3, 2013 (Fiserv).
## HAMP Activity: First Lien Modifications

HAMP is designed to lower monthly mortgage payments to help struggling homeowners stay in their homes and prevent avoidable foreclosure.

<table>
<thead>
<tr>
<th>HAMP Eligibility (As of Oct. 31, 2011)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eligible Delinquent Loans¹</td>
<td>2,539,502</td>
</tr>
<tr>
<td>Eligible Delinquent Borrowers²</td>
<td>891,542</td>
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### Trial Modifications

<p>| | |</p>
<table>
<thead>
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<tr>
<td>Trial Plan Offers Extended (Cumulative)³</td>
<td>1,984,196</td>
</tr>
<tr>
<td>All Trials Started</td>
<td>1,754,516</td>
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<tr>
<td>Trials Reported Since October 2011 Report⁴</td>
<td>19,059</td>
</tr>
<tr>
<td>Trial Modifications Canceled (Cumulative)⁵</td>
<td>764,340</td>
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<tr>
<td>Active Trials</td>
<td>80,223</td>
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### Permanent Modifications

<p>| | |</p>
<table>
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</thead>
<tbody>
<tr>
<td>All Permanent Modifications Started</td>
<td>909,953</td>
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<tr>
<td>Permanent Modifications Reported Since October 2011 Report</td>
<td>26,877</td>
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<tr>
<td>Permanent Modifications Canceled (Cumulative)⁵</td>
<td>159,205</td>
</tr>
<tr>
<td>Active Permanent Modifications</td>
<td>750,748</td>
</tr>
</tbody>
</table>

¹Estimated eligible 60+ day delinquent loans as reported by servicers as of October 31, 2011, include conventional loans:
- In foreclosure and bankruptcy.
- With a current unpaid principal balance less than $729,750 on a one-unit property, $984,200 on a two-unit property, $1,129,250 on a three-unit property and $1,403,400 on a four-unit property.
- On a property that was owner-occupied at origination.
- Originated on or before January 1, 2009.

²Estimated eligible 60+ day delinquent loans exclude:
- FHA and VA loans.
- Loans that are current or less than 60 days delinquent, which may be eligible for HAMP if a borrower is in imminent default.

³The estimated eligible 60+ day delinquent borrowers are those in HAMP-eligible loans, minus estimated exclusions of loans on vacant properties, loans with borrower debt-to-income ratios below 31%, loans that fail the NPV test, properties no longer owner-occupied, unemployed borrowers, manufactured housing loans with title/chattel issues that exclude them from HAMP, loans where the investor pooling and servicing agreements preclude modification, and trial and permanent modifications disqualified from HAMP. Exclusions for DTI and NPV results are estimated using market analytics.

⁴As reported in the monthly servicer survey of large SPA servicers through November 30, 2011.

⁵Servicers may enter new trial modifications into the HAMP system of record at any time.

Source: HAMP system of record. Servicers may enter new trial modifications into the HAMP system of record since the prior report. 12,877 were trials with a first payment recorded in November 2011.

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Note: Unless specified, exhibits in this report refer to HAMP first lien modification activity.
CitiMortgage CEO Sanjiv Das helps people keep their homes

By Stephanie Armour, USA TODAY
April 27, 2009

There is very little in Sanjiv Das' uncluttered office.

Just snapshots of his wife and his 20-year-old daughter, Natasha. No pictures of his favorite sports: golf and cricket.

Das has moved around the world, run credit card acquisitions for American Express in India, and handled the mortgage business for Citibank in Sydney. But none of it compares to what he's doing now.
I apologize for sending this email via your Santa Clara University email address. It was the only contact information I could locate for you. I understand that you are the Resolution Specialist at CitiMortgage, Inc. Our office is having difficulty obtaining information from CitiFinancial in regards to consumer complaints filed with our office relating specifically, when the Resolution Specialist or Executive Response Unit met to consumer complaints, the responses often fail to address the consumer's concern. I would like to follow up with the authors of the responses, they either fail to return our messages or are not informed on the details of the file and thus cannot provide us with even basic account information.

Would it be possible to set up a conference call to further discuss these issues?

Thank you,

Lisa S. Wolf
Deputy Attorney General
Office of the Attorney General
Consumer Protection Division
Homeowner Protection Unit
302 W. Washington St. 5th Floor
Indianapolis, IN 46204

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"You do that by integrity, and leadership is at the heart of my values," says. "The No. 1 thing I talk about are the customers. Each day, my business is to keep them in their homes, no matter what. That goes back to the values I was brought up with."

Mayor Eric J. Brewer - CitiMortgage's foreclosure rescue plan - I'd like to implement in my city

A message from: Mayor Eric J. Brewer
Mr. Das
I am the Mayor of the City of East Cleveland, Ohio, one of the cities hit hardest by foreclosure in this area. Today I read an article that you have initiated a foreclosure rescue plan. I held a foreclosure prevention summit last year with 200 attendees to help people keep from losing their homes. I'd like to work with CitiMortgage to get the word out in my area. My cell is 216-310-1110. I don't answer restricted calls as a mayor. I'd appreciate the opportunity to work with CitiMortgage in helping my residents and more. Please call.

Mayor Eric J. Brewer
City of East Cleveland
14340 Euclid Avenue
East Cleveland, OH 44112
Office: 216-681-2208
Executive Asst: Ms. Kim Woodson
Website: eastcleveland.org

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Mark Perry sent you a message on Facebook...

From: Facebook <notification@heovdhe@facebook.com>
Reply-To: noreply <noreply@facebook.com>
Date: 6/2/2009 10:29 AM
To: Sanjiv Das

--

Subject: Help

You do that by integrity, and leadership is at the heart of my values," says. "The No. 1 thing I talk about are the customers. Each day, my business is to keep them in their homes, no matter what. That goes back to the values I was brought up with."
A tale of two Das: Citi CEO, academic and mortgages

NEW YORK (Reuters) - Sanjiv Ranjan Das, a professor at California's Santa Clara University, last fall attacked the problem of "underwater" mortgages often cited as an Achilles' heel to the U.S. housing market.

HOUSING MARKET

He had a special ran: Sanjiv Das, the top executive at CitiMortgage, the nation’s fourth-largest home loan lender and servicer of $723 billion in mortgages.
Preview of Some Results

• Rate reductions are value-destroying in negative equity situations.
• Maturity extensions also destroy value.
• Principal reductions are optimal.
• Capitalization of payments (forbearance) into back-ended principal will also destroy loan value.
• Shared-appreciation mortgages improve ability to pay, mitigate moral hazard.
• Optimal modifications may be computed in closed-form in a reduced-form model.
• Restructuring returns yield investors hundreds of basis points in certainty equivalents.
• Optimization problems extend mean-variance models in two ways: (a) return distributions are endogenous; (b) and are sharply non-normal.
Who Cares?

• Borrowers
• Lenders
• Regulators
• Investors
Figure 1: Negative Equity By CBSA
(When Marker Increases Past Zero = Positive Equity)
HAMP UPDATE

June 4, 2010

Announcing HAMP Principal Reduction Alternative

Yesterday, June 3, 2010, Supplemental Directive 10-05: Modification of Loans with Principal Reduction Alternative, was issued offering mortgage relief to eligible homeowners whose homes are worth significantly less than the remaining amounts owed under their first lien mortgage loans. The Principal Reduction Alternative (PRA) guidance applies to non-GSE loans eligible for the Home Affordable Modification Program (HAMP) only.

Principal Reduction Alternative (PRA)

With this new guidance, servicers are required to evaluate all HAMP-eligible loans with a mark-to-market loan-to-value (MTMLTV) greater than 115% to determine if a principal reduction is beneficial. If the evaluation shows the net present value (NPV) for a HAMP modification using PRA is positive, servicers are encouraged to offer the principal reduction to the borrower. An updated NPV model reflecting principal reduction will be available to use for this evaluation. Additional details are as follows:

- **Effective Date** -- The PRA Effective Date (i.e., the date the principal reduction evaluation is required) will be either October 1, 2010, or the date of the HAMP NPV Model 4.0 release (whichever is later). However, servicers may immediately offer PRA for HAMP-eligible modifications as long as the reduction follows all PRA requirements.
- **Application** -- PRA is earned over a three-year period and is initially treated as a PRA Forbearance. Each year (for three years) that the borrower is in good standing on the anniversary of their trial period effective date, one-third of the original PRA forbearance amount will be reduced. This reduced amount will be applied to their unpaid principal balance.
- **Second Lien** -- Servicers participating in the Second Lien Program (2MP) will be required to provide a principal reduction on the borrower’s second mortgage in proportion to any principal reduction offered on the borrower’s first mortgage.
- **Investor Incentive** -- Investors will receive an incentive based on loan delinquency, LTV ratio, and the amount of the principal reduction. Note: Guidance on principal reduction and related investor incentives will be forthcoming for loans in active HAMP Trial Period Plans or that were permanently modified prior to June 3, 2010 (i.e., the SD 10-05 effective date).
Value Driver #1

Deadweights costs of foreclosure. Foote, Gerardi, Goette and Willen (2009) estimate: $180BN or 1% of GDP.
Restructuring requires fine-tuning of both:

**Ability** to pay

&

**Willingness** to pay
Value Driver #2
Strategic Default

Guiso, Sapienza and Zingales (2009) find that 26% of defaults are *strategic* in nature.

Cohen-Cole and Morse (2009) find that in the presence of *negative equity*, borrowers pay credit cards first, and prefer foreclosure.
Value Driver #3: Triggers

28% of owner-occupied single-family homes have negative equity (national average). States: California (33%), Arizona (37%), Nevada (40%)

Greater problems are likely to arise with cash-out refinancing [Mian & Sufi (2009); Khandani, Lo & Merton (2009)].
Game theoretic problems:

(a) Lender determines modification that maximizes value of loan given that borrower will act strategically in his best interest.

(b) Investor determines modification that maximizes risk-adjusted returns.
Model

Home value

\[
\frac{dV(t)}{V(t)} = (r - \delta)V(t) \, dt + \sigma_1 \, dZ_1(t) + \sigma_2 \, dZ_2(t)
\]

HJM

\[
df(t, T) = \alpha(t, T) \, dt + \beta(t, T) \, dZ_1(t), \quad \forall T.
\]

Correlation

\[
Corr(dV/V, df) = \frac{\sigma_1}{\sqrt{\sigma_1^2 + \sigma_2^2}}
\]
Discrete-time Implementation

\[
V(t + 1) = \begin{cases} 
V(t) \exp \left( +\sigma_1 \sqrt{h} + \sigma_2 \sqrt{h} \right), & \text{w/prob } q/2 \\
V(t) \exp \left( +\sigma_1 \sqrt{h} - \sigma_2 \sqrt{h} \right), & \text{w/prob } (1 - q)/2 \\
V(t) \exp \left( -\sigma_1 \sqrt{h} + \sigma_2 \sqrt{h} \right), & \text{w/prob } q/2 \\
V(t) \exp \left( -\sigma_1 \sqrt{h} - \sigma_2 \sqrt{h} \right), & \text{w/prob } (1 - q)/2 
\end{cases}
\]

\[
f(t + 1, T) = \begin{cases} 
f(t, T) + \alpha(t, T) \ h + \beta(t, T) \sqrt{h}, & \text{w/prob } 1/2 \\
f(t, T) + \alpha(t, T) \ h - \beta(t, T) \sqrt{h}, & \text{w/prob } 1/2 
\end{cases}
\]

\[
\sum_{t=1}^{T} \alpha(t, T) = \frac{1}{h^2} \ln \left[ \cosh \left( \sum_{t=1}^{T} \beta(t, T) h^{3/2} \right) \right], \quad \forall T
\]
Martingale system

\[ V(t) e^{(r(t) - \delta)h} = E[V(t + 1)] \]
\[ = V(t) \exp \left( +\sigma_1 \sqrt{h} + \sigma_2 \sqrt{h} \right) \times q/2 \]
\[ + V(t) \exp \left( +\sigma_1 \sqrt{h} - \sigma_2 \sqrt{h} \right) \times (1 - q)/2 \]
\[ + V(t) \exp \left( -\sigma_1 \sqrt{h} + \sigma_2 \sqrt{h} \right) \times q/2 \]
\[ + V(t) \exp \left( -\sigma_1 \sqrt{h} - \sigma_2 \sqrt{h} \right) \times (1 - q)/2 \]
Risk-neutral Probabilities

\[ q = \frac{2 \, e^{(r(t) - \delta)h} - (u_2 + u_4)}{u_1 + u_3 - u_2 - u_4} \]

\[ u_1 = \exp\left(\sigma_1 \sqrt{h} + \sigma_2 \sqrt{h}\right) \]
\[ u_2 = \exp\left(\sigma_1 \sqrt{h} - \sigma_2 \sqrt{h}\right) \]
\[ u_3 = \exp\left(-\sigma_1 \sqrt{h} + \sigma_2 \sqrt{h}\right) \]
\[ u_4 = \exp\left(-\sigma_1 \sqrt{h} - \sigma_2 \sqrt{h}\right) \]
Modeling the Mortgage

Loan balance

\[ L(t + 1) = L(t) \left[ (1 + i) - \frac{i}{1 - (1 + i)^{-(N-t)}} \right] \]

Default put

\[ P(T) = \max(0, S - V(T) - K_R) \]

Lender’s value on default

\[ B^l(T) = \phi \cdot V(T) \]

Borrower’s liability

\[ B^b(T) \]

Deadweight cost of foreclosure

\[ (1 - \phi) \cdot V(T) \]

Refinancing option

\[ B^b(t) > L(t) + S? \]
Loan balance = $300,000 @6%
Home value = $250,000

Remaining maturity = 25 years
A = $1,933 per month

\( A_{\text{max}} = $20,000 \text{ per year} \)  
\( ($1,667 \text{ per month}) \)

\[
L_0 = \frac{A_{\text{max}}}{m} \left[ \frac{1 - (1 + \frac{r_L}{m})^{-N}}{\frac{r_L}{m}} \right]
\]
Value Driver #4: Values of Iso-Service Loans
Figure 3: How loan principal and deadweight costs determine loan value. In this graph we plot the value of the loan from our model, for a range of loan principal balances, varying from $200,000 to $300,000, given a current home value of $250,000. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = 0.03$, service flow level $\delta = 0.01$, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$, foreclosure recovery rate $\phi = \{0.7, 0.9\}$, loan maturity $T = 25$ years, and a flat forward rate curve at 5%.
Value Driver #5: Modifying Maturity
Cure risk and Re-default Risk

The risk of unnecessary relief, i.e., the borrower would not have ultimately defaulted.

Providing futile relief, leading to ultimate default anyway.

\[
\text{Loan Value} = B(A) \cdot \left[ 1 - N \left( \frac{A - \mu_I}{\sigma_I} \right) \right] + \phi V_0 \cdot N \left( \frac{A - \mu_I}{\sigma_I} \right)
\]

Value of loan accounting for willingness to pay

A: borrower income available for housing service, with mean \( \mu \) and std. dev \( \sigma \).
Figure 3: Loan values for differing income risk. We vary the annual payment from $15,000 to $21,000. For each of these payments, we vary the loan rate in the set $r_L = \{5.0\%, 5.5\%, 6.0\\%\}$. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = 0.03$, service flow level $\delta = 0.01$, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$ (increased in a later example), foreclosure recovery rate $\phi = 0.7$, loan maturity $T = 25$ years, and a flat forward rate curve at 5%. The mean and standard deviation of the ability to pay (income risk) is varied across the plots: (a) top left plot, no risk; (b) top right, mean ability to pay $\$20,000$ per year, sd=$\$5,000$, moderate risk; (c) high income risk, mean=$\$17,000$, sd=$\$5,000$; (d) moderate income risk, with high range, mean=$\$20,000$, sd=$\$10,000$. 
Logit: Explaining Re-default (Value Driver #6)

### 2007

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<tr>
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<th>450-620</th>
<th>620-820</th>
<th>820-1120</th>
<th>&gt; 1120</th>
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<td>Est.</td>
<td>χ²</td>
<td>Est.</td>
<td>χ²</td>
<td>Est.</td>
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<td>ΔRate</td>
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<td>66</td>
<td>89</td>
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<tr>
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<td>330</td>
<td>382</td>
<td>459</td>
<td>545</td>
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<td>Wald Stat</td>
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### 2008

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<td>ΔTerm</td>
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<td>42.1224</td>
<td>&lt; 0.0001</td>
<td>46.1375</td>
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HAMP UPDATE

October 15, 2010

Supplemental Directive 10-14: MHA Program -- Principal Reduction Alternative Update

Today, October 15, 2010, Supplemental Directive 10-14: Making Home Affordable Program -- Principal Reduction Alternative Update was issued on HMPadmin.com. This guidance is being issued as a new Section 6.4 of Chapter II, Principal Reduction Alternative, of the Making Home Affordable Program Handbook for Servicers of Non-GSE Mortgages. Supplemental Directive 10-14 incorporates and supersedes Supplemental Directive 10-05 issued in June 2010. In addition, this Supplemental Directive includes the following new guidance:

- Servicers that forgive at least five percent of the borrower’s unpaid principal balance in conjunction with a HAMP modification will have new flexibility in the application of the alternative modification waterfall steps. This flexibility provides the ability to substitute principal forgiveness for discounted interest rates or extended terms in accordance with investor guidelines.
- Investors that enter into equity share arrangements with borrowers in conjunction with a Principal Reduction Alternative (PRA) modification may be eligible for PRA investor incentives so long as certain borrower protections are included in the equity share agreement. These arrangements provide investors with the potential to recoup some or all of the unreimbursed portion of the principal forgiveness if the property value increases in the future.
- Guidance has been issued regarding PRA consideration for loans that were in trial period plans or permanently modified under HAMP prior to the PRA Effective Date. Servicers are required to develop written policies and procedures that describe the basis on which they will retroactively identify loans potentially eligible for PRA. If the policy permits the retroactive evaluation for PRA, then those loans must be evaluated no later than January 31, 2011.
- Direction regarding the impact of applying PRA retroactively on second lien mortgage loans through the Second Lien Modification Program (2MP) is also provided.
Value Driver #7: Shared-Appreciation Mortgages

Exercise value = \[ B^b(t) - [V(t) - \theta \cdot C(V(t), K, t)] - K_R \]

Figure 8: Loan values with appreciation sharing. The annual payment is \( A = 19,000 \). The parameters used for this graph are: home value volatility parameters \( \sigma_1 = 0.02 \) and \( \sigma_2 = 0.03 \), service flow level \( \delta = 0.01 \), interest rate volatility per annum \( \beta = 0.0050 \) (i.e., 50 bps), time step \( h = 1/4 \), loan rate \( r_L = 0.06 \), relocation costs \( K_R = 0 \), foreclosure recovery rate \( \phi = 0.7 \), loan maturity \( T = 25 \) years, and a flat forward rate curve at 5%. Appreciation share \( \theta \) takes values in the set \( \{0, 0.1, 0.5\} \), and the strike of the appreciation sharing agreement is $250,000.
Value Driver #8: Default Put Exercise Region

Short Horizon Portfolios

L=225,000

L=250,000
Value Driver #9: Reduced-Form Analysis of SAMs

\[ dH_t = \mu H_t \, dt + \sigma H_t \, dZ_t \]

Home values

Normalize initial home value to 1. The option to default is ITM when \((H > L)\).

There is a home value \(D\) at which the borrower will default. \(D\) is a “default level” or default exercise barrier.

\(D\) is a function of the lender share \(\theta\), we write it as \(D(L, \theta)\).

\(D\) increases in \(L\) and in \(\theta\).
Default Barrier and Lender Share

\[ D = L \exp[-\gamma(1 - \theta)] \]

(1) The greater the willingness to pay (\(\gamma\)), the lower is the default level of home value \(D\).
(2) When \(\gamma = \infty\), the willingness to pay is infinite, the default level \(D = 0\). The borrower never defaults unless the home value goes to zero.
(3) When \(\gamma = 0\), there is no willingness to pay and the default level is \(D = L\), i.e., the borrower defaults the moment the home value drops infinitesimally below LTV at the time zero.

(1) The greater the lender’s share (\(\theta\)), the higher is the default level of home value \(D\). The likelihood of default is therefore greater.
(2) When the lender share \(\theta = 0\), the default level is \(L e^{-\gamma}\).
(3) When \(\theta = 1\), the default level is \(D = L\). The borrower defaults the moment there is negative equity.
Barrier Model Intuition

\[ D = L \exp[-\gamma(1-\theta)] \]

Region of no default and gains to SAM

\[ H_0 = 1 \]

Region of default

No default Payoff = L

Default Payoff = \( \phi D \)
A Barrier Option Decomposition

Non-default component

\[ Le^{-rT} \int_{D(L,\theta)}^\infty p(H_T|H_t > D, \forall t < T) \, dH_T \]

where \( p(H_T|H_t > D, \forall t < T) \) is the density of the terminal home value conditional on no interim default.

Default component

\[ \phi D \int_0^T e^{-rt} f(t; D) \, dt \]

where \( f(t; D) \) is the first-passage time density for \( H_t = D \).

Shared Appreciation component

\[ e^{-rT} \int_K^\infty (H_T - K) \, p(H_T|H_t > D, \forall t < T) \, dH_T \]

PDE

\[ \frac{\partial F}{\partial H}[\mu - \lambda \sigma]H + \frac{1}{2} \frac{\partial^2 F}{\partial H^2} \sigma^2 H^2 + \frac{\partial F}{\partial t} = rF \]
The Closed-Form Solution

\[ \text{LOANVAL} \equiv V(H, L, K, r, T, \phi, \theta, \mu, \lambda, \sigma, \gamma) \]

\[ = L e^{-rT} \left[ N(d'_{2}) - (D/H)^{(R/\sigma^2)-1} \cdot N(d'_{2b}) \right] + \phi D \left[ (D/H)^{b_1} \cdot N(a_1) + (D/H)^{b_2} \cdot N(a_2) \right] + \theta \left[ C_{SAM}(H, K) - D^{(R/\sigma^2)-1} \cdot C_{SAM}(D^2/H, K) \right] \]

\[ D = L \exp[-\gamma(1 - \theta)] \]

\[ d'_{2} = \frac{\ln(H/D) + (R - 0.5\sigma^2)T}{\sigma \sqrt{T}} \]

\[ d'_{2b} = \frac{\ln(D/H) + (R - 0.5\sigma^2)T}{\sigma \sqrt{T}} \]

\[ a_1 = \frac{\ln(D/H) + \sqrt{2r\sigma^2 + (R - 0.5\sigma^2)^2} \cdot T}{\sigma \sqrt{T}} \]

\[ a_2 = \frac{\ln(D/H) - \sqrt{2r\sigma^2 + (R - 0.5\sigma^2)^2} \cdot T}{\sigma \sqrt{T}} \]

\[ b_1 = \frac{(R - 0.5\sigma^2) + \sqrt{2r\sigma^2 + (R - 0.5\sigma^2)^2}}{\sigma^2} \]

\[ b_2 = \frac{(R - 0.5\sigma^2) - \sqrt{2r\sigma^2 + (R - 0.5\sigma^2)^2}}{\sigma^2} \]

\[ C_{SAM}(x, y) = xe^{-(r-R)T} N(d'_{1}) - ye^{-rT} N(d'_{1} - \sigma \sqrt{T}) \]

\[ d'_{1} = \frac{\ln(x/y) + (R + 0.5\sigma^2)T}{\sigma \sqrt{T}} \]
Fig. 2. Loan value as LTV is varied for loans with and without appreciation sharing. The parameters for the plot are as follows: willingness to pay coefficient $\gamma = 0.1$, home price volatility $\sigma = 0.04$, foreclosure fraction $\phi = 0.7$, risk-free rate $r = 0.02$, the house value growth rate $\mu = 0.04$, price of risk $\lambda = 0.25$, and the horizon of the model $T = 5$ years. The appreciation share fraction is $\theta = 0.50$ for the case when a SAM is applied, and $\theta = 0$ when there is no share appreciation.
Home Price Volatility

Fig. 4. Loan value as LTV is varied for loans with SAMs and housing price volatility is varied across \( \{\sigma = 0.04, \sigma = 0.10\} \). Both cases are with appreciation sharing. The parameters for the plot are as follows: willingness to pay coefficient \( \gamma = 0.1 \), foreclosure percentage \( \phi = 0.7 \), risk-free rate \( r = 0.02 \), the house value growth rate \( \mu = 0.04 \), price of risk \( \lambda = 0.25 \), and the horizon of the model \( T = 1 \) year. The appreciation share fraction is \( \theta = 0.50 \).
Fig. 6. Loan value as LTV is varied for loans with SAMs and willingness to pay is varied across \{\gamma = 0.01, \gamma = 0.10, \gamma = 0.20\}. All cases are with appreciation sharing. The parameters for the plot are as follows: the house value growth rate \(\mu = 0.04\), price of risk \(\lambda = 0.25\), foreclosure percentage \(\phi = 0.7\), risk-free rate \(r = 0.02\), housing price volatility \(\sigma = 0.04\), and the horizon of the model \(T = 1\) year. The appreciation share fraction is \(\theta = 0.50\).
Restructuring Coupon-bearing Mortgage Debt

Binomial tree model

\[ H_{t+h} = H_t \exp[\pm \sigma \sqrt{h}] \]

\[ q = \frac{R - d}{u - d} \]

where \( R = \exp[rh] \), \( u = \exp[+\sigma \sqrt{h}] \), and \( d = \exp[-\sigma \sqrt{h}] \).

Restructuring control variables

\[ \{ L, K, \theta, c \} \]
Value Driver #10: Restructuring Regions

Figure 1: Loan value for varying levels of $LTV$ with no upside sharing by the lender. The input parameters are $H = 1$, $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = 0.7$, volatility $\sigma = 0.04$, default risk parameter $\gamma = 0.05$, and coupon rate $c = 0.03$. We used $n = 100$ periods on the tree.
Value Driver #11: Negative Equity Cusp

Figure 2: Loan value for varying levels of $H$ with no upside sharing by the lender. The input parameters are $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = 0.7$, volatility $\sigma = 0.04$, default risk parameter $\gamma = 0.05$, and coupon rate $c = 0.03$. We used $n = 100$ periods on the tree.
Figure 4: Return distributions over a one-year holding period on various loans with no upside sharing by the lender. The input parameters are $H = 1$, $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = 0.7$, growth rate $\mu = 0.04$, volatility $\sigma = 0.04$, default risk parameter $\gamma = 0.05$, and coupon rate $c = 0.03$. We used $n = 100$ periods on the tree for loan pricing.
Figure 5: One-year returns on various loans with no upside sharing, plotted against $H_1$ (asset value at time $t = 1$). The input parameters are $H = 1$, $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = 0.7$, growth rate $\mu = 0.04$, volatility $\sigma = 0.04$, default risk parameter $\gamma = 0.05$, and coupon rate $c = 0.03$. We used $n = 100$ periods on the tree for loan pricing.
Restructuring

Figure 7: Return distribution over a one-year holding period on a restructured loan with an initial $LTV$ of 1.03 and purchase price of 0.8898. The restructuring here entails a principle write-down such that $LTV = 0.9787$ (for the restructured loans with shared appreciation, we set the strike $K = 1$). The input parameters are $H = 1$, $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = 0.7$, growth rate $\mu = 0.04$, volatility $\sigma = 0.04$, default risk parameter $\gamma = 0.05$, and coupon rate $c = 0.03$. We used $n = 100$ periods on the tree for loan pricing. The mean and standard deviation of returns for each of the four loans in order are: $E(R) = \{7.41\%, 10.91\%, 12.62\%, 14.22\%\}$, and $\sigma(R) = \{12.70\%, 4.44\%, 5.90\%, 9.05\%\}$. 
Value Driver #14: Restructured Returns Shift Up

Figure 8: One-year returns on a restructured loan with an initial $LTV$ of 1.03 and purchase price of 0.8898, plotted against $H_1$ (asset value at time $t = 1$). The restructuring here entails a principle write-down such that $LTV = 0.9787$ (for the restructured loans with shared appreciation, we set the strike $K = 1$). The input parameters are $H = 1$, $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = 0.7$, growth rate $\mu = 0.04$, volatility $\sigma = 0.04$, default risk parameter $\gamma = 0.05$, and coupon rate $c = 0.03$. We used $n = 100$ periods on the tree for loan pricing.
Figure 9: Return distribution over a one-year holding period on a restructured loan with an initial $LTV$ of 1.2 and purchase price of 0.7. The restructuring here entails a principle write-down such that $LTV = 0.9787$ (for the restructured loans with shared appreciation, we set the strike $K = 1$). The input parameters are $H = 1$, $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = 0.7$, growth rate $\mu = 0.04$, volatility $\sigma = 0.04$, default risk parameter $\gamma = 0.05$, and coupon rate $c = 0.03$. We used $n = 100$ periods on the tree for loan pricing. The mean and standard deviation of returns for each of the four loans in order are: $E(R) = \{4.07\%, 34.90\%, 36.61\%, 38.21\\%\}$, and $\sigma(R) = \{4.73\%, 4.44\%, 5.90\%, 9.05\\%\}$. 
Table 1: Expected return, standard deviation, skewness, and kurtosis over a one-year holding period on a loan with no upside sharing by the lender. The input parameters are $H = 1$, $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = 0.7$, growth rate $\mu = 0.04$, volatility $\sigma = 0.04$, default risk parameter $\gamma = 0.05$, and coupon rate $c = 0.03$. We used $n = 100$ periods on the tree for loan pricing. Panel A presents the return distributions without loan restructuring. Panels B.1-3 present the return distributions on a restructured loan assuming a principle write-down such that $LTV = 0.9787$ (when restructuring entails shared appreciation, we set the strike $K = 1$)

<table>
<thead>
<tr>
<th>LTV</th>
<th>Loan value</th>
<th>Expected return</th>
<th>Standard deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Panel A. Return distribution without restructuring</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.90</td>
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<td>-0.0076</td>
<td>0.0025</td>
<td>-5.9818</td>
<td>52.3799</td>
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<td>0.9636</td>
<td>0.0302</td>
<td>0.0704</td>
<td>-3.4455</td>
<td>17.3221</td>
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<tr>
<td>1.03</td>
<td>0.8898</td>
<td>0.0744</td>
<td>0.1264</td>
<td>-1.6729</td>
<td>4.8897</td>
</tr>
<tr>
<td>1.20</td>
<td>0.7000</td>
<td>0.0404</td>
<td>0.0450</td>
<td>1.5393</td>
<td>12.6329</td>
</tr>
<tr>
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<td></td>
<td>Panel B.1. Return distribution with restructuring, no SAM</td>
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<td></td>
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</tr>
<tr>
<td>1.03</td>
<td>---</td>
<td>0.1095</td>
<td>0.0419</td>
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<td>35.5363</td>
</tr>
<tr>
<td>1.20</td>
<td>---</td>
<td>0.3495</td>
<td>0.0419</td>
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<td>35.5363</td>
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<td>Panel B.2. Return distribution with restructuring, SAM = 0.2</td>
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<td>---</td>
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<td>22.9525</td>
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<td>1.20</td>
<td>---</td>
<td>0.3665</td>
<td>0.0570</td>
<td>-3.7903</td>
<td>22.9525</td>
</tr>
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<td></td>
<td>Panel B.3. Return distribution with restructuring, SAM = 0.5</td>
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<tr>
<td>1.03</td>
<td>---</td>
<td>0.1426</td>
<td>0.0891</td>
<td>-2.6646</td>
<td>11.7922</td>
</tr>
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<td>1.20</td>
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<td>0.3825</td>
<td>0.0891</td>
<td>-2.6646</td>
<td>11.7922</td>
</tr>
</tbody>
</table>
Optimal Restructuring

Utility function

$$U(W) = \frac{W^{1-\beta}}{(1 - \beta)} \quad W = 1 + R$$

CERTAINTY EQUIVALENT

$$\sum_i p_i (1 + R_i^r)^{1-\beta} / (1 - \beta) = \sum_i p_i ((1 + R_i^b)(1 + CE))^{1-\beta} / (1 - \beta)$$

$$CE = \left[ \frac{E[U(W^r)]]}{E[U(W^b)]]} \right]^{1/(1-\beta)} - 1$$
Maximize Loan Value / Utility

Table 2: Optimization outcomes for a single loan. We optimize a loan where the initial home value is normalized to $H = 1$, and the loan balance is $L = 1.02$, i.e., the LTV is 1.02. There is no shared appreciation on this loan at modification. Remaining loan maturity is $T = 5$ years, recovery rate on default is $\phi = .7$, home price volatility is 4%, willingness to pay is $\gamma = 0.04$, and the coupon rate on the loan is 4%. We assume a riskless rate of $r_f = 0.02$. The parameters of the loan are provided below. Returns on the loan are computed assuming an investment horizon of $\tau = 1$ year. The following cases are considered: (1) The loan return distribution and expected utility from investing in the loan for a horizon $\tau$, without restructuring. (2) The loan return distribution from writing down the debt to a level where the LTV is such that the loan value is optimized. (3) Changing the LTV such that the certainty equivalent of the restructured investment relative to that of the original loan is maximized. The mean, standard deviation, skewness, and kurtosis of the return distribution with horizon $\tau$ are provided below. The certainty equivalents ($CE$) are computed relative to the base case loan before restructuring.

<table>
<thead>
<tr>
<th>Case</th>
<th>LTV</th>
<th>Price</th>
<th>Mean</th>
<th>Stdev</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>$E[U(W)]$</th>
<th>CE (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Before Restr</td>
<td>1.0200</td>
<td>0.9133</td>
<td>0.0707</td>
<td>0.1343</td>
<td>-1.7062</td>
<td>4.9896</td>
<td>-0.4671</td>
<td>–</td>
</tr>
<tr>
<td>2: After Restr</td>
<td>0.9518</td>
<td>0.0216</td>
<td>0.1031</td>
<td>0.0265</td>
<td>-7.5029</td>
<td>92.5205</td>
<td>-0.4119</td>
<td>649</td>
</tr>
<tr>
<td>3: Max CE</td>
<td>0.9716</td>
<td>1.0139</td>
<td>0.1127</td>
<td>0.0464</td>
<td>-5.2883</td>
<td>41.4471</td>
<td>-0.4077</td>
<td>703</td>
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</table>
Diversifying Unrestructured Loans

Table 3: Comparing return distributions over a one-year holding period on a single loan versus a two-loan portfolio. The input parameters are $H = 1$, $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = 0.7$, growth rate $\mu = 0.04$, volatility $\sigma = 0.04$, default risk parameter $\gamma = 0.04$, and coupon rate $c = 0.04$ (the same configuration as used in Table 2). We used $n = 100$ periods on the tree for loan pricing. Panels A.1 and A.2 presents the return distributions without loan restructuring. Panels B.1 and B.2 present the return distributions assuming a principle write-down such that $LTV = 0.9518$. The certainty equivalents (CE) are computed relative to the base case single loan (before restructuring).

<table>
<thead>
<tr>
<th>Corr($Z_1, Z_2$)</th>
<th>Mean</th>
<th>Stdev</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>E[U(W)]</th>
<th>CE(bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A.1. Return distribution of single loan</td>
<td></td>
<td></td>
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<tr>
<td>–</td>
<td>0.0707</td>
<td>0.1343</td>
<td>-1.7062</td>
<td>4.9896</td>
<td>-0.4671</td>
<td>–</td>
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<tr>
<td>Panel A.2. Return distribution of two-loan portfolio</td>
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<tr>
<td>0.5</td>
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Diversifying Restructured Loans

### Panel B.1. Return distribution of single loan with restructuring, no SAM

<table>
<thead>
<tr>
<th></th>
<th>0.1031</th>
<th>0.0265</th>
<th>-7.5029</th>
<th>92.5205</th>
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<th>649</th>
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### Panel B.2. Return distribution of two-loan portfolio with restructuring, no SAM

<table>
<thead>
<tr>
<th>0.5</th>
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<th>0.0215</th>
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</table>
Summary of Value Drivers

1. Deadweight cost savings add returns.
2. Strategic default suggests cherry picking loans.
3. Negative equity mitigation adds value.
4. Principal write-downs work best.
5. Maturity extension and rate reductions dissipates value.
6. Reducing re-default adds value.
7. Shared appreciation adds value.
9. Closed-form reduced-form model speeds up implementation.
10. Restructuring regions suggest erring on the side of greater LTV reduction.
11. Manage the negative equity cusp for greatest boost in return.
12. Restructuring leads to dramatic distribution shift.
13. Restructured debt return distribution is highly sensitive to LTV.
14. Restructured returns shift up.
15. Restructured returns dramatically shift the distribution’s scale to the right.